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# Foundation on Layered Soil under Torsional Harmonic Vibration using Cone model

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***Abstract*****:** *This paper presents the analytical solution based cone model for machine foundation vibration analysis on layered soil. Impedance functions for a rigid massless circular foundation resting on layered soil underlain by rigid base subjected to torsional harmonic excitation are found using cone model. One-dimensional wave propagation in cones, based on the strength of material approach is used in computing dynamic stiffness and damping coefficients. Using the above coefficients the frequency –amplitude response of massive foundations are computed varying widely the influencing parameters such as, depth of the layer, material damping ratio and Poisson's ratio. The results are presented in the form of dimensionless graphs which may be useful for practicing engineers.*

***Keyword:*** *Cone model, Dynamic impedance, Circular foundation, one-dimensional wave propagation, Foundation vibration.*

## I. Introduction

The study of the dynamic response of foundations resting on or embedded in homogeneous soil subjected to torsional dynamic loading is an important aspect in the design of machine foundations and dynamic soil-structure interaction problems. This torsional dynamic loading arises when asymmetric horizontal forces resulting from windstorms, earthquake shaking, horizontal movements of the antennas of radar towers and operation of reciprocating engines, act on a superstructure. Thus, the determination of resonant frequency and maximum torsional rotation is an important aspect in the design of foundations under torsional loading. One of the key steps in the current methods of dynamic analysis of a foundation soil system to predict resonant frequency and amplitude under machine type loading is to estimate the dynamic impedance functions (spring and dashpot coefficients) of an associated rigid but massless foundation. With the help of these functions the amplitude of vibration is calculated using the equation of motion of a single degree of freedom oscillator.

Since the contribution of Reissner (1937), the harmonic response of torsionally excited foundations has been the subject of numerous investigations (Reisssner and Sagoci, 1944; Collins, 1962; and Weissmann, 1971). The fundamental problem considered in these investigations has been the analysis of the steady state response of a rigid, circular massless plate bonded to an elastic half-space and excited by a harmonically varying moment about an axis normal to the disk. Gazetas (1991) presented formulae and charts for impedances of surface and embeded foundations for all modes of vibration, which can be readily used by the practicing engineers. Ahmad and Gazetas (1992a, 1992b) presented simple expressions and charts for stiffness and radiation damping of arbitrary shaped embedded foundations particularly in torsional mode of vibration.

The cone model was originally developed by Ehlers (1942) to represent a surface disc under translational motions and later for rotational motion (Meek and Veletsos, 1974; Veletsos and Nair, 1974). Meek and Wolf presented a simplified methodology to evaluate the dynamic response of a base mat on the surface of a homogeneous half-space. The cone model concept was extended to a layered cone to compute the dynamic response of a footing or a base mat on a soil layer resting on a rigid rock. Meek and wolf (1994) performed dynamic analysis of embedded footing by idealizing the soil as a translated cone instead of elastic half-space. Wolf and Meek (1994) have found out the dynamic stiffness coefficients of foundations resting on or embedded in a horizontally layered soil using cone frustums. Also, Jaya and Prasad (2002) studied the dynamic stiffness of embedded foundations in layered soil using the same cone frustums. The major drawback of cone frustums method as reported by Wolf and Meek (1994) is that the damping coefficient can become negative at lower frequency, which is physically impossible. Pradhan etc al (2003,2004) have computed dynamic impedance of circular foundation resting on layered soil using wave propagation in cones, which overcomes the drawback of the above cone frustum method. The details of the use of cone models in foundation vibration analysis are summarized in Wolf (1994).

Numerical/semi-analytical methods though very accurate are not always warranted because of the complexities involved in the problem, particularly in the soil properties. Therefore a number of simplified approximate methods have been developed along with the exact solutions. Cone model is one of such approximate analytical methods, where in elastic half-space is truncated into a semi-infinite cone and the principle of one-dimensional wave propagation through this cone (Beam with varying cross-section) is considered. Most of the published results using cone model are confined to the determination of the dynamic response of the foundation in the form of impedance functions. To the best of author's knowledge no literature is available with regard to the parametric investigation of the foundation using cone model. In this paper studies the foundation resting on layered soil under torsional vibration is found out using wave propagation in cone.

## II. Mathematical Formulation

To study the dynamic response of foundation resting on the surface of a soil layer underlain by rigid base, a rigid mass less circular foundation of radius $r_0$ is subjected to torsional vibration as shown in Fig .1.

