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Improved Ratio Estimators of Population Mean Using Linear Combination of Auxiliary Variable Location Parameters

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Abstract
In this study we proposed an improved family of ratio estimators for estimation of population mean using mixture of root of correlation coefficient, kurtosis, skewness and variance with maximum value of the auxiliary variable. The properties associated with the proposed estimators are assessed by mean square error (MSE) and compared with the existing estimators. From the empirical study, we observed the three proposed ratio estimators have least MSEs as compared with the MSEs for the latest existing ratio estimators. And the relative efficiency (RE) of all existing over proposed ratio estimators are greater than one (>1). By these, we concluded that the proposed estimators are more efficient than the existing estimators. Hence, we strongly recommend that our proposed estimators are preferred over the existing estimators in practical applications

Key words: *Auxiliary information, Mean Square Error, Related Efficiency.*

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I. Introduction

One of the feature theoretical statistics is the creation of a large body of theory that discusses how to make good estimates from data. In the development of theory, specifically for sample survey, relatively little use has been made of the method of estimation. Estimation techniques that are mostly considered for sample survey work are ratio and the linear regression method (cochran1977). In the ratio method, an auxiliary variable correlated with variable of interest are obtained for each unit in the sample. The population total of the auxiliary variable most be known. This information are used by ratio estimator to indicates how many times one contains another. The aim in this method is to obtain increased precision by taking advantage of the correlation between the two variables. At present we assume simple random sampling. This can be attained by presenting a large number of modified ratio estimators which exploits the information by well-known values of coefficient of variation, skewness, kurtosis, median, variance etc. Likewise, the ratio weights are specified by $\frac{X}{x}$ where X is the recognized population total for the auxiliary variable and x is the agreeing estimate of the total centered on all corresponding units in the sample.

Many authors have suggested some modified ratio estimators using single auxiliary variable. For example, Sisodia and Dwivedi (1981), Singh and Kakran (1993) Tripathi and Khare (1994), Upadhyaya and Singh (1999), Singh and Tailor (2004), Kadilar and Cingi (2004,2006), ,Singh and Tailor (2003), Yan and Tian (2010), Subramani and Kumarapandiyan (2012a,2012b, 2012c, 2013), Jelani et al (2013), Adepoju and Shittu (2013), Shittu and Adepoju (2014), Kumar (2015), Abid et al (2016) Raja et al (2017), Bhushan and Misra (2017), Subzar *et'al* (2017a, 2017b, 2017c), Abbas *et'al* (2018), Subzar *et'al* (2018a, 2018b), Saddam *et'al* (2018), Subzar *et'al* (2019) and Jamiu and Eric (2020). They showed that their estimators were more efficient than the classical ratio estimator in some cases.

In the reviwed literature, Researchers have suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary variable parameters. They showed that their suggested estimators are more efficient than traditional ratio estimators in the estimation of the population mean. The some estimators are as follows;

Table 1: The existing ratio estimators of the population mean

	Classical ratio estimator introduced by Cochran (1977)	**Coefficient Of $\bar{X}$**	**Linear Combination**
1	$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X}$	1	0
Estimators proposed by Saddam *et al*. (2018)			
2	$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+S_x^2)}(\bar{X}C_x+S_x^2)$	C_x	S_x^2
3	$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+S_x^2)}(\bar{X}\beta_1+S_x^2)$	β_1	S_x^2
4	$\hat{\bar{Y}}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+S_x^2)}(\bar{X}\beta_2+S_x^2)$	β_2	S_x^2
	Estimators proposed by Muili *et al* (2019)		
	$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\mu_x-\bar{x})}{(\bar{x}+\alpha_1)}(\mu_x+\alpha_1)$	1	$M_x \times TM,$
	$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\mu_x-\bar{x})}{(\bar{x}\rho+\alpha_2)}(\mu_x\rho+\alpha_2)$	ρ	$M_x \times QD,$
	$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\mu_x-\bar{x})}{(\bar{x}C_x+\alpha_3)}(\mu_x C_x+\alpha_3)$	C_x	$M_x \times HL$
	Estimators proposed by Jamiu and Eric (2020)		
5	$\hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\phi_1)}(\bar{X}\beta_1+\phi_1)$	β_1	$Md \times TM,$
6	$\hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1+\phi_2)}(\bar{X}\beta_1+\phi_2)$	β_1	$Md \times MR,$
7	$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\phi_3)}(\bar{X}\beta_1+\phi_3)$	β_1	$Md \times HL$
8	$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\phi_1)}(\bar{X}\beta_2+\phi_1)$	β_2	$Md \times TM,$
9	$\hat{\bar{Y}}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\phi_2)}(\bar{X}\beta_2+\phi_2)$	β_2	$Md \times MR,$
10	$\hat{\bar{Y}}_{12} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\phi_3)}(\bar{X}\beta_2+\phi_3)$	β_2	$Md \times HL$

