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Synthesis of Algorithms for Evaluating Parameters of Complex Maneuvering Targets

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ABSTRACT: *This paper presents the results of building a filter algorithm with the purpose of evaluating the parameters of a complex maneuvering targets, in order to realize the guidance law when taking into account the parameter of a complex maneuvering targets on the basis of the application of four-states Kalman filter. The algorithm has a simple structure, high convergence and stability. The simulation results show that the algorithm is highly reliable, easy to implement in practice, and meets the requirements of modern guiding laws to advance the efficiency of target destruction and improve the accuracy of the guidance.*

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I. INTRODUCTION

In the proportional navigation guidance (PNG) law, the missile's acceleration n_c is proportional to the line-of-sight rotation speed $\dot{\sigma}$, proportional to the miss y and inversely proportional to the square of the time to go t_{go} [3], [4]:

$$n_c = \frac{N}{t_{go}^2}[y + \dot{y}t_{go}] = NV_c\dot{\sigma} \tag{1}$$

We see that the miss component in the PNG law (1) has no parameters describing the maneuverability of the target. This does not mean that the PNG law does not hit the target, but it does not mean that the guidance law is not optimal for a maneuvering target.

If the maneuverability target is a function of time, we can calculate the miss precisely and generate a new guidance law model which is an augmented proportional navigation guidance law (APNG) [1], [2], [5], [7]. The mathematical expression then of miss contains the target maneuver component, the target's acceleration $\ddot{y}_T$.

$$n_c = \frac{N}{t_{go}^2}[y + \dot{y}t_{go} + \frac{1}{2}\ddot{y}_T t_{go}^2] = NV_c\dot{\sigma} + \frac{1}{2}N\ddot{y}_T \tag{2}$$

The expression (2) of the advanced proportional navigation guidance law consists of two components, one proportional to the line-of-sight rotation speed and the other proportional to the target's acceleration.

When the target is complex maneuvering targets, if the target's maneuverability form is known, we can construct an optimal guidance law even if the target maneuver is in complex form [10], [11], [12].

One of the solutions to advance the ability to destroy complex maneuvering targets is to improve the guidance law by adding to the expression of the guidance law the parameters of the target including target acceleration, target acceleration derivative and the target's maneuvering frequency.

Currently, real equipments can only determine the position and velocity of the target. Therefore, in order to realize modern guidance laws [6], [8], [13], [14], we need to measure or evaluate the parameters in the expression of the guidance law. In addition to the parameters like the PNG law, we need to evaluate parameters such as the target's acceleration, the change in the target's acceleration (the derivative of the target's acceleration) and the target's maneuvering frequency.

By applying Kalman filter theory, [9] has proposed a filter algorithm to evaluate the parameters of the maneuvering target. However, we only stop at evaluating the target acceleration but have not evaluated the derivative of the target acceleration.

When the target maneuver is complex form with constant maneuvering frequency, we can develop the filtering problem by expanding the state space to 4 states, in addition to evaluating the acceleration parameter of the target we also evaluate the parameter of the acceleration derivative of the target.

Therefore, on the basis of the application of Kalman filter theory, the paper proposes a method to evaluate the parameters of a complex maneuvering targets. To evaluate the acceleration derivative parameter of the target, we can build a four-state linear Kalman filter.

II. BUILDING A COMPLEX MANEUVERING TARGET PARAMETERS DETERMINATION ALGORITHM

If we had a priori information that the target maneuver was sinusoidal in nature, one would think that a better Kalman filter could be designed. To design a Kalman filter optimized to estimate the states of a weaving target, we must first express the sinusoidal target motion in some statistical fashion. First we have to transform the sinusoidal motion of the target that the Laplace transform of a sinusoidal signal is given by [1], [3], [8]:

$$£\left(sin\omega t\right)=\frac{\omega}{s^2+\omega^2} \tag{3}$$

Therefore, if we assume that the target maneuver is sinusoidal in shape and that the starting time is still uniformly distributed over the flight time. Here the input to the sinusoidal transfer function is white noise u_s with spectral density Ω_s :

$$\Omega_s=\frac{\omega^2 n_{TMAX}^2}{t_F} \tag{4}$$

Where:

ω - Maneuvering frequency of the target

n_{TMAX} - The peak of the sinusoidal maneuver

t_F - The flight time

Homing loop diagram with a sinusoidal maneuvering target is shown in figure 1. In this guidance system we measure noisy relative position y^* (due to noise v).