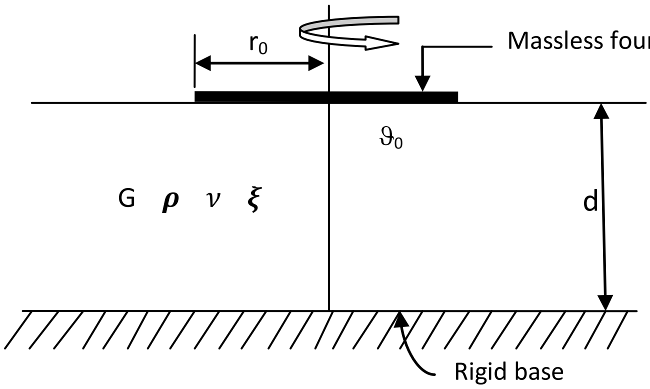


Fig 1 Massless foundation soil system under torsional harmonic excitation

The depth of the layer 'd' has the shear modulus 'G', Poisson's ratio '$\nu$',mass density '$\boldsymbol{\rho}$',hysteretic damping '$\boldsymbol{\xi}$'. The interaction moment Mo and the corresponding rotational displacement $\vartheta_0$ are assumed to be harmonic i.e.Mo = $|Mo| e^{i\omega t}$ and $\vartheta_0 = |\vartheta_0| e^{i\omega t}$. The dynamic impedance of the massless foundation (disk) is expressed by

$$\bar{K}(a_0) = \frac{M_0}{\vartheta_0} = K[k(a_0) + ia_0 c(a_0)] \tag{1}$$

Where, $\bar{K}(a_0) =$ dynamic impedance, $k(a_0) =$ spring coefficient, $c(a_0) =$ damping coefficient. $a_0 = \omega r_0 / c_s$, dimensionless frequency, $c_s = \sqrt{G/\rho}$, shear wave velocity of the soil. $K$ = static stiffness coefficient of disk on homogeneous half-space. Using the equations of dynamic equilibrium, the dynamic displacement amplitude of the foundation with mass $m$ and subjected to a harmonic moment M is expressed as

$$|\vartheta_0| = \left| M/K\left[k(a_0) + ia_0 c(a_0) - (Gr_0^{\,3}/K)\cdot Ba_0^{\,2}\right]\right| \tag{2}$$

Where $|\vartheta_0|$ = dynamic rotational displacement amplitude under the foundation resting on two-layerd soil system, $|M|$ = moment amplitude, $B_\vartheta = \dfrac{I_\vartheta}{\rho r_0^{\,5}}$, the mass ratio, with $I_\vartheta = \dfrac{m r_0^{\,2}}{2}$.

Dynamic displacement amplitude given in Eq. (2) can be expressed in the non-dimensional form as given below,

$$\left|\frac{\vartheta_0 G r_0^{\,3}}{M}\right| = \left|\frac{K}{G r_0^{\,3}}\left[k(a_0) + ia_0 c(a_0)\right] - Ba_0^{\,2}\right|^{-1} \tag{3}$$

Magnification factor i.e. the ratio of dynamic displacement to the static displacement is expressed be

$$\left|M\right| = \left|\vartheta_0 /(M/K)\right| = \left|[k(a_0) + ica_0(a_0) - 3/16 B_\vartheta {a_0}^2]\right|^{-1} \tag{4}$$

