

Design and Computational Analysis of the Propulsion System for the Tiangong Butterfly-Wing Micro-Bionic Flapping-wing Aircraft

Ding Bowen, Gao Bowen, Cheng Qiming, Liu Haiming

(School of Mechanical and Vehicle Engineering, Changchun University, Changchun, Jilin 130022)

Abstract: To address the transmission synchronization challenge of a kilogram-class flapping-wing aircraft, this study proposes a transmission scheme for the Tian Gong butterfly wing, comprising two motors, a closed gear train, and two rocker arms, followed by comprehensive computational analyses of geometry, kinematics, structural strength, and power output. Two 615 hollow-cup reduction motors are driven by gear pairs with a module of 0.6 mm and a tooth ratio of 11:34, respectively, to the left and right sides; the two driving gears and the two driven gears mesh with each other to form a synchronous closed-loop mechanism. The lengths of the crank, connecting rod, rocker arm, and frame are 6.49 mm, 19.37 mm, 8.18 mm, and 20.00 mm, respectively. Based on the specified motor parameters, the maximum mechanical output power of a single motor is 1.16 W; the flapping frequency under full-throttle no-load conditions is 9.71 Hz, while the equilibrium flapping frequency under assumed load conditions is 9.43 Hz. The four-bar linkage satisfies the Grashof criterion, with a calculated total swing angle of 105.0° and a minimum transmission angle of 35.0° . Under the selected load and material parameters, the root bending stress during the start-up locked condition is 38.2 MPa, exceeding the assumed allowable value of 33 MPa; therefore, verification of start-up current limiting and gear strength is required. The design mass is 19.49 g, the wingspan is 380 mm, the induced power estimated according to momentum theory is 0.160 W, and the required flight power ranges from 0.25 W to 0.32 W. The above results pertain to design calculations; actual flight performance, material allowable values, and power supply chain specifications require validation via traceable experimental measurements.

Keywords: bionic flapping-wing aircraft; butterfly; gear transmission; crank-rocker mechanism; motor load line; 3D-printed PLA; lightweight design

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I. Introduction

1.1 Research Background on Bionic Flight

Through billions of years of evolution, nature has given rise to countless exquisite biological motion mechanisms, among which the flight capability of insects is particularly remarkable. Butterflies, as highly representative flying organisms in nature, exhibit elegant flight postures and exceptional maneuverability, underpinning complex aerodynamic principles. Under low Reynolds number conditions ($Re < 10^4$), the steady-state aerodynamic theories traditionally employed by conventional fixed-wing or rotor aircraft often fail; however, insects such as butterflies can leverage unsteady aerodynamic mechanisms—including leading-edge vortices (LEV), rotational circulation, and wake capture—to generate lift far exceeding the predictions of steady-state theories [1]. These unsteady mechanisms enable micro-aerial vehicles to maintain stable lift output even at extremely low flight speeds, thereby providing a novel theoretical framework for the design of micro-aerial vehicles (MAVs).

The application of bionics in aircraft design aims to draw inspiration from biological flight mechanisms to overcome the design limitations of conventional aircraft. The core characteristic of butterfly flight lies in the coupling effect between its wing-flapping motion and its flexible wing surfaces. Unlike birds, butterfly wings possess exceptional flexibility and intricate pleated structures, enabling significant passive deformation during flapping. This deformation not only helps delay airflow separation but also enhances propulsion efficiency by storing and releasing elastic potential energy [2]. Therefore, conducting in-depth research on the biomechanical mechanisms of butterfly flight and translating these insights into engineered mechanical structures holds considerable scientific significance for advancing the development of miniature bionic aircraft.

1.2 Current Research Status of Micro-Wing Aircraft

Micro-flapping-wing aircraft (Flapping-Wing MAV), as a pivotal branch of bionic aircraft technology, have achieved remarkable research progress both domestically and internationally in recent years. The RoboBee project at Harvard University in the United States, the DelFly series at Delft University of Technology in the Netherlands, and research teams from Chinese universities—including Beihang University and Nanjing University of Aeronautics and Astronautics—have all attained breakthrough results in the aerodynamic characteristics, structural design, and control strategies of flapping-wing aircraft [3]. These studies demonstrate that flapping-wing aircraft possess inherent aerodynamic advantages in low Reynolds number environments, and their flight modes closely resemble those of insects, endowing them with exceptional stealth capabilities and environmental adaptability.