Table 2: The relative constant and the mean square error of the ratio estimator

	Constant	**Bias**	**Mean Square Error (MSE)**
For Cochran (1977) estimators			
1	$\hat{R} = \frac{\bar{Y}}{\bar{X}}$	$\beta(\hat{\bar{Y}}_r) = \frac{1-f}{n}\frac{1}{\bar{Y}}(RS_x^2-\rho S_x S_y)$	$MSE = \frac{1-f}{n}(S_y^2+R^2S_x^2-2R\rho S_x S_y)$
For Saddam *et al.* (2018)			
2	$R_1 = \frac{\bar{Y}C_x}{\bar{X}C_x+S_x^2}$	$\beta(\hat{\bar{Y}}_1) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_1^2$	$MSE(\hat{\bar{Y}}_1) = \frac{1-f}{n}(R_1^2S_x^2+S_y^2(1-\rho^2))$

3	$R_2 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + S_x^2}$	$\beta(\hat{\bar{Y}}_2) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_2^2$	$MSE(\hat{\bar{Y}}_2) = \frac{1-f}{n}(R_2^2 S_x^2 + S_y^2(1-\rho^2))$
4	$R_3 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + S_x^2}$	$\beta(\hat{\bar{Y}}_3) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_3^2$	$MSE(\hat{\bar{Y}}_3) = \frac{1-f}{n}(R_3^2 S_x^2 + S_y^2(1-\rho^2))$
For Muili *et al* (2019)			
4	$R_4 = \frac{\bar{Y}}{\bar{X} + \alpha_1}$	$\beta(\hat{\bar{Y}}_4) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_4^2$	$MSE(\hat{\bar{Y}}_4) = \frac{1-f}{n}(R_4^2 S_x^2 + S_y^2(1-\rho^2))$
5	$R_5 = \frac{\bar{Y}\rho}{\bar{X}\rho + \alpha_2}$	$\beta(\hat{\bar{Y}}_5) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_5^2$	$MSE(\hat{\bar{Y}}_5) = \frac{1-f}{n}(R_5^2 S_x^2 + S_y^2(1-\rho^2))$
6	$R_6 = \frac{\bar{Y}C_x}{\bar{X}C_x + \alpha_3}$	$\beta(\hat{\bar{Y}}_6) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_6^2$	$MSE(\hat{\bar{Y}}_6) = \frac{1-f}{n}(R_6^2 S_x^2 + S_y^2(1-\rho^2))$
For Jamiu and Eric (2020)			
7	$R_7 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \phi_1}$	$\beta(\hat{\bar{Y}}_7) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_7^2$	$MSE(\hat{\bar{Y}}_7) = \frac{1-f}{n}(R_7^2 S_x^2 + S_y^2(1-\rho^2))$
8	$R_8 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \phi_2}$	$\beta(\hat{\bar{Y}}_8) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_8^2$	$MSE(\hat{\bar{Y}}_8) = \frac{1-f}{n}(R_8^2 S_x^2 + S_y^2(1-\rho^2))$
9	$R_9 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \phi_3}$	$\beta(\hat{\bar{Y}}_9) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_9^2$	$MSE(\hat{\bar{Y}}_9) = \frac{1-f}{n}(R_9^2 S_x^2 + S_y^2(1-\rho^2))$
10	$R_{10} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \phi_2}$	$\beta(\hat{\bar{Y}}_{10}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$	$MSE(\hat{\bar{Y}}_{10}) = \frac{1-f}{n}(R_{10}^2 S_x^2 + S_y^2(1-\rho^2))$
11	$R_{11} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \phi_3}$	$\beta(\hat{\bar{Y}}_{11}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$	$MSE(\hat{\bar{Y}}_{11}) = \frac{1-f}{n}(R_{11}^2 S_x^2 + S_y^2(1-\rho^2))$
12	$R_{12} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \phi_3}$	$\beta(\hat{\bar{Y}}_{12}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2$	$MSE(\hat{\bar{Y}}_{12}) = \frac{1-f}{n}(R_{12}^2 S_x^2 + S_y^2(1-\rho^2))$