## III. Rotational Cone Model

Fig.2a shows wave propagation in cones beneath the disk of radius $r_0$ resting on a layer underlain by a rigid base under torsional harmonic excitation $M_0$. The shear waves emanate beneath the disk and propagate at velocity c equal to shear wave velocity $c_s$. These waves reflect back and forth at the rigid base and free surface, spreading and decreasing in amplitude. Let the rotational displacement of the (truncated semi-infinite) cone, modeling a disk with same torsional moment $M_0$ on a homogeneous half-space with the material properties of the layer be denoted as □ with theFig.2a shows wave propagation in cones beneath the disk of radius $r_0$ resting on a layer underlain by a rigid base under torsional harmonic excitation $M_0$. The shear waves emanate beneath the disk and propagate at velocity c equal to shear wave velocity cs. These waves reflect back and forth at the rigid base and free surface, spreading and decreasing in amplitude. Let the rotational displacement of the (truncated semi-infinite) cone, modeling a disk with same torsional moment $M_0$ on a homogeneous half-space with the material properties of the layer be denoted as □ with the value □$_0$ under the disk, Fig.2b, the parameters of which are given in Table 1. This rotational displacement □$_0$ is used to generate the rotational displacement of the layer □ with its surface value □$_0$, Fig.2a. Thus, □$_0$ can also be called as the generating function. The first downward wave propagating in a cone with apex 1 (height $z_0$ and radius of base $r_0$), which may be called as the incident wave and its cone will be the same as that of the half-space, as the wave generated beneath the disk does not know if at a specific depth a rigid interface is encountered or not. Thus the aspect ratio defined by the ratio of the height of cone from its apex to the disk is made equal for cone of the half-space and first cone of the layer. Since the incident wave and subsequent reflected waves propagate in the same medium (layer), the aspect ratio of the corresponding cones will be same. Thus knowing the height of the first cone, from the geometry, the height of other cones corresponding to subsequent upward and downward reflected waves are found as shown in Fig.2a. The rotational displacement amplitude of the first downward incident wave (propagating in a cone with apex 1) at a depth z, which is inversely proportional to the square and cube of the distance from the apex of the cone and expressed in frequency domain as

$$\bar{\vartheta}(z,\omega) = \left[\frac{1}{\left(1+\frac{z}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{z}{z_0}\right)^3} - \frac{1}{\left(1+\frac{z}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{z}{c}}\,\bar{\vartheta}_0(\omega) \tag{5}$$

The rotational displacement of the incident wave at the interface equals

$$\bar{\vartheta}(d,\omega) = \left[\frac{1}{\left(1+\frac{d}{z_0}\right)^2} + \left\{\frac{1}{\left(1+\frac{d}{z_0}\right)^3} - \frac{1}{\left(1+\frac{d}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{d}{c}}\,\bar{\vartheta}_0(\omega) \tag{6}$$

Enforcing the boundary condition that the rotation at rigid base vanishes, the rotational displacement of the first reflected upward wave propagating in a cone with apex 2 ( Fig.2a) is given by

$$\bar{\vartheta}(2d-z,\omega) = \left[\frac{1}{\left\{1+\frac{2d-z}{z_0}\right\}^2} + \left\{\frac{1}{\left\{1+\frac{2d-z}{z_0}\right\}^3} - \frac{1}{\left\{1+\frac{2d-z}{z_0}\right\}^2}\right\} * \frac{1}{1+\frac{i\omega z_0}{c}}\right] * e^{i\omega\frac{2d-z}{c}} * \overline{\vartheta_0} \tag{7}$$

At the free surface the rotational displacement of the upward wave derived by substituting z = 0 in Eqn.7 equals

$$\bar{\vartheta}(2d,\omega) = \left[\frac{1}{\left\{1+\frac{2d}{z_0}\right\}^2} + \left\{\frac{1}{\left\{1+\frac{2d}{z_0}\right\}^3} - \frac{1}{\left\{1+\frac{2d}{z_0}\right\}^2}\right\} * \frac{1}{1+\frac{i\omega z_0}{c}}\right] * e^{i\omega\frac{2d-z}{c}} * \overline{\vartheta_0} \tag{8}$$