However, existing research on micro-pterostats has primarily focused on numerical simulations of aerodynamic mechanisms and wind tunnel experiments; nevertheless, numerous challenges remain in the development of engineering prototypes. Taking gram-scale (<100 g) pterostats as an example, the highly integrated design of their power systems, transmission mechanisms, and control units continues to be a critical constraint on their advancement. In particular, the trade-off among synchronization, lightweighting, and power density in transmission systems has yet to be fully resolved. Currently, most micro-pterostats employ a single-motor configuration to drive both wings, utilizing complex differential mechanisms or flexible couplings to transmit motion. This approach not only increases the overall system mass but also makes the system susceptible to phase synchronization failures between the left and right wings due to mechanical play, which in turn compromises flight stability [4].

1.3 Limitations and Challenges of the Existing Technology

Despite significant progress in the research on micro-flapping-wing aircraft, three core challenges remain in their practical engineering applications:

Transmission synchronization challenge: K-class flapping-wing aircraft require extremely high phase synchronization between their left and right wings. Any phase difference between the flapping motions of the left and right wings can generate a yawing moment, leading to loss of aircraft control. While existing single-motor drive schemes can fundamentally ensure synchronization, they suffer from an overly long transmission chain and low mechanical efficiency; in contrast, the dual-motor independent drive scheme offers a compact structure, but relies on electronic synchronization control, making it difficult to achieve high-precision real-time closed-loop control on K-class platforms, while also increasing the computational load and mass of the flight control system [5].

The trade-off between lightweight design and structural strength: The takeoff weight of micro-aerial vehicles is typically only in the tens of grams, making every milligram of weight reduction critically important. However, their propulsion mechanisms must withstand high-frequency (>10 Hz) reciprocating loads, which imposes stringent requirements on the fatigue strength and stiffness of the materials. Conventional metal fabrication techniques struggle to achieve lightweight manufacturing of complex structures at micro scales, whereas the mechanical properties of conventional 3D-printed materials often fail to meet the demands of high-frequency transmission [6].

The challenge of power matching: micro-hollow-cup motors exhibit limited power density, and their efficiency declines sharply at high rotational speeds. The core difficulty in transmission system design lies in optimizing the geometric parameters of the crank-rocker mechanism to satisfy the requirement for a large sweep angle (>90°) while ensuring that the minimum transmission angle remains within an appropriate range, thereby maximizing mechanical transmission efficiency. Furthermore, the precise matching between the power unit and the aerodynamic load directly determines whether the aircraft can achieve stable hover or forward flight. The power chain of the micro-winged aircraft encompasses the full-chain energy conversion—from battery discharge, motor output, transmission losses, to aerodynamic induced power—where cumulative losses at any stage may prevent the system from achieving its desired flight performance. According to momentum theory estimates, the induced power of an aircraft with a wingspan of 380 mm under indoor conditions is approximately 0.160 W, whereas the required flight power, accounting for form drag losses, ranges from 0.25 W to 0.32 W. A single 615 hollow-cup motor delivers a maximum mechanical output power of 1.16 W under full-throttle conditions; the combined output of two such motors amounts to 2.32 W, which theoretically satisfies the power requirement. However, in actual flight operations, the effective output power of the motors is subject to multiple factors—including voltage droop, power degradation due to temperature rise, gear meshing losses (with single-stage efficiency 0.95), and fluctuations in aerodynamic loads—meaning the actual usable power may be significantly lower than the theoretical value. Furthermore, the voltage holding capability of a 2S 7.4 V/60 mAh lithium polymer battery under high-rate discharge, the compatibility between the average operating current (calculated as 0.180 A for this model) and battery capacity, as well as the safety of gliding and landing operations following motor shutdown under fail-safe protection mechanisms, all require rigorous validation through bench tests and flight trials [7].

