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The unsuitability of the application of Pythagorean Theorem of Exhaustion Method, in finding the actual length of the circumference of the circle and Pi

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Abstract: *There is only one geometrical method called Exhaustion Method to find out the length of the circumference of a circle. In this method, a regular polygon of known number and value of sides is inscribed, doubled many times until the inscribed polygon exhausts the space between the polygon and circle as limit. In the present paper, it is made clear, that the value for circumference, i.e. 3.14159265358…. of polygon, attributing to circle is a lower value than the real value, and the real value is* $\frac{14-\sqrt{2}}{4}$ *= 3.14644660942 adopting error-free method.*

Key words: *Circle, corner length, diameter, diagonal, polygon, side, square.*

I. Introduction

The **Holy Bible** has said π value is 3. The formula for circumference of a circle is πd, where π is a constant and d is the diameter. When the diameter is 1 the circumference is equal to π value.

When, a hexagon is inscribed in a circle of ½ as its radius, the perimeter of hexagon is equal to 3. It means, the circumference is greater than 3, because hexagon is an inscribed entity in the circle. With the unit diameter of circle, circumference and/or π value is exchangeable, because, both are represented by a single number. There were many values for π. $\sqrt{10}$ = 3.16… (**Chung Hing,** 250 AD), $\frac{142}{45}$ = 3.155… (**Wang Fau,** 250 AD), 3.14159… (**Liu Hui,** 263 AD), 3.1415 (**Aryabhata,** 499 AD). From **Francois Viete** (1579) onwards the π value 3.1415926… has become an accepted value.

It is a well established fact that the π value is 3.14159265358… However, two things which are associated with this number have not satisfied some scholars of mathematics. They are 1. 3.14159265358… is a borrowed number of polygon, attributed to circle, believing in the logic of limitation principle and being an approximate number, made super computers too have failed to find the exact value and 2. Assertively, this number has said, squaring a circle impossible, being it is a transcendental umber. High school students, now and then ask, when mathematics is an exact science, how is it possible we have many values are being used, for example $\frac{22}{7}$, 3.14, 3.1416, 3.14159265358….

Nature has been kind. Exact π value has been found at last. A few papers in support of the exact π value have been published (in Reference). Surprisingly the new π value is struggling very hard still for its approval. All the time, the work done on π has been cited, saying, the past generation or the present π workers could **not be wrong**. Thus, the new π value (through was discovered 16 years ago, in March 1998 is yet to be accepted.

From March 1998, with the discovery of $\frac{14-\sqrt{2}}{4}$ = 3.14644660942, this worker has been struggling to find evidences in support of new π value, and also struggling **much more** in search of an error in the derivation of 3.14159265358…

There are some similarities and some differences between present and new π values.

The similarity is, in Exhaustion method a polygon is inscribed in a circle and in the new method a circle is inscribed in a square.

The differences are many: 1. Present π value 3.14159265358… represents the perimeter/ area of the polygon and **attributed** to circle; the new π value represents the area of the inscribed circle only in a square (called Siva method in Reference). Here, square helps but does not lend its value to circle. 2. Present π value is derived applying Pythagorean theorem meant for straight lines. New π value is derived adopting entirely a **new approach** for which no previous proofs/ statements are available. 3. Present π value is a transcendental number and new value is an algebraic number. 4. Present π value says squaring of a circle is an impossible idea and whereas the new π value squared a circle. It also, circled a square (i.e., constructing a circle whose circumference and/or area equal to the perimeter and/ or area of a square respectively. For example, if the side of a square is 1, the perimeter would be 4 and its area would be 1. Circling a square means we have to find out a radius geometrically of circle whose length of the circumference is 4 and/ or area of the circle is 1.

In this paper, present π value-to some decimal places – is obtained using the new π value $\frac{14-\sqrt{2}}{4}$, showing, circle, polygon and square are **not** different geometrical entities and their interrelationship, if understood correctly, may help in the derivation of present π value from the new π value also.