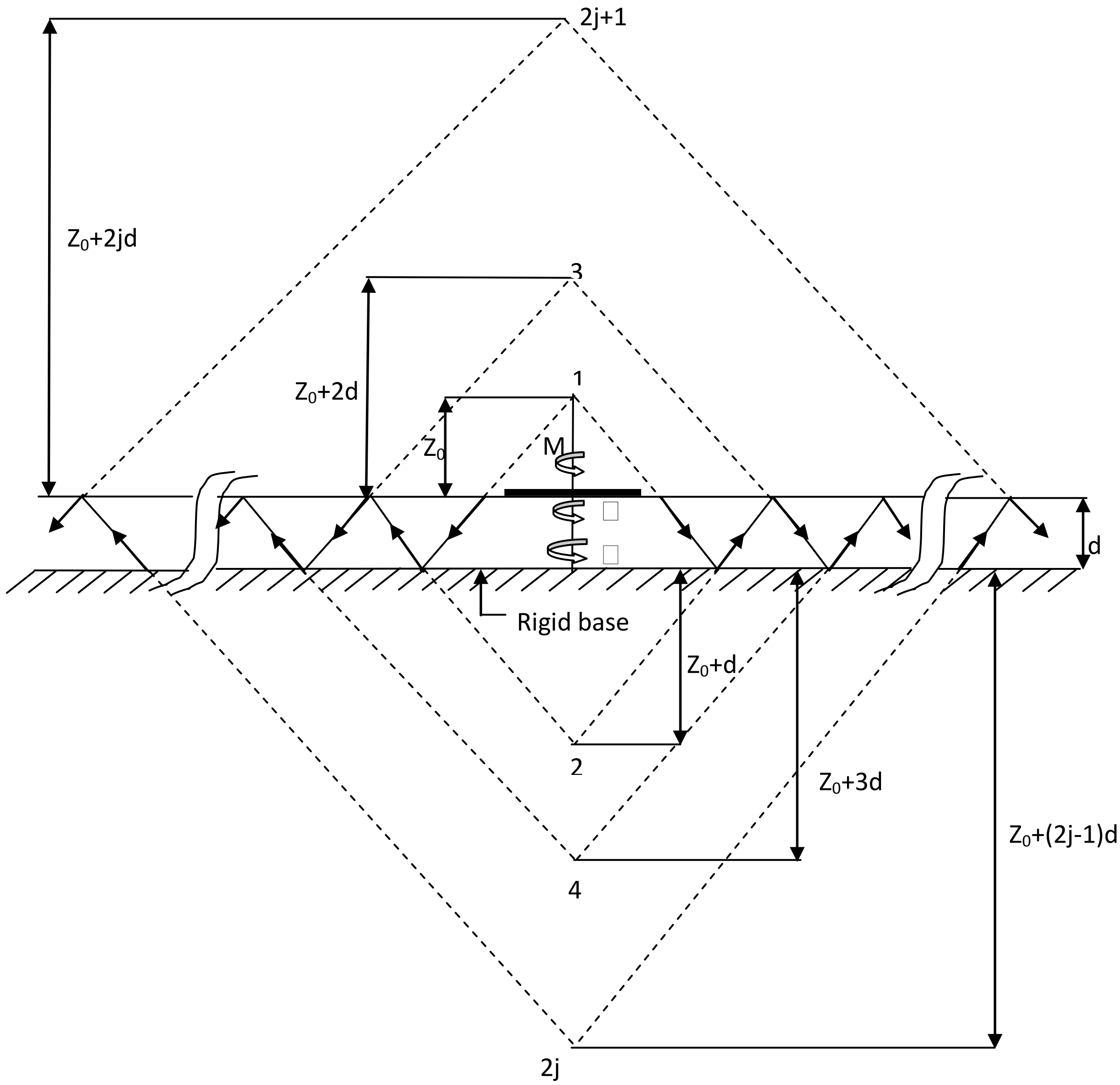


Fig 2(a) Wave propagation in cone for the layer

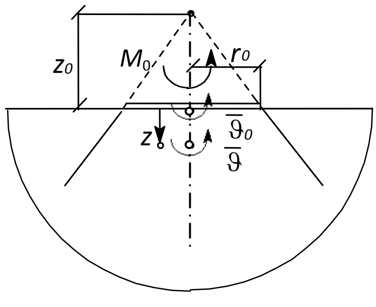


Fig 2(b) wave propagation in 1st cone

Enforcing compatibility of the amplitude and of elapsed time of the reflected wave's rotational displacement at the free surface, the rotational displacement of the downward wave propagating in a cone with apex 3 is formulated as

$$\overline{\vartheta}(2d+z,\omega)=\left[\frac{1}{\left\{1+\frac{2d+z}{z_0}\right\}^2}+\left\{\frac{1}{\left\{1+\frac{2d+z}{z_0}\right\}^3}-\frac{1}{\left\{1+\frac{2d+z}{z_0}\right\}^2}\right\}*\frac{1}{1+\frac{i\omega z_0}{c}}\right]*e^{i\omega\frac{2d-z}{c}}*\overline{\vartheta_0} \tag{9}$$

In this way the waves propagate in their respective cones and their corresponding rotational displacements are found out. The sum of all the down and up waves gives the resulting rotational displacement in the layer soil.

