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Advancements in Forecasting Model via Iterative Fuzzy Relations and Label Propagation Algorithm

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***ABSTRACT*: *Many forecasting models relying on fuzzy time series (FTS) employ fixed interval lengths. The drawback of this static interval length is that historical data are crudely categorized into intervals, even when their variance is not significant. This paper introduces a novel forecasting model that combines the iterative fuzzy relation and Label Propagation Algorithm (LPA). Initially, the authors utilize the label propagation algorithm to partition historical data into clusters and adjust them into intervals of varying lengths. Subsequently, based on these new intervals, the proposed method is used to fuzzify all historical data, identify iterative fuzzy relations, construct Iterative Fuzzy RelationGroups(IFRGs), and calculate forecasted output through an enhanced defuzzification phase. To assess the performance of theproposed model, datasets of "killed" in car road accidents in Belgium and historical enrolment data from the University of Alabama are employed. Comparing with established methods in literature, particularly first-order and high-order FTS forecasting model, reveals that the suggested FTSFM achieves higher forecasting accuracy and is better suited to handle incomplete data.*

Keywords:*Fuzzy time series, forecasting, Iterative fuzzy relation, LPA clustering, enrolments,car road accidents*

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I. INTRODUCTION

Throughout history, the prediction of future time series events has captivated attention across various domains. Many forecasting models have been utilized to address diverse issues, including enrolment forecasting [1-6], crop prediction [7, 8]and stock market trends [9, 10], among others. Traditional forecasting methods encounter challenges when historical data are expressed through linguistic values. For instance, Song and Chissom[1, 2] introduced both time-invariant FTS and time-variant FTS models to forecast University of Alabama enrolments, yet these methods suffered from significant computational burdens, particularly with larger fuzzy relationship matrices. In response to this challenge, Chen [3] proposed the first-order FTS model, employing fuzzy relationship groups to streamline computational complexity. This model replaced max-min composition operations with simpler arithmetic calculations, thereby enhancing forecasting accuracy. Subsequently, there have been extensive studies on FTS aimed at refining forecasting precision across numerous applications. Huarng [5] devised a method, based on Chen's work [3], incorporating a heuristic function to improve predictions for University of Alabama enrolments. Expanding on his prior research, Chen further introduced several forecast models based on high-order fuzzy time series to address enrolment forecasting issues [6],[11]. Yu introduced refinement relation models [4] and weighting schemes [9], targeting enhanced forecasting accuracy for both stock indices and enrolments.

Other innovative approaches emerged in the literature to advance forecasting methods. Ref. [12] proposed a new forecast model using trapezoidal fuzzy numbers. Ref.[13] introduces a forecasting mode based on fuzzy logical relationships and genetic algorithms for forecasting stock market. Additionally, alternative techniques for identifying best intervals and interval lengths, such as automatic clustering techniques such as K-means clustering [14], automatic clustering techniques [15, 16], optimization techniques [17]. Huarng [18] demonstrated the impact of interval length variations on forecast accuracy. He improved previous methods by employing ratio-based lengths, resulting in enhanced forecasting accuracy.

Recent studies, such as those in [19-22] introduced hybrid forecasting models that combine particle swarm optimization with FTS, aiming to determine optimal interval lengths and adjust intervals accordingly.

This study introduces a hybrid forecasting model that merges IFRGwith the LPA algorithm. Experimental findings indicate that the proposed method achieves a superior average forecasting accuracy in comparison to current methods. Furthermore, empirical results demonstrate that the high-order FTS model outperforms the first-order FTS model, exhibiting a lower forecast error.

The rest of this paper is organized as follows. In Section 2, we provide a brief review of FTS model and LPA algorithm. In Section 3, we present a hybrid model for forecasting problems based on LPA algorithm and the IFRGs through the experiments of forecasting enrolment of the university of Alabama. Then, the experimental results are shown and analysed in Section 4. Finally, conclusions are presented in Section 5.