II. Overall Plan and Design Specifications

2.1 Design Specifications

Table 1: Design Objectives and Calculated Values

Indicator Item	target value	design value	unit	proof technique
Takeoff mass	≤ 20	19.49	g	Electronic balance (0.01 g)
Wingspan	≤ 400	380	mm	Steel ruler / Visual measurement
Expand the projection envelope	$\leq 400 \times 400$	380×295	mm	Top-view silhouette calibration
Swimming frequency	1 – 10, continuously adjustable	9.43 (full throttle); 4.79 (55% throttle)	Hz	High-speed camera: 480 fps
Total swing angle of the joystick	100~110	105.0	°	Frame-by-frame angle measurement
Left-right phase difference	≤ 2	0 (Mechanical lock)	°	Bilateral frame-to-frame comparison
Phase difference between the fore and hind wings	90 ± 5	90° (corresponding to the eccentric pin angle of 70.2°)	°	Three-point fitting within the same frame
Unilateral wing area	110~120	112	cm ²	CAD Projection Measurement
Airspeed	≥ 5	12 – 17 (ideal upper limit)	min	Time from full charge to low-voltage alarm
Average operating current	≤ 0.30	0.180(model)	A	Current shunt sampling at 20 Hz
Flight noise at 1 m	≤ 60	—	dB(A)	Sound level meter – A-weighting / slow range
Noise reduction for quadrotors of the same mass class	≥ 10	—	dB(A)	Isotope high-contrast study

2.2 System Composition and Overall Layout

The complete aircraft consists of five subsystems: power system, transmission system, airframe, control system, and frame. ① The power unit comprises two 615 hollow-cup two-stage reduction motors and one set of 2S 7.4 V/60 mAh lithium polymer batteries; ② The transmission unit consists of one set of 11:34 gear pair on each side (left and right), an end-face eccentric crank, connecting rods, and two sets of rocker arms (front and rear); ③ The airframe features segmented butterfly wings composed of $\Phi 0.8$ mm carbon fiber ribs and 0.03 mm PET film; ④ The control unit comprises a FlySky FS-i6X transmitter and an MXR62 V3 receiver, with both motors driven directly via the same control channel; ⑤ The frame is a PLA-printed hollow structure that also serves as the battery clamp, tail rod, and motor mount.

2.3 Transmission Scheme and Its Kinematic Consequences

Two motors are arranged symmetrically about an axis; each motor's output shaft is equipped with one 11-tooth driving gear, and the two driving gears mesh with each other. Each driving gear further meshes with one 34-tooth driven gear, and the two driven gears also maintain meshing with each other, thereby forming a closed kinematic chain. The mechanical characteristics of this configuration are listed in Table 2, and a comparative analysis of the configurations is presented in Table 3.

Table 2: Kinematic Characteristics of the Closed Gear Chain

feature	Conclusion	according to
Bilateral symmetry	Forced constant-speed reverse operation; the phase is uniquely determined by the installation angle, eliminating the need for electronic synchronization.	Eqs. (7) – (10), Table 6
Dual-machine operation	Power sharing (1.16 W per side), rather than degrees of freedom control	Equation (5)

feature	Conclusion	according to
Turning capability	Not applicable: the rotational speeds of the two machines are locked by the kinematic chain, making it impossible to establish a speed difference.	Section 3.5.4
Assembly constraints	All four gear sets must be installed simultaneously; otherwise, gear pre-tensioning may become stuck.	Section 3.7 Tolerances

Table 3: Comparison of flutter transmission mechanism configurations

Institution Type	Bipennal symmetry	Adjustability for convenience	transmission efficiency	Motion smoothness	Compact design	Conclusion
Gear – Rocker mechanism (this solution)	Optimal (forced engagement lock)	Easy adjustment of eccentricity and pin angle	High (single-stage: 0.95)	Excellent	Middle	put to use
crank block	Middle	Middle	Middle	Middle	good people	Not adopted (due to high linear friction and lack of self-locking synchronization)
Double-crank four-bar linkage	difference	Easy	Gao	difference	good people	Does not employ (synchronous dependency on motor consistency)