$$\vartheta(z,\omega)=\left[\frac{1}{\left(1+\frac{z}{z_0}\right)^2}+\left\{\frac{1}{\left(1+\frac{z}{z_0}\right)^3}-\frac{1}{\left(1+\frac{z}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{z}{c}}\,\overline{\vartheta}_0(\omega)$$

$$+\sum_{j=1}^{\infty}(-1)^j\left[\frac{1}{\left(1+\frac{2jd-z}{z_0}\right)^2}+\left\{\frac{1}{\left(1+\frac{2jd-z}{z_0}\right)^3}-\frac{1}{\left(1+\frac{2jd-z}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{2jd-z}{c}}\,\overline{\vartheta}_0(\omega)$$

$$+\sum_{j=1}^{\infty}(-1)^j\left[\frac{1}{\left(1+\frac{2jd+z}{z_0}\right)^2}+\left\{\frac{1}{\left(1+\frac{2jd+z}{z_0}\right)^3}-\frac{1}{\left(1+\frac{2jd+z}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{2jd+z}{c}}\,\overline{\vartheta}_0(\omega) \tag{10}$$

Where *j* is the number of impingement of waves at the rigid boundary.
At the free surface the rotational displacement of the foundation is obtained by setting $z=0$ in Eq. (10) as given below.

$$\vartheta(z,\omega)=$$

$$1+2\sum_{j=1}^{\infty}(-1)^j\left[\frac{1}{\left(1+\frac{2jd}{z_0}\right)^2}+\left\{\frac{1}{\left(1+\frac{2jd}{z_0}\right)^3}-\frac{1}{\left(1+\frac{2jd}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{2jd}{c}}\,\overline{\vartheta}_0(\omega) \tag{11}$$

$$\overline{\vartheta}_0(\omega)=H(\omega)\,\vartheta_0(\omega) \tag{12}$$

Where

$$H(\omega)=$$

$$1+2\sum_{j=1}^{\infty}(-1)^j\left[\frac{1}{\left(1+\frac{2jd}{z_0}\right)^2}+\left\{\frac{1}{\left(1+\frac{2jd}{z_0}\right)^3}-\frac{1}{\left(1+\frac{2jd}{z_0}\right)^2}\right\}\cdot\frac{1}{1+i\omega\frac{z_0}{c}}\right]\cdot e^{-i\omega\frac{2jd}{c}}\,\overline{\vartheta}_0(\omega) \tag{13}$$

$H(\omega)$ given by Eq.(13) may be called as rotational displacement transfer function, the value of which at $\omega=0$ gives the static stiffness of the layer normalized by the static stiffness of the homogeneous half-space with material properties of the layer. In numerical evaluation of the above transfer function, the summation of series over *j* is worked out up to a finite term as the displacement amplitude of the waves vanish after a finite number of impingement.

## IV. Dynamic Impedance

The interaction moment rotation relationship for a massless disk resting on the homogeneous half-space using the cone model can be written as

$$M_0(\omega) = K\big(k(a_0) + i\omega c(a_0)\big)\, \vartheta_0(\omega) \tag{14}$$

Where $k(a_0)$=spring coefficient and $C(a_0)$ = dashpot coefficient, the values of which are given in Table 1. K=Static stiffness of homogeneous half-space with material properties of top layer.
Using Eq. (12) in Eq. (14), then obtain the interaction moment rotation relationship for the layer-rigid base system as

$$M_0(\omega) = K\big(k(a_0) + i\omega\, c(a_0)\big)H(\omega)\cdot \vartheta_0(\omega) \tag{15}$$

$$\overline{K}(\omega) = \frac{M_0(\omega)}{\vartheta_0(\omega)} = K\big(k(a_0) + i\omega c(a_0)\big)H(\omega) \tag{16}$$

Table 1 The parameters of cone model under torsional vibration

| Cone Parameters | Parameter Expressions under Torsional Vibration |
|---|---|
| Aspect Ratio $\frac{z_0}{r_0}$ | $\frac{9\pi}{32}$ |
| Static stiffness coefficient $K$ | $\frac{3\rho\, c^2 I_0}{z_0}$ |
| Polar moment of inertia $I_0$ | $(\pi/2){r_0}^4$ |
| Normalized spring coefficient $k(a_0)$ | $1-\frac{1}{3}\frac{a_0^2}{\left(\frac{r_0 c}{z_0 c_s}\right)^2 + a_0^2}$ |
| Normalized damping coefficient $c(a_0)$ | $\frac{z_0}{3r_0}\frac{c_s}{c}\cdot\frac{{a_0}^2}{\left(\frac{r_0 c}{z_0 c_s}\right)^2 + {a_0}^2}$ |
| Dimensionless frequency $a_0$ | $\frac{\omega r_0}{c_s}$ |
| Appropriate wave velocity $c$ | $c = c_s$, for all values of Poisson's ratio (ν) |