II. FUZZY TIME SERIES AND LPA ALGORITHM

2. 1. Definitions of fuzzy time series

Song and Chissom introduced the concepts of fuzzy time series [1, 2] in 1993, wherein the fuzzy time series values are expressed using fuzzy set. Let $U=\{u_1, u_2, \ldots, u_n\}$ be an universe of discourse; a fuzzy set A of U is defined as:

$$A_i = \frac{\mu_{A_i}(u_1)}{u_1} +, \ldots, + \frac{\mu_{A_i}(u_2)}{u_2}, \ldots, + \frac{\mu_{A_i}(u_n)}{u_n}$$

where $\mu_{A_i}: U \rightarrow [0,1]$ is the membership function of A_i , $\mu_{A_i}(u_i)$ indicates the degree of membership of u_i in the fuzzy set A, $\mu_{A_i}(u_i) \ \epsilon \ [0, 1]$ and $1 \leq i \leq n$. General definitions of fuzzy time series are given as follows:

Definition 1: Fuzzy time series [1, 2]

Let $Y(t)(t = 0, 1, 2, \ldots)$, a subset of real numbers, be the *UD* on which the fuzzy sets $f_i(t)$ $(i = 1,2 \ldots)$ are defined. $F(t)$ is a collection of $f_1(t), f_2(t), \ldots f_i(t), \ldots$, then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2: Fuzzy logical relationship– FLG [1-4]

The relationship between $F(t)$ and $F(t-1)$ can be denoted by $F(t-1) \rightarrow F(t)$. Let A_i = $F(t)$ and A_j = $F(t-1)$, the relationship between $F(t)$ and $F(t-1)$ is denoted by fuzzy logical relationship $A_i \rightarrow A_j$; where A_i and A_j refer to the current state and the next state of fuzzy relations.

Definition 3: Iterative fuzzy relation Group **(**IFRG**)**[9]

Similar fuzzy relationships sharing the same fuzzy set in the current state can be organized into a collective fuzzy relationship group. Assuming the existence of relationships as follows:

$$A_i \rightarrow A_k;\ A_i \rightarrow A_m;\ A_i \rightarrow A_k;\ \ldots\ldots$$

There for, based on [9], these fuzzy relationships can be grouped into the same IFRG as: $A_i \rightarrow A_k, A_m, A_k \ldots$

2.2 *Label Propagation Algorithm (LPA)*

LPA is a semi-supervised machine learning algorithm used primarily for clustering tasks in graphs or networks was introduced by Raghavan, Albert, and Kumara [23] in 2007. It operates on the principle of propagating labels among connected nodes within a graph. To cluster a dataset of numerical values into groups. This algorithm assumes an input dataset X_i (i = 1, n) with n observations and provides the data belonging to each cluster that LPA has classified.

The algorithm is composed of the following steps

Input: Data set of X_i (i = 1 to n)

Output: Clusters obtained by the LPA algorithm

Begin

Step 1: Create a graph from the input data
- ✓ *Construct a graph G from dataset* X_i
- ✓ *Each data point in* X_i *corresponds to a node in graph G*
- ✓ *Define relationships between nodes based on distance using a distance matrix*

Step 2: Initialize initial labels for each node
- ✓ *Assign each node a unique initial label*

Step 3: Repeat until convergence (*reach the maximum number of iterations*):

For each node in the graph:
- ✓ *Gather labels from neighboring nodes*
- ✓ *Update the node's label to the most frequent label among its neighbors*
- ✓ *If multiple labels have the same frequency, randomly choose one label*

Step 4: Create clusters from the final labels of each node
- ✓ *Group data points based on the final labels obtained after convergence*

Step 5: Output
- ✓ *Display the clusters obtained by the LPA algorithm*

End

III. FORECASTING MODEL BASE ON LPAALGORITHM AND IFRG

In this section, the fuzzy time series forecasting model is built upon iterative fuzzy relation groups and the LPA algorithm. During the first phase, the study utilizes LPA clustering to efficiently categorize the collected data into discrete clusters. Subsequently, these clusters undergo adjustments to generate intervals of varying lengths, detailed in Subsection 3.1. After establishing these intervals, the historical data is converted into fuzzy sets, forming iterative fuzzy relation groups essential for computing the output forecasting results, as demonstrated in Subsection 3.2. To assess the effectiveness of the proposed model, historical enrollment data [3] is employed to exemplify the first-order FTS forecasting process.