II. The proposed ratio estimators

In this research we proposed six new modified ratio estimators by adapting the methodology of Saddam *et'al* (2018) and Jamiu and Eric (2020) using the linear combination of root of correlation coefficient, coefficient kurtosis and skewness. And population variance with maximum of the supporting variable, on the assumption that the supporting variable is related to the study variable and any linear combination of the information on the supporting variable can be utilized to propose new estimators that can be used to estimate the population mean with a reduced MSE, this is given as follows:

Table 3: The proposed ratio estimators of the population mean

	Proposed estimators	A	G
1	$\hat{\bar{Y}}_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}A_1 + G)}(\bar{X}A_1 + G)$	$\sqrt{\rho}$	$M_x S_x^2$
2	$\hat{\bar{Y}}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}A_2 + G)}(\bar{X}A_2 + G)$	$\sqrt{\beta_1}$	$M_x S_x^2$
3	$\hat{\bar{Y}}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}A_3 + G)}(\bar{X}A_3 + G)$	$\sqrt{\beta_2}$	$M_x S_x^2$

III. Derivations of Mean Square Errors

The MSE for the first proposed estimator can be obtained by using Taylor's series technique as follows:

$$h(\bar{x},\bar{y}) \cong h(\bar{X},\bar{Y}) + \frac{\partial h(c,d)}{\partial c}|_{\bar{X},\bar{Y}}\ (\bar{x}-\bar{X}) + \frac{\partial h(c,d)}{\partial d}|_{\bar{X},\bar{Y}}\ (\bar{y}-\bar{Y})$$

(3.32)

Where $h(\bar{x},\bar{y}) = \hat{R}_{pi}$ and $h(\bar{X},\bar{Y}) = R$

As shown in Wolter (1985), equation (3.32) can be applied to the proposed estimators in order to obtain the MSEs of the proposed family of estimators given above.

$$\hat{R}_{pi} - R \cong \frac{\partial(\bar{y}+b(\bar{X}-\bar{x})}{\partial\bar{x}(\bar{x}A+G)}|_{\bar{X},\bar{Y}}\ (\bar{x}-\bar{X}) + \frac{\partial(\bar{y}+b(\bar{X}-\bar{x})}{\partial\bar{y}(\bar{x}A+G)}|_{\bar{X},\bar{Y}}\ (\bar{y}-\bar{Y})$$

$$\cong -\left(\frac{\bar{y}A}{(\bar{x}A+G)^2} + \frac{b(\bar{X}A+G)}{(\bar{x}A+G)^2}\right)|_{\bar{X},\bar{Y}}\ (\bar{x}-\bar{X}) + \frac{1}{(\bar{x}A+G)}|_{\bar{X},\bar{Y}}\ (\bar{y}-\bar{Y})$$

$$E(\hat{R}_{pi}-R)^2 \cong \frac{(\bar{Y}A+B(\bar{X}A+G))^2}{(\bar{X}A+G)^4}V(\bar{x}) - \frac{2(\bar{Y}A+B(\bar{X}A+G))}{(\bar{X}A+G)^3}Cov(\bar{x},\bar{y}) + \frac{1}{(\bar{X}A+G)^2}V(\bar{y})$$

$$\cong \frac{1}{(\bar{X}A+G)^2}\left\{\frac{(\bar{Y}A+B(\bar{X}A+G))^2}{(\bar{X}A+G)^2}V(\bar{x}) - \frac{2(\bar{Y}A+B(\bar{X}A+G))}{(\bar{X}A+G)}Cov(\bar{x},\bar{y}) + V(\bar{y})\right\}$$

Where $B = \frac{S_{xy}}{S_x^2} = \frac{\rho S_x S_y}{S_x^2} = \frac{\rho S_y}{S_x}$, Note that we omit the difference of $(E(b)-B)$, because this assumes a line through the origin, as in the unbiased case for design-based ratio estimator. In Sukhatme (1954),this condition for bias to vanish for SRS makes sense because weighted least square (WLS) regression and ordinary (homoscedastic) least square (OLS) regression are both unbiased for *b*. That is, a derivation of the design based ratio estimator which shows it is unbiased when we have a linear regression through the origin with the regression coefficient being homoscedastic.