If this idea is accepted, an algebraic number like $\frac{14-\sqrt{2}}{4}$ is also, capable of giving rise to, a transcendental number like 3.14159265358…. It leads to the creation of a new doubt, is 3.14159265358… is really a transcendental number ?

There are 3 examples cited below, linking present and new π values 3.14159265358… and 3.14644660942… = $\frac{14-\sqrt{2}}{4}$.

To drive to the point an elaborate explanation is chosen and here and there repetitions too appear.

II. PROCEDURE

Example-1

The following formula gives the present π value upto 5 decimal places.

When my work on deriving the new value of π equal to $\frac{14-\sqrt{2}}{4}$ = 3.1464466.. was submitted, I was advised by many to think how to derive the present π value 3.1415926… cautioning me as the new one was wrong.

In obedience, I hereby submitted the following formula giving the number exact upto five decimal places.

$3+\frac{9\pi}{200}=3.14159...$ where $\pi=\frac{14-\sqrt{2}}{4}$

To get the same value (3.14159…) from the Classical method of **Archimedes,** we have to travel a long way using the following general formula to calculate the length of the perimeter of the inscribed polygon of 1536 sides starting from 6 sides, in a circle

Side = $s_{2n}=\sqrt{\left[2r^2-r\sqrt{\left(4r^2-s_n^2\right)}\right]}$

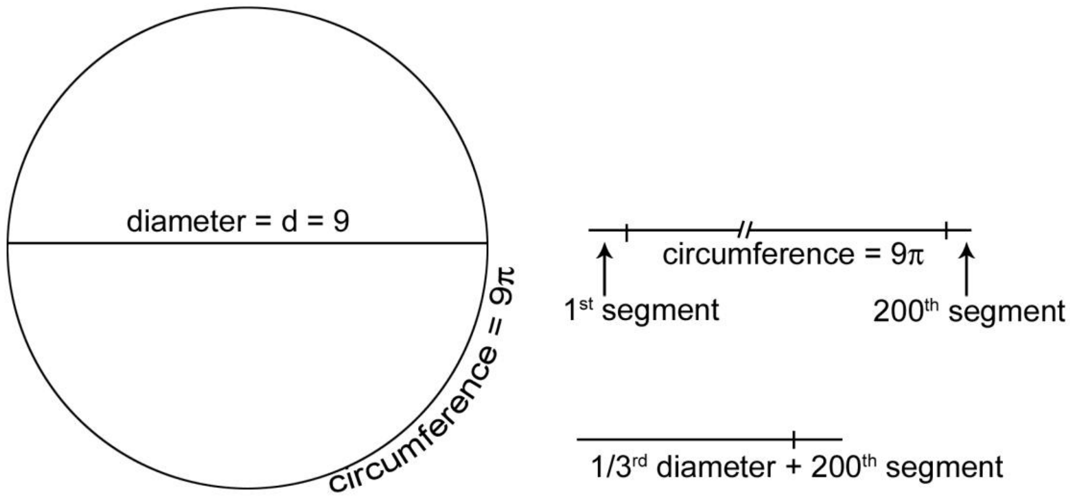
A geometrical line segment for 3.14159…

Let us draw a circle with diameter 9 and radius 9/2. Cut the circumference at one point and straighten it and further divide it into 200 equal segments.

Similarly, divided the diameter into 3 equal segments.

1/3rd diameter + one segment length of circumference

$$\left(\frac{9}{3}\right) \quad + \quad \left[\frac{9\pi}{200}=\frac{9\left(\frac{14-\sqrt{2}}{4}\right)}{200}\right] = 3.14159\ldots$$



Example-2

Archimedes inscribed a polygon of **96 sides** with the circle and gave perimeter of the polygon as equal to 3 10/71 = 3.140845… When the diameter of the superscribed circle is 1 unit, the perimeter = π value. **Prof. Constantine Karapappopulos of University of Patra, Patra, Greece** has suggested a formula for the Archimedian minimum as equal to $3+\frac{1}{3}\left(1-\frac{\sqrt{3}}{3}\right)$ in the range of $3\frac{10}{71}$ to $3\frac{10}{70}$