3.1 Creating intervals based on the LPA algorithm

The algorithm composed of four steps is introduced step-by-step with the same dataset [3].

Step 1: Apply the LPA algorithm to partition the time series data into$q$ clusters and sort the data in clusters in an ascending sequence. In this paper, we set q=7 clusters, the clustering results are as follows:
{13055,13563,13867}; {14696}; {15311, 15145, 15163};{15433};{15460, 15497, 15603, 15861, 15984, 16388}; {16807, 16859, 16919}; {18150, 18876, 18970, 19328, 19337}.

Step 2: Calculate the cluster center

In this step, we use clustering techniques [17]to generate cluster center(Center_k) from clusters according to Eq. (1)as follows:

$$\mathbf{Center_k}=\frac{\sum_{i=1}^{n} d_i}{n} \tag{1}$$

where $d_i$is a datum in cluster_k, n denotes the amount of data in cluster_k and $1 \le k \le q$.

Step 3: Adjust the clusters into intervals according to the follow rules.

Assume that $Center_k$ and $\text{Center}_{k+1}$are adjacent cluster centers, then the upper bound $Cluster_UB_k$of $cluster_k$and the lower bound $clusterLB_{k+1}$of $cluster_{k+1}$can be calculated as follows:

$$Cluster\_UB_k=\frac{Center_k + Center_{k+1}}{2} \tag{2}$$

$$Cluster\_LB_{k+1}= Cluster\_UB_k \tag{3}$$

where k = 1,.., q-1. Since there are no clusters preceding the first one and no clusters following the last one, the lower bound, $Cluster\_LB_1$, of the initial cluster, and the upper bound, $Cluster\_UB_q$, of the final cluster can be determined in the following manne:

$$Cluster\_LB_1=\boldsymbol{Center_1} - (\boldsymbol{Center_1} - Cluster\_UB_1) \tag{4}$$

$$Cluster\_UB_q=\boldsymbol{Center_q} + (\boldsymbol{Center_q} - Cluster\_LB_q) \tag{5}$$

Step 4: Let each cluster Cluster_k form an interval $interval_k$, which means that the upper bound $Cluster\_UB_k$ and the lower bound $Cluster\_LB_k$ of the cluster $cluster_k$ are also the upper bound $interval\_UBound_k$ and the lower bound $interval_LBound_k$of the interval $interval_k$, respectively. Calculate the middle value Mid_value_k of the interval $interval_k$as follows:

$$Mid\_value_k=\frac{interval\_LB_k + interval\_UB_k}{2} \tag{6}$$

where $interval_LBound_k$and $interval\_UBound_k$ are the lower bound and the upper bound of the interval$interval_k$, respectively, with k = 1,..,q.

3.2. Forecasting model based on the first – order FTS

In this section, we present a hybrid method for forecasting enrolments based on the LPA algorithm and iterative fuzzy relation groups. The proposed method is now presented as follows:

Step 1: Partition the universe of discourse$U$ into intervals

Applying the LPA clustering technique, we obtained 7 intervals as shown in Table 1, and the number of data points distributed across each interval is listed in Figure 1 below.

