International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 11, Issue 9 [September. 2022] PP: 80-83

On Finding Integer Solutions to Non-homogeneous Ternary Cubic Equation $7(x^2+y^2)-6xy=11z^3$

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Abstract:
This paper concerns with the problem of obtaining non-zero distinct integer solutions to the non-homogeneous ternary cubic equation $7(x^2+y^2)-6xy=11z^3$. Different sets of integer solutions are illustrated.

Keywords: non-homogeneous cubic,ternary cubic , integer solutions

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Date of Submission: 07-09-2022 Date of Acceptance: 22-09-2022

I.INTRODUCTION:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-32] for a few problems on bi-quadratic equation with 3 unknowns. This paper concerns with yet another interesting bi-quadratic diophantine equation with three variables given by $2(x^2+y^2)-xy=57z^4$ for determining its infinitely many non-zero distinct integral solutions

II. METHOD OF ANALYSIS:

The non-homogeneous ternary cubic equation under consideration is

$$7(x^2+y^2)-6xy=11z^3 \quad (1)$$

Introduction of the linear transformations

$$x=u+2v,\ y=u-2v,\ z=2w \quad (2)$$

in (1) leads to

$$u^2+10v^2=11w^3 \quad (3)$$

We solve (3) through different ways and using (2) obtain different sets of solutions to (1).

Way 1:

Let

$$w=a^2+10b^2 \quad (4)$$

Write 11 on the R.H.S. of (3) as

$$11=(1+i\sqrt{10})(1-i\sqrt{10}) \qquad (5)$$

Substituting (4) & (5) in (3) and employing the method of factorization,

consider

$$u+i\sqrt{10}v=\left(1+i\sqrt{10}\right)\left(a+i\sqrt{10}b\right)^3 \qquad (6)$$

On equating the real and imaginary parts in (6), and employing (2),the values of x, y, z

are given by

$$\left.\begin{aligned} x &= 3a^3-90ab^2-24a^2b+80b^3 \\ y &= -a^3+30ab^2-36a^2b+120b^3 \\ z &= 2a^2+20b^2 \end{aligned}\right\} \qquad (7)$$

Thus,(4) and (7) represent the integer solutions to (1).

Note 1:

The integer 11 on the R.H.S. of (3) is also represented as

$$11=\frac{(29+i7\sqrt{10})(29-i7\sqrt{10})}{(11)^2}$$

Repetition of the above process leads to a different set of integer solutions to (1).

Way 2:

Rewrite (3) as

$$u^2+10v^2=11w^3*1 \qquad (8)$$

Consider 1 on the R.H.S. of (8) as

$$1=\frac{(3+i2\sqrt{10})(3-i2\sqrt{10})}{49} \qquad (9)$$

Following the analysis similar to Way1,and replacing a by 7A ,b by7B,

thevalues of x, y, z satisfying (1) are given by

$$\begin{aligned} x &= -343A^3+10290AB^2-12348A^2B+41160B^3 \\ y &= -1323A^3+39690AB^2-2352A^2B+7840B^3 \\ z &= 98A^2+980B^2 \end{aligned}$$

Note 2:

The integer 1 on the R.H.S. of (8) is also expressed as

$$1=\frac{(10r^2-s^2+i\sqrt{10}\,2rs)\,(10r^2-s^2-i\sqrt{10}\,2rs)}{(10r^2+s^2)^2},$$

$$1=\frac{(1+i6\sqrt{10})\,(1-i6\sqrt{10})}{19^2}$$

Repeating the above process, different sets of solutions to (1) are obtained.

Way 3:

In (2),

Assume

$$u=11^2m(m^2+10n^2)$$
$$v=11^2n(m^2+10n^2)$$
$$w=11(m^2+10n^2)$$

The values of x,y,z are

$$x=11^2\left(m^3+10mn^2+2m^2n+20n^3\right)$$
$$y=11^2\left(m^3+10mn^2-2m^2n-20n^3\right)$$
$$z=22m^2+220n^2$$

In (2), Assume

$$u=11^2m\left(m^2-30n^2\right)$$
$$v=11^2n\left(3m^2-10n^2\right)$$
$$w=11\left(m^2+10n^2\right)$$

The values of x,y,z are

$$x=11^2\left(m^3-30mn^2+6m^2n-20n^3\right)$$
$$y=11^2\left(m^3-30mn^2-6m^2n+20n^3\right)$$
$$z=22m^2+220n^2$$

III.CONCLUSION:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous cubic diophantine equation with three unknowns given by $7(x^2+y^2)-6xy=11z^3$.One may search for other sets of integer solutions to the considered equation as well as other choices of the third degree diophantine equations with multi-variables

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