## V. Results and Discussions

### (a) Comparison Of Dynamic Impedance

A rigid circular foundation resting on a layer over rigid base with $d$=2$r_0$, ν = 1/3, and $\xi$ = 5% is examined for torsional mode of vibration. The impedance functions normalized by $K_L(1 + 2i\xi)$ with $K_L$, the static stiffness of the layer-rigid base system under a given mode and $\xi$, material damping ratio, are computed using the cone model for the above cases. The results thus computed for the frequency range $a_0$ = 0 to 6, are compared with the reported results of Gazetas (1983) obtained by a more rigorous analytical method which is presented in Fig.3. Excellent agreement is observed in both stiffness and damping coefficients in the lower frequency range ($a_0 \le 1.5$). But in the higher frequency range the trends of the predicted stiffness and damping coefficients are found to be almost similar though there is some variation in magnitude.

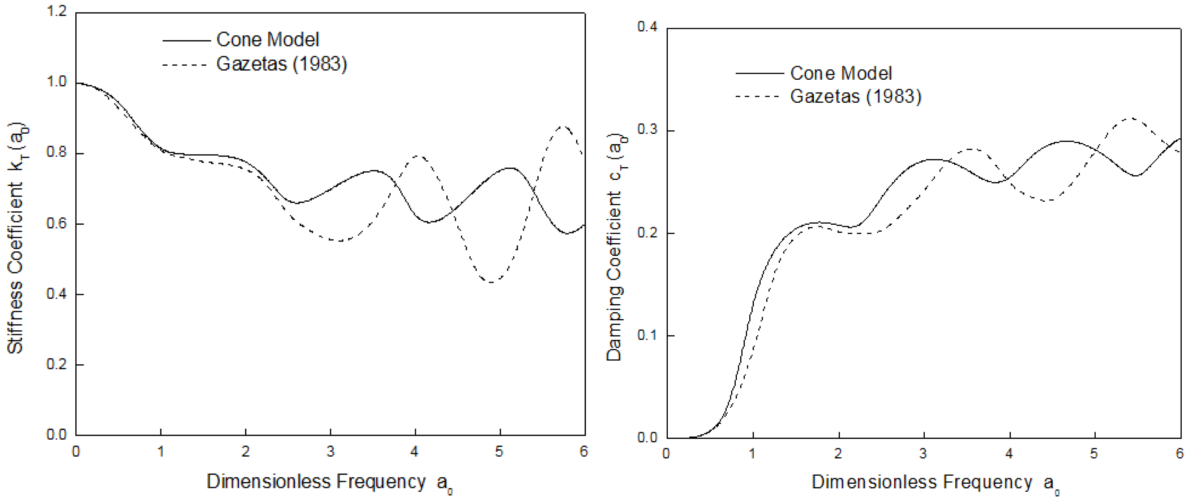


Fig. 3 Comparison of normalized torsional impedance functions of rigid circular foundation on layer over rigid base ($d$ = $2r_0$, ν = 1/3, and ξ = 0.05)

**(b) Parametric Study of Static Stiffness**

In this case the static stiffness of circular foundation is studied varying the depth of the layer, i.e. $d/r_0$ ratio from 0.5 to 10. The values of Poisson's ratio (ν) considered are 0.0, 0.3 and 0.49. The normalized static stiffness, rotational degrees of freedom, are presented in Fig.4. It is observed from this figure that the Poisson's ratio under torsional degree of freedom is found to be independent of Poisson's ratio. The static stiffness of the foundation is found to be more when the depth of the layer is less (Fig.4). With the increase in the depth of the layer the stiffness decreases and it approaches to half-space value at a specific depth depending on the degree of freedom.