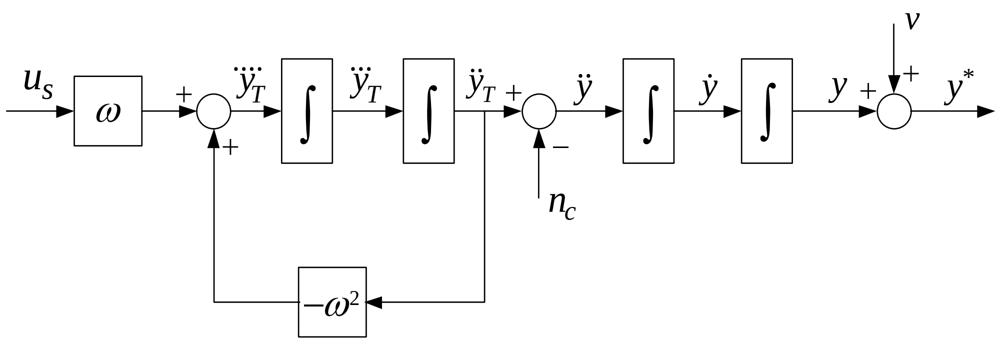

Fig. 1 Homing loop model for Kalman filter to be designed for sinusoidal target maneuver

The linear four-state Kalman filter will estimate relative position y, relative velocity $\dot{y}$, target acceleration $\ddot{y}_T$ and the speed of change of the target's acceleration i.e. the derivative of the target's acceleration $\dddot{y}_T$.

The homing loop model of fig 1 assumes that the achieved missile acceleration n_c and the target maneuver frequency ω, are both known and do not have to be estimated.

The model of fig 1 can be expressed in state space form as:

$$\begin{cases}\dot{y}=\dot{y}\\ \ddot{y}=\ddot{y}_T-n_c\\ \ddot{y}_T=\ddot{y}_T\\ \dddot{y}_T=-\omega^2\ddot{y}_T+\omega u_s\end{cases}$$

$$\begin{bmatrix}\dot{y}\\ \ddot{y}\\ \ddot{y}_T\\ \dddot{y}_T\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&-\omega^2&0\end{bmatrix}\begin{bmatrix}y\\ \dot{y}\\ \ddot{y}_T\\ \dddot{y}_T\end{bmatrix}+\begin{bmatrix}0\\-1\\0\\0\end{bmatrix}n_c+\begin{bmatrix}0\\0\\0\\ \omega u_s\end{bmatrix} \tag{5}$$

The systems dynamics matrix of the preceding equation can be written by inspection and is given by:

$$F=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&-\omega^2&0\end{bmatrix} \tag{6}$$

The fundamental matrix can be derived from the systems dynamics matrix according to:

$$\Phi(t) = \pounds^{-1}\left\{[sI - F]^{-1}\right\} \tag{7}$$

Where: I the fundamental matrix and $\pounds^{-1}$ the Laplace transform domain.
We have:

$$sI - F = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 0 & 0 & s+\omega^2 & 0 \end{bmatrix} \tag{8}$$

Therefore, the fundamental matrix in the Laplace transform domain can be expressed as:

$$\Phi(s) = (sI - F)^{-1} = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 0 & 0 & s+\omega^2 & 0 \end{bmatrix}^{-1} \tag{9}$$

From the preceding equation (9), we can see that first we must take the inverse of a four-by-four matrix *4x4* and then take its inverse Laplace transform to find the fundamental matrix in the time domain. After considerable algebra, the continuous fundamental matrix turns out to be:

$$\Phi(t) = \begin{bmatrix} 1 & t & \frac{1-\cos\omega t}{\omega^2} & \frac{\omega t - \sin\omega t}{\omega^3} \\ 0 & 1 & \frac{\sin\omega t}{\omega} & \frac{1-\cos\omega t}{\omega^2} \\ 0 & 0 & \cos\omega t & \frac{\sin\omega t}{\omega} \\ 0 & 0 & -\omega\sin\omega t & \cos\omega t \end{bmatrix} \tag{10}$$

By replacing time t with the sampling time T_s, we obtain the discrete form of the fundamental matrix Φ_k as:

$$\Phi_k = \begin{bmatrix} 1 & T_s & \frac{1-\cos x}{\omega^2} & \frac{x - \sin x}{\omega^3} \\ 0 & 1 & \frac{\sin x}{\omega} & \frac{1-\cos x}{\omega^2} \\ 0 & 0 & \cos x & \frac{\sin x}{\omega} \\ 0 & 0 & -\omega\sin x & \cos x \end{bmatrix} \tag{11}$$