Table 1: **The intervals and midpoint of each interval**

No	Intervals	Mid_interval
1	[12894.5, 14095.5]	13495
2	[14095.5, 14951.2]	14523.35
3	[14951.2, 15319.7]	15135.45
4	[15319.7,15615.9]	15467.8
5	[15615.9, 16330.3]	15973.1
6	[16330.3,17896.9]	17113.6
7	[17896.9,19967.5]	18932.2

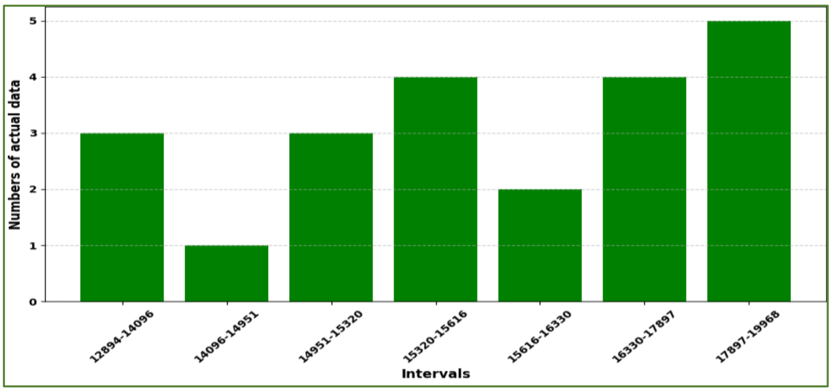

Fig.1 Distribution of Enrollment Data into Intervals

Step 2:Define the fuzzy setsA_i .

Every interval outlined in Step 1 signifies a linguistic variable associated with enrollment. With 7 intervals, there exist 7 corresponding linguistic terms. Each of these linguistic terms defines a fuzzy set A_i $(1 \le i \le 7)$and its definition is described in Eq. (7)

$$\begin{aligned} A_1 &= \frac{1}{u_1} + \frac{0.5}{u_2} + \cdots + \frac{0}{u_7} \\ A_2 &= \frac{0.5}{u_1} + \frac{1}{u_2} + \cdots + \frac{0}{u_7} \\ &\text{------------------------------} \\ A_7 &= \frac{0}{u_1} + \cdots + \frac{0.5}{u_6} + \frac{1}{u_7} \end{aligned} \tag{7}$$

Here, the symbol "+" indicates the operation of union.The value 0, 0.5 and 1 indicate the grade of membership of u_j in the fuzzy set A_i .

Step 3: Fuzzy all timeserial data

In order to fuzzify all historical data, it's necessary to assign a corresponding linguistic value to each interval first. The simplest way is to assign the linguistic value with respect to the corresponding fuzzy set that each interval belongs to with the highest membership degree. For example, the historical enrolment of year 1972 is 13563, and it belongs to interval u_1 because 13563 is withinu_1= [12894.5, 14095.5]. So, we then assign the linguistic value or the fuzzy set A_2corresponding to interval u_2 to it. In the same way, we can complete fuzzified results of the enrolments are listed in Table 2.

Table 2: **Fuzzified historical enrolment data of the University of Alabama**

Year	Actual data	Fuzzy sets
1971	13055	A_1
1972	13563	A_1
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1991	19337	A_7
1992	18876	A_7

Step 4: Create all m – order fuzzy relationships ($m \ge 1$).

Based on Definition 2 and 3. To establish a m-order fuzzy relationship, we should find out anyrelationship which has the $F(t-m), F(t-m+1), \ldots, F(t-1) \to F(t)$,where $F(t-m), F(t-m+1), \ldots, F(t-1)$ and $F(t)$ are called the current state and the next state, respectively. Then a m - order fuzzy relationship in the training phase is got by replacing the corresponding linguistic values. For example, supposed $m = 1$ from Table 2, a fuzzy relation $A_1 \to A_2$ is got as $F(1971 \to F(1972)$. So on, we get the first-order fuzzy relationships are shown in Table 3, where there are 22 relations; the first 21 relations are called the trained patterns, and the last one is called the untrained pattern (in the testing phase). For the untrained pattern, relation 22 has the fuzzy relation$A_7 \to$ # as it is created by the relation $F(1992) \to F(1993)$, since the linguistic value of $F(1993)$is unknown within the historical data, and this unknown next state is denoted by the symbol '#'.