$$MSE(\bar{y}_{pi}) = (\bar{X}A+G)^2 E(\hat{R}_{pi}-R)^2$$

$$\cong \frac{(\bar{Y}A+B(\bar{X}A+G))^2}{(\bar{X}A+G)^2}V(\bar{x}) - \frac{2(\bar{Y}A+B(\bar{X}A+G))}{(\bar{X}A+G)}Cov(\bar{x},\bar{y}) + V(\bar{y})$$

$$\cong \frac{\bar{Y}^2A^2 + 2B(\bar{X}A+G)\bar{Y}A + B(\bar{X}A+G)}{(\bar{X}A+G)^2}V(\bar{x}) - \frac{2\bar{Y}A+2B(\bar{X}A+G)}{(\bar{X}A+G)}Cov(\bar{x},\bar{y}) + V(\bar{y})$$

$$\cong \frac{(1-f)}{n}\left\{\left(\frac{\bar{Y}^2A^2}{(\bar{X}A+G)^2} + \frac{+2B\bar{Y}A}{(\bar{X}A+G)} + B^2\right)S_x^2 - \left(\frac{2\bar{Y}A}{(\bar{X}A+G)} + 2B\right)S_{xy} + S_y^2\right\}$$

$$\cong \frac{(1-f)}{n}(R_{pi}^2S_x^2 + 2BR_{pi}S_x^2 + B^2S_x^2 - 2R_{pi}S_{xy} - 2BS_{xy} + S_y^2)$$

$$\cong \frac{(1-f)}{n}(R_{pi}^2S_x^2 + 2R_{pi}\rho S_xS_y + \rho^2S_y^2 - 2R_{pi}\rho S_xS_y - 2\rho^2S_y^2 + S_y^2)$$

$$\cong \frac{(1-f)}{n}(R_{pi}^2S_x^2 - \rho^2S_y^2 + S_y^2)$$

$$MSE(\bar{y}_{pi}) \cong \frac{(1-f)}{n}(R_{pi}^2S_x^2 + S_y^2(1-\rho^2))$$

Similarly, the relative constant and the bias are obtained from the derivation of MSEs as

Table 4: The **Relative constant, Bias and MSE of the proposed estimators table**

	Relative Constant	Bias	MSE
1	$R_{p1} = \frac{\bar{Y}A_1}{\bar{X}A_1 + G}$	$\beta(\hat{\bar{Y}}_{p1}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{p1}^2$	$MSE(\bar{y}_{p1}) \cong \frac{(1-f)}{n}(R_{p1}^2 S_x^2 + S_y^2(1-\rho^2))$
2	$R_{p2} = \frac{\bar{Y}A_2}{\bar{X}A_2 + G}$	$\beta(\hat{\bar{Y}}_{p2}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$	$MSE(\bar{y}_{p2}) \cong \frac{(1-f)}{n}(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2))$
3	$R_{p3} = \frac{\bar{Y}A_3}{\bar{X}A_3 + G}$	$\beta(\hat{\bar{Y}}_{p3}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{p3}^2$	$MSE(\bar{y}_{p3}) \cong \frac{(1-f)}{n}(R_{p3}^2 S_x^2 + S_y^2(1-\rho^2))$

IV. Efficiency comparison of the Proposed Estimators

The efficiency condition under which the proposed ratio estimators will have least mean square error compared with the classical ratio estimator and prevailing adjusted ratio estimators to evaluate the finite population mean would be shown algebraically.

i) The proposed estimators $\hat{\bar{Y}}_{pj}$ will be better than the classical estimator $\bar{Y}_r$

iff $$MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_r)$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \leq \frac{1-f}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y)$$

$$R_{pj}^2 S_x^2 - \rho^2 S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y \leq 0$$

$$\rho S_y - RS_x)^2 - R_{pj}^2 S_x^2 \leq 0$$

$$(\rho S_y - RS_x + R_{pj} S_x)(\rho S_y - RS_x - R_{pj} S_x) \leq 0$$

Condition 1: $(\rho S_y - RS_x + R_{pj} S_x) \leq 0$ and $(\rho S_y - RS_x - R_{pj} S_x) \leq 0$

After solving the condition 1,we get

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right)$$

Hence, $MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_r)$ when the above given inequality hold.