With $x = \omega T_s$

The discrete measurement equation can be written by inspection of fig 1 as:

$$y_k^* = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_k \\ \dot{y}_k \\ \ddot{y}_{T_k} \\ \dddot{y}_{T_k} \end{bmatrix} + v_k \tag{12}$$

Which means that the discrete measurement matrix H_k is given by:

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

The continuous control matrix $G(t)$ can be written by inspection of the original state space equation:

$$G(t) = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

The continuous control matrix G_k can be written by inspection of the original state space equation $G(t)$ as:

$$G_k = \int_0^{T_s} \Phi(t) G(t) dt = \begin{bmatrix} -0,5\,T_s^2 \\ -T_s \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

After some algebra the discrete control matrix $Q(t)$ becomes:

$$Q(t) = E[w\ w^T\ ] = E\left\{ \begin{bmatrix} 0 & 0 & 0 & \omega u_s \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega u_s \end{bmatrix} \right\} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega^2 \Omega_s \end{bmatrix} \tag{16}$$

Finally, the continuous process noise matrix Q_k can be written from the system state space equation $Q(t)$ by inspection as:

$$Q_k = \int_0^{T_s} \Phi(t) Q(t) \Phi^T(t) dt = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} \end{bmatrix} \tag{17}$$

Where:

$$\begin{aligned}
&Q_{11} = \frac{\Omega_s}{\omega^5}\left[0.333x^3 - 2\sin x + 2x\cos x + 0.5x - 0.25\sin 2x\right];\ Q_{12} = \frac{\Omega_s}{\omega^4}\left[0.5x^2 - x\sin x + 0.5\sin^2 x\right] \\
&Q_{13} = \frac{\Omega_s}{\omega^3}\left[\sin x - x\cos x - 0.5x + 0.25\sin 2x\right];\ Q_{14} = \frac{\Omega_s}{\omega^2}\left[\cos x + x\sin x - 0.5\sin^2 x - 1\right] \\
&Q_{22} = \frac{\Omega_s}{\omega^3}\left[1.5x - 2\sin x + 0.25\sin 2x\right];\ Q_{23} = \frac{\Omega_s}{\omega^2}\left[1 - \cos x - 0.5\sin^2 x\right] \\
&Q_{24} = \frac{\Omega_s}{\omega}\left[\sin x - 0.5x - 0.25\sin 2x\right];\ Q_{33} = \frac{\Omega_s}{\omega}\left[0.5x - 0.25\sin 2x\right] \\
&Q_{34} = 0.5\Omega_s \sin^2 x;\ Q_{44} = \omega\Omega_s\left[0.5x + 0.25\sin 2x\right]
\end{aligned} \tag{18}$$

Where:

$$\Omega_s = \frac{\omega^2 \ddot{y}_T^2}{t_F} \qquad x = \omega T_s \tag{19}$$

Recall that the discrete Kalman filtering equation is given by:

$$\hat{x}_k = \Phi_k \hat{x}_{k-1} + G_k u_{k-1} + K_k (y_k^* - H\Phi_k \hat{x}_{k-1} - HG_k u_{k-1}) \tag{20}$$

Substitution of the appropriate matrices into the preceding matrix difference equation yields (20):