Table 3: **The first-order fuzzy logical relationships**

No	Years_status	Fuzzy logical relations
1	1971→1972	$A_1 \to A_1$
2	1972 → 1973	$A_1 \to A_1$

	1973 → 1974	$A_1 \rightarrow A_2$
--	---------	--------
20	1990 → 1991	$A_7 \rightarrow A_7$
21	1981 → 1982	$A_7 \rightarrow A_7$

Step 5: Establish all fuzzy logical relationship groups

In previous studies [1-3], [11], [13] the repeated FLRs were simply ignored when fuzzy relationships were established. But, according to the Definition 3, the iterative fuzzy relations can be used to indicate how the FLR may appear in the future. Therefor, based on the information provided in Table 3, we can establishall the IFRGs are listed in Table 4.

Table 4: **Completed all first-order iterative fuzzy relation groups**

No group	IFRGs
1	$A_1 \rightarrow A_1$
2	$A_1 \rightarrow (2A_1)$
3	$A_1 \rightarrow (2A_1), A_2$
--	-----
20	$A_7 \rightarrow (3A_7)$
21	$A_7 \rightarrow (4A_7)$

Step 6: Defuzzify and calculate the forecasting values

To calculate the forecast output for allIFRGs, we use [21] for the trained patterns in the training phase.
For the training phase, we can compute all forecasted values for recurrence fuzzy relationship groups based on the global information of fuzzy relationships with the local information latest fuzzy set appear in current state (ILF)according to Eq.(8).

$$\text{Forecasted\_value} = w_1 * \textbf{\textit{Global\_}inf} + w_2 * \textbf{\textit{Local\_inf}} \quad (8)$$

where,the ***Global_inf*** efers to the comprehensive information derived from the fuzzy groups established in Step 5, as indicated by equation (9):

$$\boldsymbol{Global\_inf} = \frac{m_{t1} + m_{t2} + \cdots + m_{tp}}{p} \quad (9)$$

Here, $m_{k1}, m_{k2}, \dots, m_{kp}$ are the midpoints value of intervals $u_1, u_{2,} \dots, u_p$ with respect to p linguistic values existing in the next states.
The ***Local_inf***represents specific information obtained through the ILF scheme. This scheme is an estimation method based on the subsequent state and the most recent past within the current state, calculated in accordance with equation (10).

$$\boldsymbol{Local_{inf}} = Lv_{tk} + \frac{Uv_{tk} - Lv_{tk}}{2} * \frac{m_{tk} - m_{t-1}}{m_{tk} + m_{t-1}} \quad (10)$$

Here, m_{t-1} and m_{tk} are midpoints of the fuzzy intervals u_{t-1} and u_{tk} with respective to A_{t-1} and A_{tk}; Lv_{tk}, Uv_{tk} denote the lower bound and upper bound of interval u_{tk} , t is forecasting time
Based on the aforementioned forecast rule, we complete forecasted results for enrolments from 1971 to 1992 based on first-order fuzzy time series model with 7 intervals are listed in Table 5.

Table 5: **The complete forecasted results based on the first–order FLRwith 7 intervals**

Year	Actual data	Fuzzy set	Forecasted value
1972	13563	A_1	13194.5
1973	13867	A_1	13194.5
1974	14696	A_2	1406.09
----	-----	----	-----
1991	19337	A_7	18414.6
1992	18876	A_7	18414.6

To estimate the forecasting accuracy, theRoot Mean Square Error (RMSE) used as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=m}^{n}(F_i - R_i)^2} \quad (10)$$

Where, R_i denotes actual data at year *i*, F_i is forecasted value at year *i*, *n* is number of the forecasted data, $m$is order of the fuzzy logical relationships.

IV. EXPERIMENTAL RESULTS

In this paper, we apply the proposed method to forecast the enrolments of University of Alabama with the whole historical data[3]**Error! Reference source not found.**, the period from 1971 to 1992 and we also apply the proposed method to forecast the average rice production (thousand ton/ year) of Viet Nam between 1990-2010 in which it taken from https://www.gso.gov.vn/default.aspx?tabid=717.