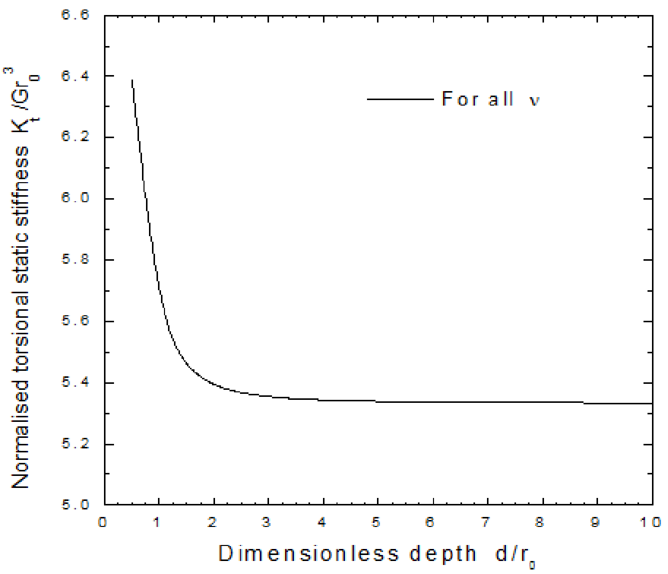


Fig. 4 Normalized static stiffness of circular foundation resting on a layer over rigid base with variation of $d/r_0$ for various values of ν.

**(c) Parametric Study of Dynamic Impedance**

Results for the dynamic impedance functions of a rigid circular disk on the surface of a soil layer of finite depth over rigid base are presented in Figs. 5 and 6. Fig. 5 shows the effect of $d/r_0$ ratio on the dynamic stiffness coefficient, k ($a_0$) and damping coefficient, c ($a_0$) for a single value of hysteretic material damping ratio, ξ = 0.05; and Fig. 6 shows the sensitivity of k ($a_0$) and c ($a_0$) to the variation of material damping ratio,ξ, for $d/r_0$ = 2. The variation of stiffness and damping coefficients with frequency shows a strong dependent on $d/r_0$ ratio (Fig. 5). k ($a_0$) and c ($a_0$) are not smooth functions as on a homogeneous half-space, but exhibit undulations (peaks and valleys) associated with the natural frequencies of the soil layer. The stiffness and damping coefficients in torsion modes are observed to be relatively smooth functions of $a_0$, rapidly approaching their corresponding half-space curves as $d/r_0$ ratio approaches 4.

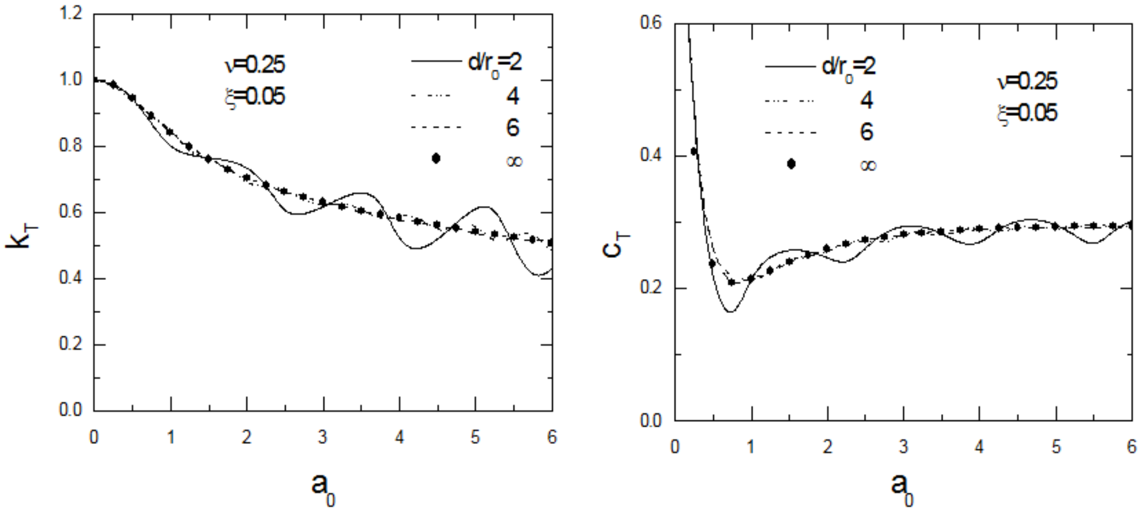


Fig. 5 Variation of impedance functions with depth of the layer for a rigid circular foundation resting on a layer over bedrock