$$\begin{aligned}
\begin{bmatrix} \hat{y}_k \\ \hat{\dot{y}}_k \\ \hat{\ddot{y}}_{T_k} \\ \hat{\dddot{y}}_{T_k} \end{bmatrix} &= \begin{bmatrix} 1 & T_s & \frac{1-\cos x}{\omega^2} & \frac{x - \sin x}{\omega^3} \\ 0 & 1 & \frac{\sin x}{\omega} & \frac{1-\cos x}{\omega^2} \\ 0 & 0 & \cos x & \frac{\sin x}{\omega} \\ 0 & 0 & -\omega \sin x & \cos x \end{bmatrix} \begin{bmatrix} \hat{y}_{k-1} \\ \hat{\dot{y}}_{k-1} \\ \hat{\ddot{y}}_{T_{k-1}} \\ \hat{\dddot{y}}_{T_{k-1}} \end{bmatrix} + \begin{bmatrix} -0.5T_s^2 \\ -T_s \\ 0 \\ 0 \end{bmatrix} n_{c_{k-1}} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} \\
&\times \left[y_k^* - \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & T_s & \frac{1-\cos x}{\omega^2} & \frac{x - \sin x}{\omega^3} \\ 0 & 1 & \frac{\sin x}{\omega} & \frac{1-\cos x}{\omega^2} \\ 0 & 0 & \cos x & \frac{\sin x}{\omega} \\ 0 & 0 & -\omega \sin x & \cos x \end{bmatrix} \begin{bmatrix} \hat{y}_{k-1} \\ \hat{\dot{y}}_{k-1} \\ \hat{\ddot{y}}_{T_{k-1}} \\ \hat{\dddot{y}}_{T_{k-1}} \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -0.5T_s^2 \\ -T_s \\ 0 \\ 0 \end{bmatrix} n_{c_{k-1}} \right]
\end{aligned} \tag{21}$$

We can multiply out the terms of the preceding matrix equation to yield the Kalman filter scalar equations:

$$\begin{aligned}
RES_k &= y_k^* - \hat{y}_{k-1} - T_s\hat{\dot{y}}_{k-1} - \frac{1-cosx}{\omega^2}\hat{\ddot{y}}_{T_{k-1}} - \frac{x-sinx}{\omega^3}\hat{\dddot{y}}_{T_{k-1}} + 0.5T_s^2 n_{c_{k-1}} \\
\hat{y}_k &= \hat{y}_{k-1} + T_s\hat{\dot{y}}_{k-1} + \frac{1-cosx}{\omega^2}\hat{\ddot{y}}_{T_{k-1}} + \frac{x-sinx}{\omega^3}\hat{\dddot{y}}_{T_{k-1}} - 0.5T_s^2 n_{c_{k-1}} + K_1 RES_k \\
\hat{\dot{y}}_k &= \hat{\dot{y}}_{k-1} + \frac{sinx}{\omega}\hat{\ddot{y}}_{T_{k-1}} + \frac{1-cosx}{\omega^2}\hat{\dddot{y}}_{T_{k-1}} - T_s n_{c_{k-1}} + K_2 RES_k \\
\hat{\ddot{y}}_{T_k} &= cosx(\hat{\ddot{y}}_{T_{k-1}}) + \frac{sinx}{\omega}\hat{\dddot{y}}_{T_{k-1}} + K_3 RES_k \\
\hat{\dddot{y}}_{T_k} &= -\omega sinx(\hat{\ddot{y}}_{T_{k-1}}) + cosx(\hat{\dddot{y}}_{T_{k-1}}) + K_4 RES_k
\end{aligned} \tag{22}$$

With $x = \omega T_s$

The gain K_k of the filter is determined from solving the Ricatti matrix equation.

$$K_k = M_k H_k^T \left(H_k M_k H_k^T + R_k\right)^{-1} \tag{23}$$

$$P_k = \left(I - K_k H_k\right) M_k$$

$$M_k = \Phi_k P_{k-1} \Phi_k^T + Q_k \tag{24}$$

III. SIMULATION RESULTS AND ANALYSIS

Algorithm for the four-state linear Kalman filter were programmed as part of the homing loop and linearized missile-target engagement. The simulation has a single time constant representation of the flight control system with a 3g maneuvering target and a maneuvering frequency of 2 rad/s. Nominally, there is 1 mr of measurement noise on the line of sight angle and the closing velocity is $2700\ (m/s)$ to reflect a ballistic target engagement. The guidance law options for this filter are either proportional navigation, augmented proportional navigation, optimal guidance.