The proposed formula for π = $\left(\frac{14-\sqrt{2}}{4}\right)$ gives the circumference of the circle as equal to

$$3+\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)$$

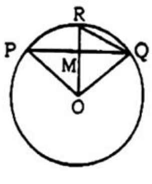
$$3+\left\{\frac{1}{3}\left(1-\frac{\sqrt{3}}{3}\right)\right\} \quad \text{x} \quad \left\{\frac{3\sqrt{3}\left(\sqrt{2}-1\right)}{2\sqrt{2}\left(\sqrt{3}-1\right)}\right\} \quad = \quad 3+\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)=\frac{14-\sqrt{2}}{4}$$

Perimeter of Polygon x Circle connecting link = Circumference of Circle

$\frac{3\sqrt{3}\left(\sqrt{2}-1\right)}{2\sqrt{2}\left(\sqrt{3}-1\right)}$ is obtained by the calculation method. One need not set aside it on the reason that the above factor is based on the calculation method. This factor or **circle connecting link** can be viewed as a **clue** or guiding factor for further analysis. On close observation one finds the denominator and the numerator of the link are **symmetrical** and give us a right relationship between the inscribed polygon and the superscribed circle.

Example-3

Classical method involves the inscription of polygon in the circle. Starting with the known perimeter of a regular polygon (here we start with a regular hexagon) of n sides inscribed in a circle, the perimeter of the inscribed regular polygon of 2 n sides can be calculated by the application of Pythagorean theorem. Let C be a circle with centre O and radius r, and let PQ = s be a side of a regular inscribed polygon of n sides having a known perimeter. Then the apothem, OM = u is given by $\sqrt{r^2-\left(\frac{s}{2}\right)^2}$ hence the sagetta, MR = v = r – u is known. Then the side RQ = w of he inscribed polygon of 2n sides is found from $w=\sqrt{v^2+\left(\frac{s}{2}\right)^2}$ hence the perimeter of this polygon is known.



S. No.	**n**	**First Side PQ = s**	**ns Perimeter = Circumference = π**	**s/2 = t**	**Apothem** $\sqrt{r^2 - t^2} = u$	**Sagetta r – u = v**	**Succeeding side** $\sqrt{v^2 + t^2} = w$
1.	6	0.5	3.0	0.25	0.433012701	0.066987299	0.258819045
2.	12	0.258819045	3.1058828544	0.129409522	0.482962913	0.017037087	0.13052619
3.	24	0.13052619	3.132628565	0.065263095	0.49572243	0.00427757	0.065403128
4.	48	0.065403125	3.139350157	0.032701564	0.498929461	0.001070539	0.032719082
5.	96	0.032719082	3.141031907	0.016359541	0.499732293	0.000267707	0.016361731
6.	192	0.016361731	3.141452431	0.00818065705	0.499933069	0.000066931	0.008181139495
7.	384	0.008181139495	3.141557566	0.004090569748	0.499983267	0.000016733	0.004090603972
8.	768	0.004090603972	3.14158385	0.002045301986	0.499995816	0.000004184	0.002045306262
9.	1536	0.002045306262	3.141590424	0.001022653133	0.499998654	0.0000010 4	0.001022653668
10.	3072	0.001022653668	3.141592067	-	-	-	-

Let us analyse the above Table. The calculation is started with a regular polygon of 6 sides having the perimeter equal to 3. The sides of polygon are doubled for 9 times and now the inscribed polygon has 3072 sides and its perimeter = circumference of circle = π value is equal to 3.141592067… Thus, the number of sides are increased 512 times finally $\frac{3072}{6} = 512$. The length of the perimeter has increased from $6\left(\frac{1}{2}\right)$ = 3.000000 to **3.141592067**… The correct value is 3.14159265358… From the above Table, it is possible to find the exact length only upto 3.141592 i.e. 6 decimal places only with 3072 sides of the polygon.