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right) \text{ or } \left(\frac{RS_x - \rho S_y}{S_x}\right) \leq R_{pj} \leq \left(\frac{\rho S_y - RS_x}{S_x}\right)$$

ii) The proposed estimators $\hat{\bar{Y}}_{pj}$ will be better than the existing estimator $\hat{\bar{Y}}_i$

iff $$MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_i)$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \leq \frac{1-f}{n}(R_i^2 S_x^2 + S_y^2(1-\rho^2))$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2$$

$$R_{pj}^2 \leq R_i^2$$

V. Relative Efficiency

The relative efficiency of the proposed ratio estimators for population mean to the classical and existing estimators can be measured base on percentage. And the formula is as follows

$$PRE = \frac{MSE(existing - estimator)}{MSE(proposed - estimator)} \times 100$$

(3.33)

Equation (3.33) gives the formula for estimating the percentage relative efficiencies of the proposed family of estimators in this research.

VI. Empirical Study

The performances of the proposed ratio estimators are evaluated and compared with the mentioned ratio estimators in chapter three by using the data of the following populations.

Population 1 are test result of individuals with their ages (X) and systolic blood pressure (Y) while Population 2 is an experimental data of hybrid soybean plant height (X) and its yield (Y) conducted by Jigawa State Agricultural and Rural Development Authority. We apply the proposed and existing estimators as follows;

Table 5: Characteristics of the Populations

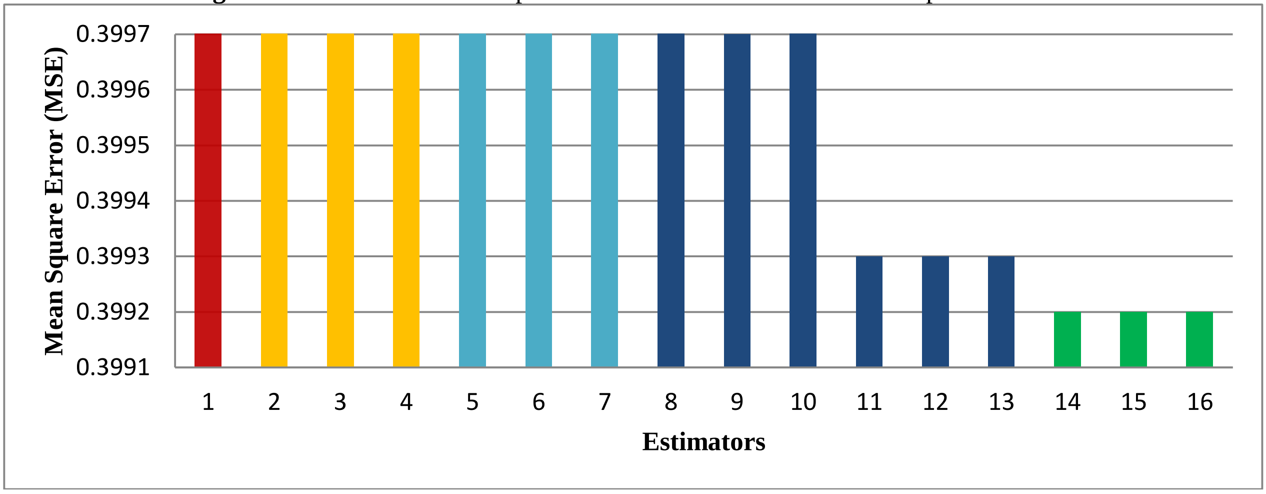
	Population I		**Population II**	
Variable	**(X)**	**(Y)**	**(X)**	**(Y)**
Population Total N	100	100	100	100
Sample Total n	70	70	70	70
Mean	55.18	149.20	127.89	15.51
Correlation ρ	0.92	0.92	0.87	0.87
S. Variance S^2	141.60	604.40	156.64	0.33
Coef. Variation C	21.57	16.48	9.79	3.73
Skewness β_1	0.66	-0.51	0.02	-0.05
Kurtosis β_2	- 0.31	-1.04	-0.11	-1.19
Tri-mean TM	54.64	150.08	127.90	15.51
Median Md	55.00	155.00	129.00	15.51
Mid Range MR	60.00	100.00	132.50	15.50
H. Lehmann HL	55.00	100.00	129.00	15.49
Maximum M_x	82.00	185.00	166.00	16.48
Q. deviation QD	53.50	149.63	127.63	15.50