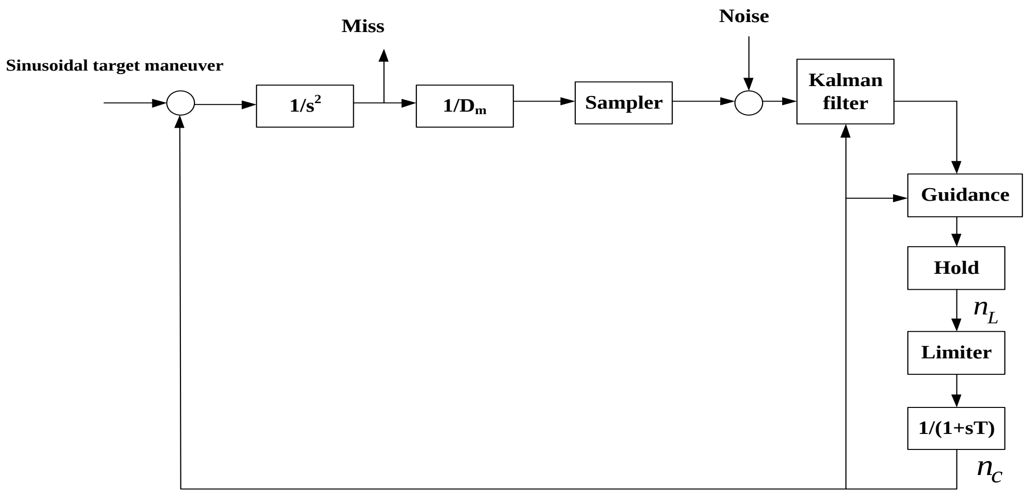

Fig. 2 Guidance system model for miss distance analysis

System input parameter:

Target acceleration level: $n_T = 3g\ (m/s^2)$

Missile velocity: $V_M = 900\ (m/s)$

Measurement noise (seeker): $\sigma_{Noise} = 0,1\ (mr)$

Closing velocity: $V_c = 2700\ (m/s)$

Autopilot time constant: $T = 0,5\ (s)$

Target maneuver frequency: $\omega = 2\ (rad/s)$

Flight time: $t_F = 10\ (s)$

Sampling time: $T_s = 0,01\ (s)$

Guidance law [3], [7], [8]:

$$n_L = \frac{N}{t_{go}^2}[y + \dot{y}t_{go} + \frac{1 - cos\omega t_{go}}{\omega^2}\ddot{y}_T + \frac{\omega t_{go} - sin\omega t_{go}}{\omega^3}\dddot{y}_T - n_C T^2(e^{-x} + x - 1)] \quad (25)$$

With $x = \frac{t_{go}}{T}$

t_{go} - The time to go

T - Autopilot time constant

The scaling coefficient of the guidance law [3], [7], [8] :

$$N = \frac{6x^2(e^{-x} - 1 + x)}{2x^3 + 3 + 6x - 6x^2 - 12xe^{-x} - 3e^{-2x}} \quad (26)$$

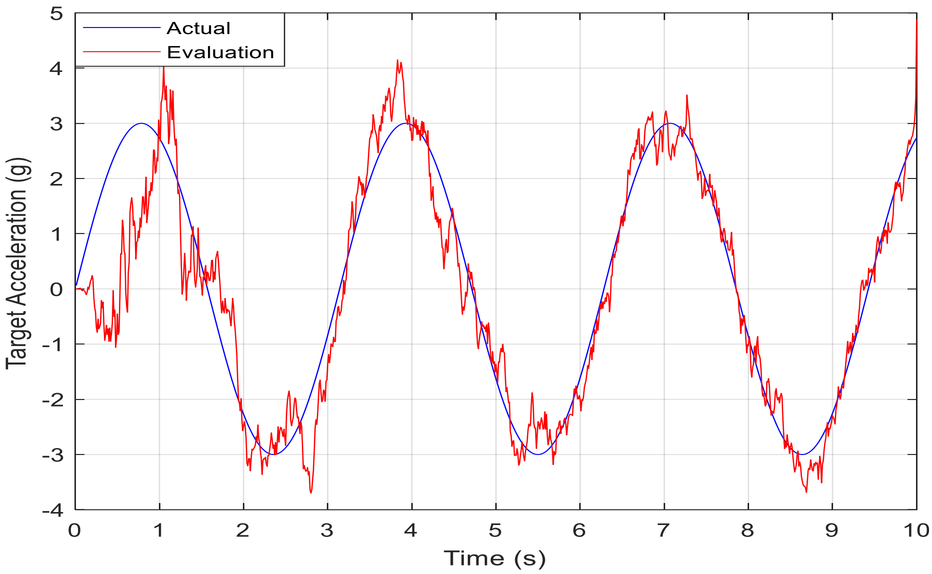

Fig. 3 Target acceleration

When the target's maneuvering frequency information is wrong. The four-state linear Kalman filter works well if the target maneuver frequency is known. In figure 3-4, it can be seen that we have a very good evaluate of the target’s acceleration and the target’s acceleration derivative after a short time interval.