ABCD = Square; EFGR = Circle; AB = Side = NQ = diameter = 1;

AC = Diagonal = $\sqrt{2}$; AN = QC = Corner length;

Diagonal – diameter = AC – NQ = $\sqrt{2} - 1$; Hexagon = STGUVE

In the Classical method of calculating perimeter/ circumference/ π in the above table, number of sides of the inscribed polygon plays an important role. The derivation is started with 6 sides and ended with 3072 sides. The perimeter of the hexagon is 3 and the perimeter of the polygon of 3072 sides has increased to 3.141592067…

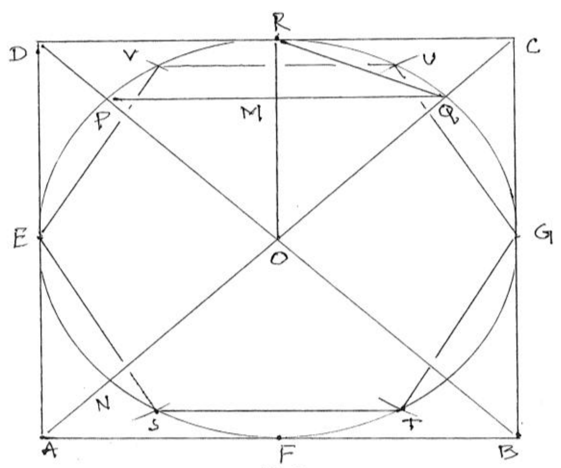


Fig.2

Starting no. of sides of the polygon = 6
Ending no. of sides of the polygon = 3072
Starting perimeter = 3
Ending perimeter = 3.141592067
Present π value = 3.14159265358

New π value = $\frac{14-\sqrt{2}}{4}$ = 3.14644660942…

In the following formula a relation is shown between two π values: present π value and new π value as in the above two examples.

New π value – $\left\{(\text{Diagonal} - \text{diameter})\left(\frac{\text{Square of the starting no. of sides}}{\text{Ending no. of sides}}\right)\right\}$

= Present π value (correct upto 6 decimals)

$$\frac{14-\sqrt{2}}{4} - \left\{\left(\sqrt{2}-1\right)\left(\frac{6\times 6}{3072}\right)\right\} = \frac{899-67\sqrt{2}}{256}$$ = 3.14159254422…

The lengthy process involved in the above Exhaustion method is represented in a single formula $\frac{899-67\sqrt{2}}{256}$.

In the Exhaustion method Pythagorean theorem is applied. $\sqrt{3}$ invariably appears in every aspect of calculation. The result 3.14159254422… obtained from the above formula using $\sqrt{2}$ is much more accurate at 7^{th} decimal place than what it is (3.141592067…) obtained in the Exhaustion method in the table.
In the Exhaustion method RQ is the **hypotenuse** and is the side of the inscribed polygon.
And in the above formula diagonal – diameter = corner lengths on either side of the diameter also play an important role. Hence, it has become possible that the new π value has given the present π value (to some decimals).
Let us understand, in much more clear terms, the above formula.
There are four factors in the Exhaustion method. They are

- No. of sides of the hexagon = 6
- Perimeter of the hexagon = ½ x 6 = 3
- No. of sides of the final polygon = 3072
- Perimeter of the final polygon = 3.141592067

1. Let us divide final sides by the sides of the beginning polygon i.e. hexagon $\frac{3072}{6} = 512$.
2. Let us divide corner length AN + QC into 512 parts.
   AB = Side = NQ = Diameter = 1
   AC = diagonal = $\sqrt{2}$
   Corner length = AC – NQ = diagonal – diameter = $\sqrt{2}-1$
   Let us divide above length into 512 parts = $\frac{\sqrt{2}-1}{512}$ = 0.00080901086
3. Multiply the above value with the no. of sides of the beginning polygon (hexagon) $\left(\frac{\sqrt{2}-1}{512}\right)6$ = 0.00485406516
4. Deduct the above value from the new π value $\frac{14-\sqrt{2}}{4}$ = 3.14644660942 which gives the present π value (to some decimals).
   $$\frac{14-\sqrt{2}}{4} - \left(\frac{6\sqrt{2}-6}{512}\right) = \frac{899-67\sqrt{2}}{256}$$ = 3.14159254424
5. So, the corner length $\sqrt{2}-1$ is divided into 512 parts.