Table 6: The Statistical Analysis for the MSE Of The Estimators For The Populations

Estimators	**Population 1**			**Population II**		
	Constant	**Bias**	**MSE**	**Constant**	**Bias**	**MSE**
Cochran (1977)						
$\hat{\bar{Y}}_r$	2.7039	0.0298362	4.8507	0.12127	0.000638	0.01025
Saddam *et al* (2018)						
$\hat{\bar{Y}}_1$	2.4164	0.0238283	3.9544	0.10780	0.000504	0.00817
$\hat{\bar{Y}}_2$	0.5532	0.0012489	0.5855	0.00194	0.000000	0.00035
$\hat{\bar{Y}}_3$	-0.3715	0.0005632	0.4832	-0.01196	0.000006	0.00044
Muili *et al* (2019)						
$\hat{\bar{Y}}_4$	0.0329	0.0000044	0.3999	0.00072	0.000000	0.00035
$\hat{\bar{Y}}_5$	0.0336	0.0000046	0.3999	0.00127	0.000000	0.00035
$\hat{\bar{Y}}_6$	0.2912	0.0003461	0.4508	0.01283	0.000007	0.00046
Jamiu *et al* (2020)						
$\hat{\bar{Y}}_7$	0.0324	0.0000043	0.3998	0.00001	0.000000	0.00035
$\hat{\bar{Y}}_8$	0.0295	0.0000036	0.3997	0.00001	0.000000	0.00035
$\hat{\bar{Y}}_9$	0.0322	0.0000042	0.3998	0.00001	0.000000	0.00035
$\hat{\bar{Y}}_{10}$	0.0153	0.0000010	0.3993	-0.00010	0.000000	0.00035
$\hat{\bar{Y}}_{11}$	0.0139	0.0000008	0.3993	-0.00010	0.000000	0.00035
$\hat{\bar{Y}}_{12}$	0.0152	0.0000009	0.3993	-0.00010	0.000000	0.00035
Proposed (2021)						
$\hat{\bar{Y}}_{p1}$	0.0123	0.0000006	0.3992	0.000005	0.000000	0.00034

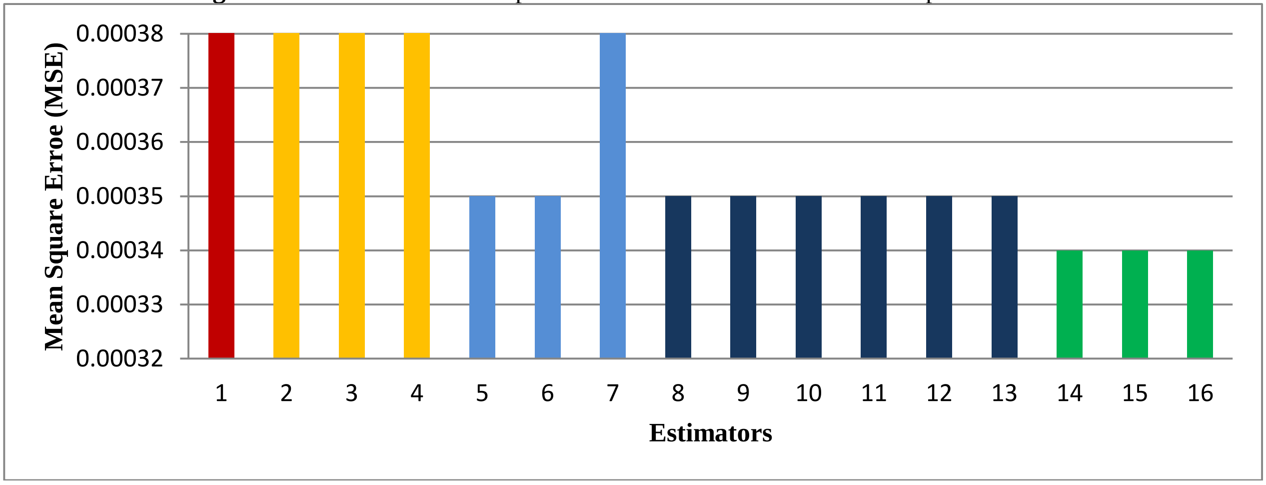
$\hat{\bar{Y}}_{p2}$	0.0104	0.0000004	0.3992	0.000008	0.000000	0.00034
$\hat{\bar{Y}}_{p3}$	0.0071	0.0000002	0.3992	0.000000	0.000000	0.00034