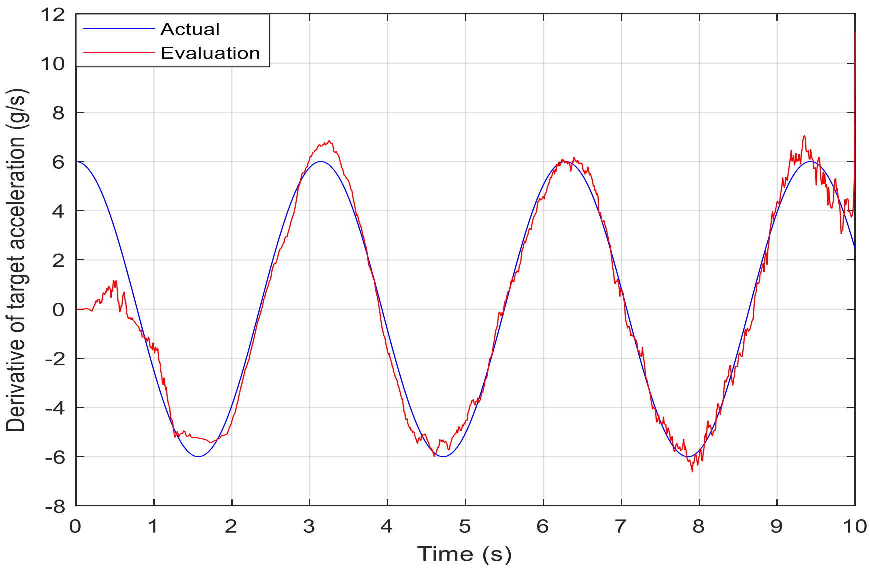

Fig. 4 Derivative of target acceleration

Kalman filter estimates of target acceleration and Derivative of target acceleration improve significantly (fig 3-4). In fact, we can see that the estimates of these states are nearly perfect in the low-noise environment. Small evaluation error (fig 5-6).