III. Drive System Design

3.1 Power Source Characteristics and Load Line

3.1.1 Motor Parameters

Table 4: Key Parameters of the 615 Hollow Cup 2-Stage Reduction Motor

parameter	numeric value	parameter	numeric value
Model/Dimensions	615 Hollow Cup Grade 2 reduction motor: outer diameter $\Phi 6$ mm, total length ≤ 12 mm	rated voltage	7.4 V
Built-in gear ratio	26.5 (Level 2)	starting voltage	≤ 1 V
No-load speed (gearbox output end)	1800 ± 180 r/min	No-load speed of the motor (estimated)	$\approx 4.77 \times 10^4$ r/min
Locked-rotor (peak) torque	$250 \text{ g} \cdot \text{cm} = 24.5 \text{ N} \cdot \text{mm} = 0.0245 \text{ N} \cdot \text{m}$	no-load current	60 mA
locked rotor current	1100 mA	Single-unit quality	1.3 g
Output shaft diameter	$\Phi 2.0$ (active gear hole: $\Phi 2.06$)	Core electromagnetic efficiency	≈ 0.80

3.1.2 Speed – Torque Load Curve

For the linear DC motor model (ignoring winding inductance and temperature rise), there is an expression at the output end of the gearbox:

$$T_n = T_{st1} - \frac{n}{n_0} \quad (1)$$

$$I_n = I_0 + \frac{I_{st} I_0 T_n}{T_{st}} \quad (2)$$

$$P(n) = T(n) \cdot 2\pi n / 60 \quad (3)$$

$$\frac{dP}{dn} = 0 \Rightarrow n = \frac{n_0}{2}, \quad P_{min} = \frac{T_{st} \cdot \pi n_0}{120} \quad (4)$$

Substituting $n_0 = 1800$ r/min, $T_{st} = 250 \text{ g} \cdot \text{cm}$, $I_0 = 60$ mA, and $I_{st} = 1100$ mA, we obtain

$$P_{max} = 1.16W \quad (5)$$

At this point, $n = 900$ r/min, $T = 125 \text{ g} \cdot \text{cm}$, $I = 0.58$ A, and the electrical input power is 4.29 W.

3.1.3 Relationship between flutter frequency and throttle position

The external gear ratio and the no-load rotational speed at full throttle are as follows:

$$i = \frac{z_2}{z_1} = \frac{34}{11} = 3.0909$$

$$n_{crank} = \frac{n_0}{i} = 582 \text{ r/min}$$

$$f_0 = \frac{n_{crank}}{60} = 9.71 \text{ Hz}$$
(6)

The crank pulley frequency must be accounted for using an external transmission ratio of 11:34. By simultaneously considering the assumed power requirement and the motor load curve, the operating points of the model listed in Table 5 can be obtained; these operating points, however, are not based on actual flight test data.

Table 5: Throttle duty cycle - pulse frequency - torque - current - range (combined for both motors; required power: 0.25 W)

throttle	Sampling rate / Hz	Crank speed / (r/min)	Torque per side / (g • cm)	Current/A	continuation of the journey/min
1.00	9.43	566	21.0	0.177	20.4
0.80	7.41	445	26.7	0.192	18.8
0.70	6.38	383	31.0	0.203	17.7
0.60	5.33	320	37.1	0.220	16.4
0.55	4.79	288	41.3	0.231	15.6
0.40	3.01	181	65.7	0.297	12.1