4.1. Experimental results forforcasting enrolments

The University of Alabama's real enrollment [3] serve as the basis for a comparative analysis during the training. To assess the forecasting efficacy, the suggested model is pitted against established models across different orders and intervals. The projected accuracy of the proposed approach is gauged by evaluating the RMSE value in Equation (10).

In order to verify the forecasting effectiveness of the proposed model for the first – order FRLGsunder different number of intervals, five FTS models in the SCI model Error! Reference source not found., the C96 model [3], the H01 model [4], model Error! Reference source not found. and model in work [24] are examined and compared. A comparison of the forecasting results among these models is shown in Table 6. It is obvious that the proposed model getsthe smallestRMSE value of 439.6among all the compared models with different number of intervals. The major difference between thecompared modelsand ourmodel is thedefuzzification forecasting rules and interval technique used.

Table 6: **A comparison of the forecasted results for the first-order IFRGs with 7 intervals**

Year	Actual data	SCI [1]	C96 [3]	H01 [5]	Model [10]	Model [24]	Our model
1971	13055	---	---	---	---	---	---
1972	13563	14000	14000	14000	13680.75	14537	13512
-----	-----	----	-----	-----	----	----	----
1991	19337	19000	19000	19500	18972.15	19217	19182
1992	18876	19000	19000	19149	18972.15	19217	19182
MSE		**650.4**	**638.36**	**476.03**	**431.64**	**512.18**	**368.2**

To evaluate the forecasting performance of the high-order fuzzy time series, four existing forecasting models—specifically, the C02 [6], CC06b [11], HPSO [25], and AFPSO [21] models—were used for comparison with our proposed model. The comparative forecasting results are detailed in Table 7, where each forecasting model utilizes seven intervals. More visually, the comparative RMSE values for each order among the forecasting models are also displayed in Figure 2. Analysis of Table 7 reveals the superior precision of the proposed model over the other four models across all parameters, demonstrating both the best and average fitted accuracies among the five models. Notably, the proposed method consistently yields the lowest RMSE values across various orders of fuzzy time series, recording values of 162.9, 135.2, 119.4, 111.2, 104.8, 96.1, and 107.6 for 3rd to 9th-order fuzzy time series, respectively. Specifically, for the 8th-order FTS model, the proposed approach achieves the smallest RMSE value of 96.1. Moreover, the average RMSE value of the proposed model stands at 121.43, showcasing its superiority in comparison to all other examined forecasting models

Table 7: **A comparison of the RMSEvalue under various high-order FTS models with seven intervals.**

Order	C02 [6]	CC06b [11]	HPSO [25]	AFPSO [21]	Our model
3	294.4	176.4	177.9	176.6	162.9
4	299	178.9	152.5	142	135.2
5	307.5	157.9	153.4	142.7	119.4
6	313.4	164.3	153.9	149.3	111.2
7	322.6	164.2	143.7	135.9	104.8
8	319.7	149.6	130.8	121.6	**96.1**
9	320.6	136.9	134.1	123.5	107.6
Average RMSE	**311.2**	**161.7**	**150.2**	**142.67**	**121.43**

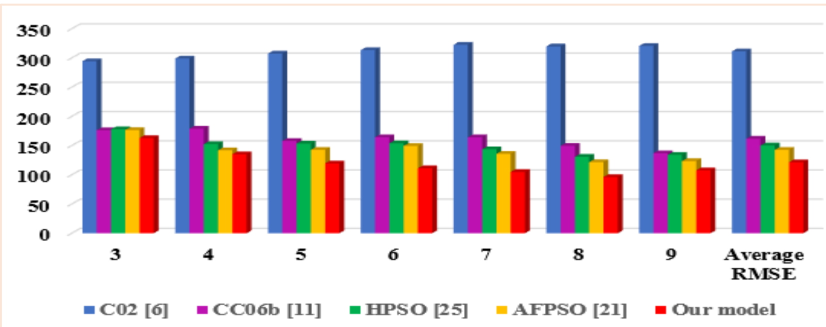

Fig.2. Representation of RMSE values between our proposed model and existing models.