Diagram 1: The Chart below represents the MSE of Estimators for Population 1



Key: •Cochran Estimator • Sadam Estimators •Muili Estimators •Jamiu Estimators •Proposed Estimators

Diagram 2: The Chart below represents the MSE of Estimators for Population 2

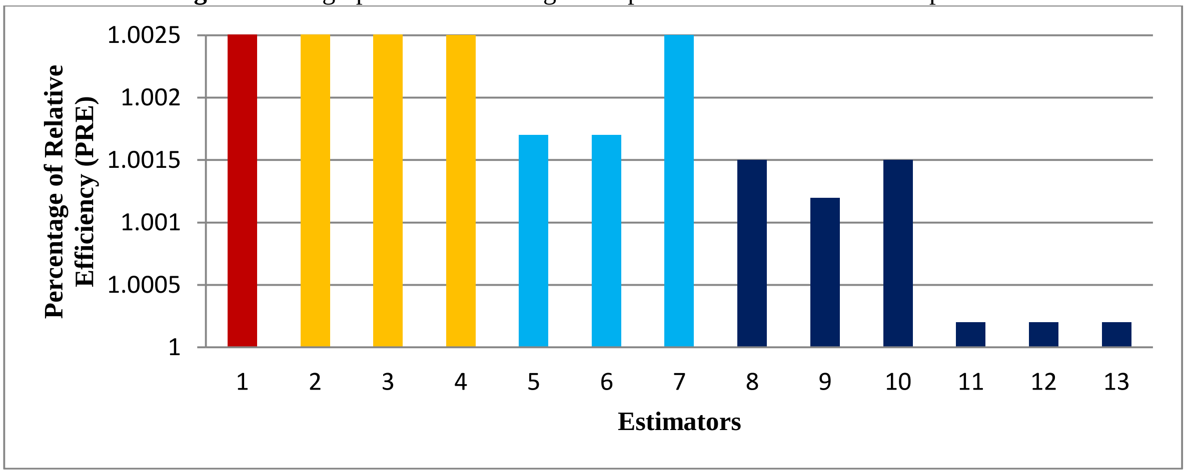


Key: •Cochran Estimator • Sadam Estimators •Muili Estimators •Jamiu Estimators •Proposed Estimators

Table 7: RE of the Existing to the Proposed Estimators For The Population 1 and 2

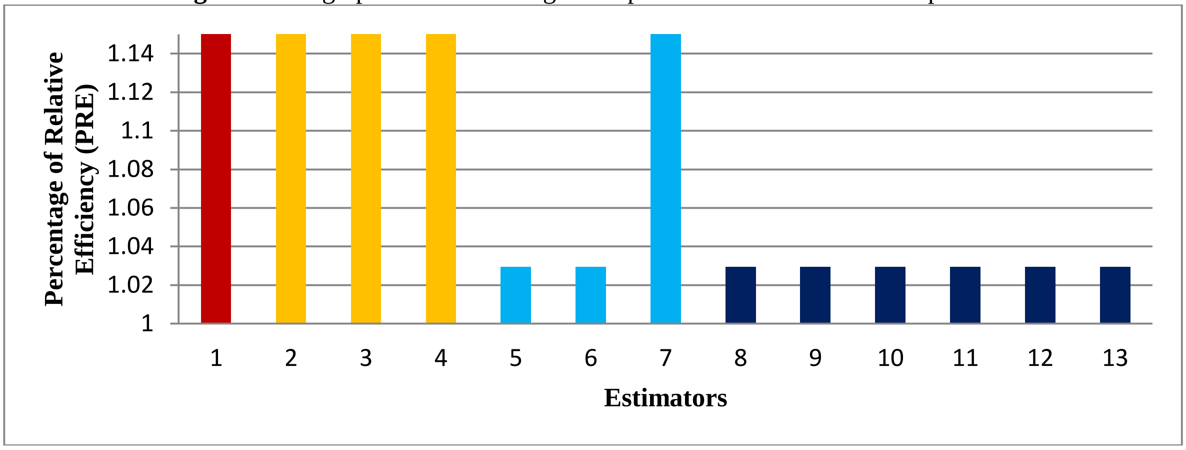
Estimators	**Population 1**				**Population 2**		
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$		$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$
Cochran (1977)							
$\hat{\bar{Y}}_r$	12.1510	12.1510	12.1510		30.1447	30.1447	30.1447
Saddam *et al* (2018)							
$\hat{\bar{Y}}_1$	9.9058	9.9058	9.9058		24.0294	24.0294	24.0294
$\hat{\bar{Y}}_2$	1.4666	1.4666	1.4666		3.9706	3.9706	3.9706
$\hat{\bar{Y}}_3$	1.2104	1.2104	1.2104		1.2941	1.2941	1.2941
Muili *et al* (2019)							
$\hat{\bar{Y}}_4$	1.0017	1.0017	1.0017		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_5$	1.0017	1.0017	1.0017		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_6$	1.1292	1.1292	1.1292		1.3594	1.3594	1.3594
Jamiu *et al* (2020)							
$\hat{\bar{Y}}_7$	1.0015	1.0015	1.0015		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_8$	1.0012	1.0012	1.0012		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_9$	1.0015	1.0015	1.0015		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_{10}$	1.0002	1.0002	1.0002		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_{11}$	1.0002	1.0002	1.0002		1.0294	1.0294	1.0294
$\hat{\bar{Y}}_{12}$	1.0002	1.0002	1.0002		1.0294	1.0294	1.0294