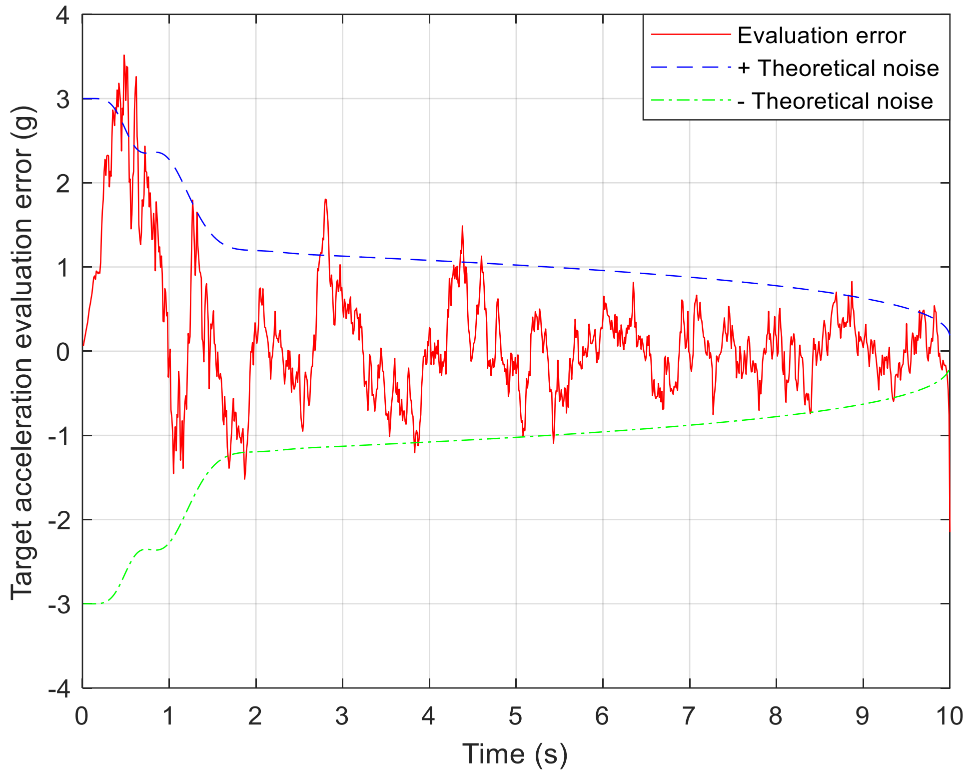

Fig. 5 Target acceleration evaluation error

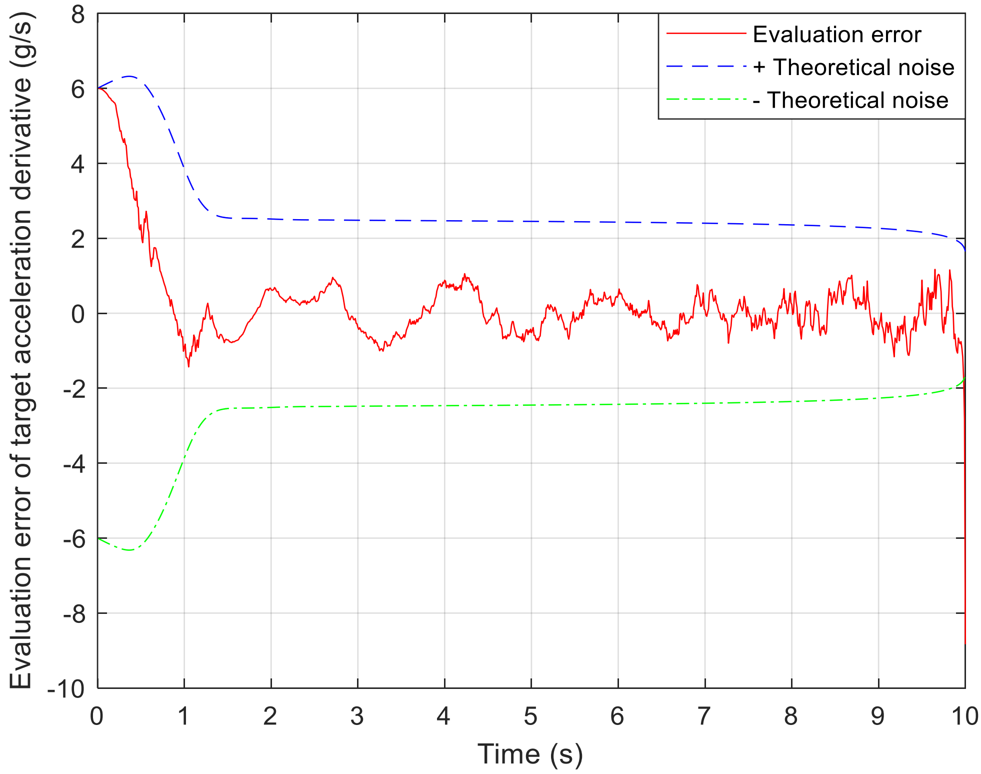

Fig. 6 Evaluation error of target acceleration derivative

If the target maneuver frequency information is false, the evaluation of the four-state linear Kalman filter of the target's acceleration and the derivative of the target's acceleration will indeed deteriorate.

Suppose, when the targe's maneuvering frequency is in real 2 rad/s, but the four-state linear Kalman filter assumes that the target's maneuverimg frequency is 1 rad/s. In this case, the filter is underestimating the maneuvering frequency of the target relative to the real.