4.2. Experimental results forforecasting the rice production in VietNam

In this section, we apply the proposed method for forecasting the rice production from 1990 to 2010.The forecasted results of proposed model under number of interval equal to 14 and various orders are listed in Table 8. Table 8 illustrates that the performance of the proposed model notably enhances as the number of orders within the same interval increases.Specifically, the proposed model achieves the lowest RMSE value of 127.5 with a 4th-order fuzzy logical relationship.

Table 8: **The completed forecasting results for rice production data of Viet Nam under different orders of FTS.**

Years	**Actual data**	***Forecasted results***			
		2nd-order	3rd-order	4th-order	5th-order
1990	19225.1				
1991	19621.9				
1992	21590.4	21052.75			
1993	22836.5	22494	22926.5		
1994	23528.2	23832.25	22912.5	23609.75	
1995	24963.7	25076.5	24524	25009.25	24928.25
----	-----	-----	-----	----	-----
2010	39988.9	39046	39617	39807.75	39959.5
RMSE		**369.34**	**297.37**	**127.5**	**139.75**

The graph demonstrates the trend of rice production predictions using the high-order fuzzy time series model, allowing for a direct visual comparison between the predicted values and the real data, as depicted in Figure 3.

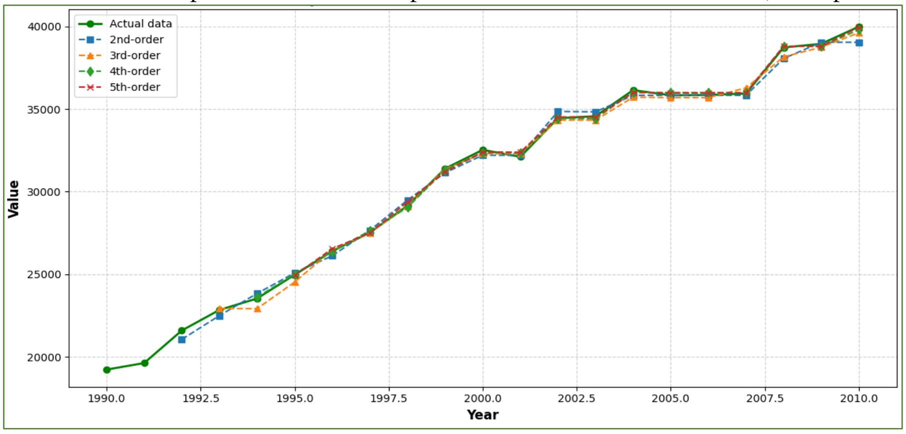

Fig. 3. The curves of the actual data and forecasted values for forecasting rice production

From Figure 3, it can be seen that the forecasted trend of proposed model is close to the actual data based on the high - order FTS.

To sum up, demonstrations above show that the proposed model outperform the existing models based on high - order FTS model with various number of intervals for forecasting the different problems.

V. CONCLUSION

In this study, a forecasting model integrating a fuzzy time series approach with iterative fuzzy relations and the LPA clustering algorithm is introduced. The utilization of the LPA algorithm enhances the delineation of more appropriate partitions within the universe of discourse, leveraging the recurrence numbers of fuzzy relations and resulting in noteworthy enhancements in forecasting accuracy. The efficacy of the proposed method is substantiated through its application to two distinct datasets: enrollment data from the University of Alabama and a dataset pertaining to rice production. Comparative analyses, delineated comprehensively in Tables 6, 7, and 8, exhibit the superior performance of the proposed model over existing methods relying on first-order FTS and high-order FTS across various interval lengths. While the evaluation primarily focuses on enrollment forecasting and rice production prediction, we posit that its applicability extends to diverse forecasting domains encompassing population trends, weather predictions, forecasts related to car accidents, and analogous scenarios. Future research endeavors will delve deeper into exploring these potential applications.

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