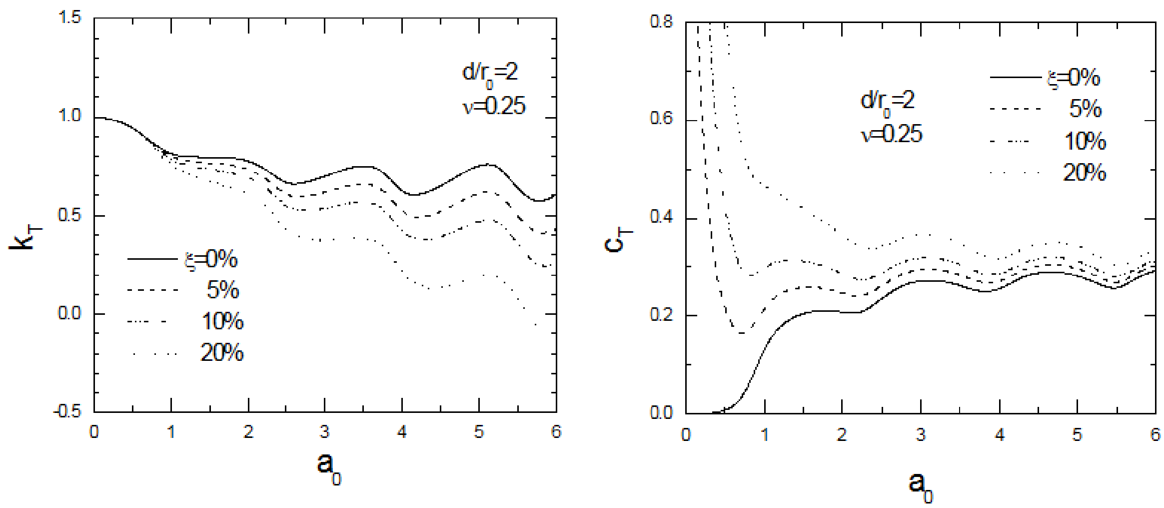


Fig. 6 Variation of impedance functions with variation in material damping ratio for a rigid circular foundation resting on a layer over bedrock.

## VI. Conclusions

In contrast to rigorous methods, which address the very complicated wave pattern consisting of body waves and generalized surface waves working in wave number domain, the proposed method based on wave propagation in cones considers only one type of body wave depending on the mode of vibration i.e. shear wave for torsional degree of freedom. The sectional property of the cones increases in the direction of wave propagation downwards preserving physical insight. Thus, the model provides physical insight which is often obscured by the complexity of rigorous numerical solutions, exhibit adequate accuracy, easier to use and offers a cost-effective tool for the design foundations under dynamic loads.

Based on parametric study, the following conclusion can be drown

- The static stiffness in case of torsional degree of freedom is found to be independent of Poisson's ratio.
- The static stiffness of the foundation is found to be more when the depth of the layer is less
- The stiffness and damping coefficients in torsion modes are observed to be relatively smooth functions of $a_0$, when $d/r_0$ ratio approaches 4

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NOTATIONS

| | |
|---|---|
| $a_0$ | dimensionless frequency $(\omega r_0 / c_s)$ |
| $B_\vartheta = \frac{I_\vartheta}{\rho r_0^5}$ | non-dimensional mass ratio in torsional mode |
| $I_\vartheta = \frac{m r_0^2}{2}$ | mass moment of inertia of the foundation about the axis of rotation in torsion |
| $I_0 = (\pi/2) r_0^4$ | Polar moment of inertia |
| $K$ | static stiffness of the disk on homogeneous half-space |
| $K_L$ | static stiffness of the disk on layered soil |
| $\bar{K}(a_0)$ | non-dimensional dynamic impedance |
| $\bar{K}(\omega)$ | frequency dependent dynamic impedance |

| | |
|---|---|
| $k(a_0)$ | normalized stiffness coefficient |
| $M$ | harmonic torsional moment on foundation |
| $\lvert M \rvert$ | torsional moment amplitude on the foundation |
| $M_0$ | harmonic interaction moment |
| $M_0(\omega)$ | frequency dependent harmonic interaction moment |
| $m$ | mass of the foundation or total vibrating mass (mass of foundation plus machine) in case of machine foundation |
| $r_0$ | radius of circular foundation or radius of equivalent circle for non-circular foundation |
| $\left\lvert \frac{\vartheta_0 G r_0{}^3}{M} \right\rvert$ | non-dimensional rotational amplitude |

Greeks

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|---|---|
| $\bar{\vartheta}$, $\bar{\vartheta}(z,\omega)$ | rotational harmonic displacement at depth *z* for homogeneous half-space |
| $\bar{\vartheta}_0$, $\bar{\vartheta}_0(\omega)$ | rotational harmonic surface displacement for homogeneous half-space |