6. In this process corner length $\sqrt{2}-1$ is taken for consideration.
7. In the Exhaustion method the hypotenuse RQ in Fig.2 is taken for consideration and proceeded successively for many times.
8. AC is the hypotenuse of the triangle DAC of Fig.2 and RQ is the hypotenuse of the triangle RMQ.
9. **How does the length over and above 3 diameters of the circumference of the circle is arrived in deriving the new π value ?**
   The answer is very simple.
   Let us divide the corner length QC only of Fig-2 by $\sqrt{2}$ and add to the 3 diameters.
   Side = AB = diameter NQ = 1
   Corner length QC = $\frac{\text{Diagonal} - \text{diameter}}{2}$
   $= \frac{\text{AC} - \text{NQ}}{2} = \frac{\sqrt{2}-1}{2} = \text{QC}$
   Divide QC by $\sqrt{2}$
   $= \left(\frac{\sqrt{2}-1}{2}\right)\frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{2\sqrt{2}} = \frac{2-\sqrt{2}}{4}$
   Now, the length of circumference = 3diameters + $\frac{2-\sqrt{2}}{4}$
   $= 3 + \frac{2-\sqrt{2}}{4} = \frac{14-\sqrt{2}}{4}$
10. When we compare two ways of arriving the **exact length** of circumference of a circle, it is clear in Exhaustion method the perimeter of the inscribed polygon is increased slowly by doubling the number of sides of the previous polygon. Thus, the number of sides have been increased from 6 to 3072, it means it has been increased **512 times**. In other words, we have divided corner length into 512 parts.
    In the second approach in the arrival of length of the circumference, the corner length is divided at **one stroke** with $\sqrt{2}$ = $\frac{\text{Corner length QC}}{\sqrt{2}}$
    So, 3 sides + $\frac{\text{QC}}{\sqrt{2}}$ = $3 + \left\{\left(\frac{\sqrt{2}-1}{2}\right)\frac{1}{\sqrt{2}}\right\}$ = $\frac{14-\sqrt{2}}{4}$
11. Thus there are two values for the length of the circumference of the circle.
    1. 3.14159265358… of Exhaustion method and
    2. 3.14644660942… of Gayatri method
12. We can thus visualize a diagram of 2, containing a hexagon whose perimeter STGUVE is 3, next to hexagon, 3.14159265358 of inscribed polygon of 3072 sides and further next and the circle EFGR, whose circumference is equal to 3.14644660942… with radius ½.

III. Conclusion

There are now two π values 3.14159265358… and 3.14644660942 … = $\frac{14-\sqrt{2}}{4}$. In the Exhaustion method, perimeter of the polygon is attributed to the circumference of the circle. As the inscribed polygon is smaller one, the value 3.14159265358… must be **lesser** than the exact length of the circumference of circle. This paper clearly shows the **calculation error** involved in the arrival of the actual length of the circumference (= π) of the circle. Hence, Exhaustion method is not suitable in arriving the π of the circle. However, this study shows that both the π values have a **common-ness in their nature**. In Exhaustion method while doing calculations involving squaring, square root etc. and only **a few** decimals have been taken. All the numbers are **infinite numbers**. So, the **prolongation** of round-off-error is universal throughout the calculations. And this has resulted in a **lower** value 3.14159265358… instead of the actual value. However, the new work has tried to overcome the error supposed to be in the Exhaustion method by adopting **Gayatri method, Siva method, Jesus**

method etc. and found the **actual length** of the circumference and real π value, i.e. $\frac{14-\sqrt{2}}{4}$ = 3.14644660942…

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