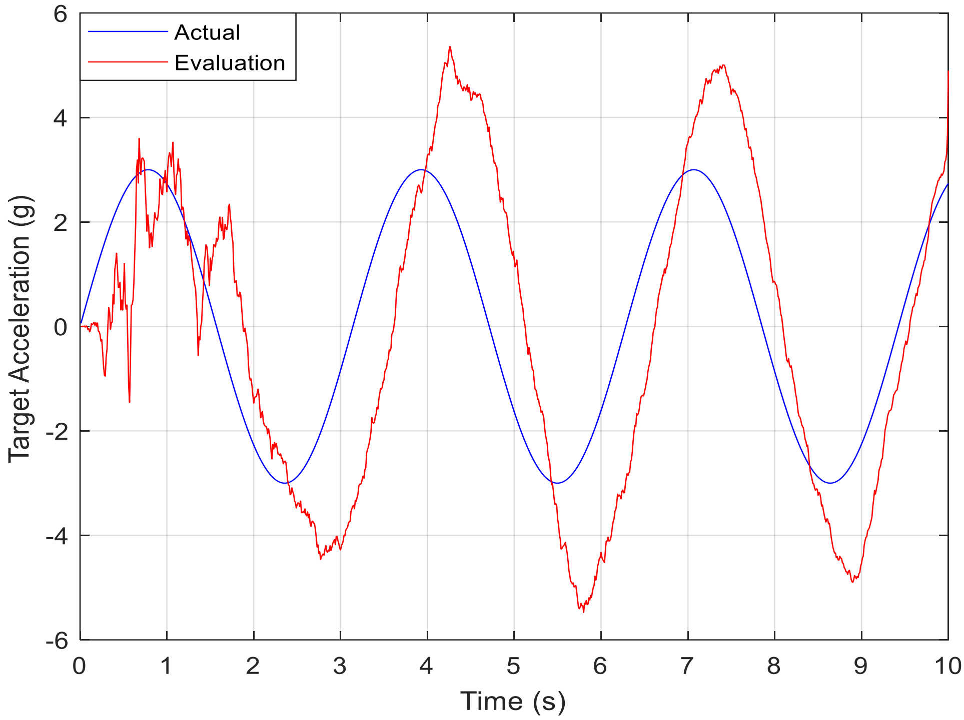

Fig. 7 Acceleration of the target when the filter underestimating the target's maneuvering frequency

When the target's maneuvering frequency is underestimated (figure 7), the evaluation of the four-state linear Kalman filter of the target's acceleration has a significant error with the actual target's acceleration.

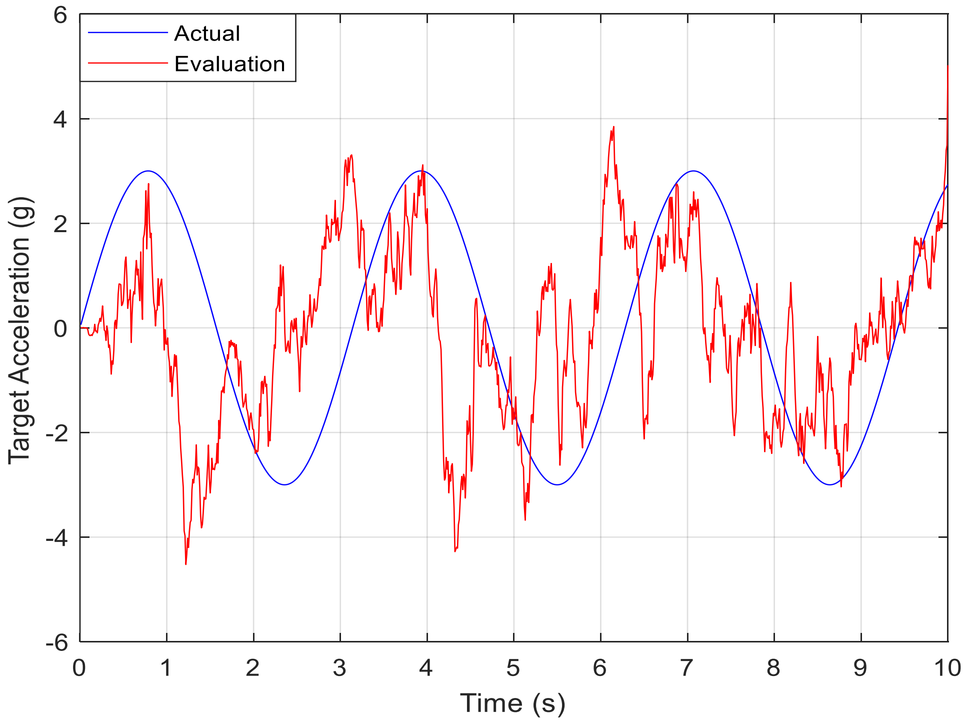

Fig. 8 Target's acceleration when overestimating the target's maneuvering frequency

When the four-state linear Kalman filter assumes that the target maneuver frequency is 4 rad/s. In this case, the filter is overestimating the target's maneuvering frequency relative to the real. At this point, it is almost impossible to evaluate the target's acceleration.

IV. CONCLUSIONS

The four -state Kalman filter is capable of estimating parameters such as relative position, relative velocity, acceleration of the target and derivative of target acceleration. Therefore, the corresponding guidance laws can be used in combination with three-state Kalman filter to create a missile control loop which are proportional navigation, augmented proportional navigation, optimal guidance and other modern guidance laws, but the target maneuvering frequency must be known in advance. The four-state linear Kalman filter works well if the target maneuver frequency is known.

To evaluate the target acceleration derivative without knowing the target's maneuver frequency in advance, a five-state Kalman filter can be developed. This algorithm will be presented in the next paper.

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