Figure 3: RE graphs of the Existing to Proposed Estimators For the Population 1



Key: ●Cochran Estimator ● Sadam Estimators ●Muili Estimators ●Jamiu Estimators ●Proposed Estimators

Figure 4: RE graphs of the Existing to Proposed Estimators For the Population 2



Key: ●Cochran Estimator ● Sadam Estimators ●Muili Estimators ●Jamiu Estimators ●Proposed Estimators

VII. Discussion of Results

From the above tables and graphs of the user populations, we observed that the three proposed ratio estimators have the least MSEs as compared with the MSEs for the Classical ratio estimator introduced by Cochran (1940) and the latest existing ratio estimators suggested by Saddam et al. (2018) Jamiu et al (2019) and Jamiu and Eric (2020) which indicated that the proposed estimators are more efficient than the classical and existing ratio estimators in the population 1 and 2.

From empirical study, comparisons were made simultaneously for the determinations of relative efficiency (RE). All the results are greater than 1, these steal revealed that the proposed ratio estimators are more preferable to the existing ratio estimators in the application.

VIII. Conclusion and Implication

In this paper, we have proposed three modified ratio estimators using the linear combination of parameters and auxiliary information. Practical studies revealed that the MSEs' of the proposed estimators are less than that of the prevailing estimators under the study populations. We have also observed that the proposed estimators are more productive than the existing estimators in the literature based on the RE values.

As the proposed estimators perform better than the existing estimators in the literature as their mean square errors are less than that of the existing estimators. Hence, we strongly recommend that our proposed estimators are preferred over the existing estimators in practical applications.

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APPENDIX

N	-	Population size
n	-	Sample size
$f = \frac{n}{N}$	-	Sampling fraction
Y	-	Study/Main variable
X	-	Auxiliary/Supporting variable
$\bar{Y}$	-	Mean of Y
$\bar{X}$	-	Mean of X
y, x	-	Sample totals
$\bar{y}, \bar{x}$	-	Sample means
S_y, S_x	-	Standard deviations
S_y^2, S_x^2	-	Variances
S_{xy}	-	Covariance
C_x	-	Coefficient of variation
ρ	-	Correlation coefficient
β_1	-	Population skewness
β_2	-	Population kurtosis
$\beta(.)$	-	Bias of the estimator
$\bar{Y}_i$	-	Existing modified ratio estimator of $\bar{Y}$
$\bar{Y}_{pi}$	-	Proposed modified ration estimator of $\bar{Y}$
R	-	Ratio
$MSE(.)$	-	Mean square error of the estimator
RE	-	Relative Efficiency
D_i	-	Deciles of X
Md	-	Median of X
DM	-	Deciles mean
QD	-	Quartiles deviation
TM	-	Tri-mean
HL	-	Hodges-Lehmann estimator (median)
MR	-	Mid-range
Kt	-	Kurtosis
Sk	-	Skewness
i, j	-	Subscript for the estimators