3.2 Gear Parameters and Geometric Verification

Table 6: Gear geometric parameters (comparison of drawing specifications and calculated values)

project	driving gear Z_1	driven gear Z_2	test and verify
modulism	0.6 mm	0.6 mm	Parameter tables for both diagrams
number of teeth z	11	34	Parameter tables for both diagrams
angle of pressure α	20°	20°	Parameter tables for both diagrams
Crown height coefficient / Clearance coefficient	1.0/0.25	1.0/0.25	standard series
Diametral circle $d = mz$	6.60	20.40	Icon: $\phi 6.60 / \phi 20.40$
Diameter at the crown of the tooth, $d_a = m(z + 2ha^*)$	7.80	21.60	Drawing indicates $\phi 7.80 / \phi 21.60$
Root circle diameter: $d_f = m(z - 2.5)$	5.10	18.90	calculated value
breadth of tooth b	2.2 (Total Width)	2.1	The design value for b is set to 2.2 mm.
bore	$\phi 2.06$ (motor shaft $\phi 2$)	$\phi 6.0$ (output shaft, bushing transitions to $\phi 3$)	Two images
End-face features	—	Eccentric pin 6.49; angle specifications: 18°, 15°	34T graph
material	PLA	PLA (FDM printing)	title bar
Code/Scale	HD-02 / 5 : 1	HD-10 / 2 : 1	title bar

The three center distances together constitute both the design datum and the assembly datum:

$$a_{11} = m \cdot z_1 = 6.600 \text{ mm} \quad (\text{两主动齿轮互啮}) \quad (7)$$

$$a_{12} = \frac{mz_1z_2}{2} = 13.500 \text{ mm} \quad (\text{主-从动}) \quad (8)$$

$$a_{22} = m \cdot z_2 = 20.400 \text{ mm} \quad (\text{两从动齿轮互啮}) \quad (9)$$

$$i = \frac{z_2}{z_1} = 3.0909 \quad (10)$$

The hole coordinate positions of the frame (HD-12) are D1 (−3.30,0), D2 (3.30,0), B1 (−10.20, −11.60), B2 (10.20, −11.60), C1 (−10.00,8.40), C2 (10.00,8.40), A1 (−16.80,7.28), and A2 (16.80,7.28). From these coordinates, the distances D1 − D2 = 6.600 mm, D1 − B1 = D2 − B2 = 13.497 mm, and B1 − B2 = 20.400 mm were calculated, which precisely match Equations (7) − (9) (with deviations $\leq 3 \mu\text{m}$), thereby confirming that the gear layout and the frame design are mutually validated.

3.3 Analysis and Handling of Root Cutting

The minimum number of teeth for a standard spur cylindrical gear without root cutting, and the minimum required positive profile shift, are as follows:

$$Z_{min} = \frac{2h_a^*}{\sin^2 \alpha} = \frac{2}{\sin^2 20^\circ} = 17.10 \quad (11)$$

$$x_{min} = h_a^* - \frac{z_1 \sin^2 \alpha}{2} = 1 - \frac{11 \times 0.11698}{2} = 0.357 \quad (12)$$

The minimum number of teeth required by the theory to prevent undercutting, given that the 11-tooth driving gear has a tooth height lower than the standard full tooth height, is specified here. The existing tooth tip circle dimensions alone are insufficient to conclusively prove that the tooth profile remains un-shifted; the tooth root profile and the shift amount shall be verified using the original tooth profile diagram or physical measurements. Subsequently, positive and negative height shift schemes may be compared, and the center distance, interference, and tooth root strength shall be rechecked.

3.4 Root Bending Strength and Tooth Surface Contact Strength

The most unfavorable torque transmitted by the pinion—the starting stall torque $T_1 = 24.5 \text{ N} \cdot \text{mm}$ —was taken as the verification operating condition (converted to the shaft end during stable flight, this value ranges from 6.8 to 20.4 g · cm), with a tooth width $b = 2.2 \text{ mm}$:

$$F_t = \frac{2T_1}{d_1} = \frac{2 \times 24.5}{6.60} = 7.42 \text{ N} \quad (13)$$

$$\sigma_F = \frac{K \cdot F_t \cdot Y_{Fa} \cdot Y_{Sa}}{bm} \leq \sigma_F \quad (14)$$

$$\sigma_H = Z_E \cdot \sqrt{\frac{F_t}{bd_1} \cdot \frac{u+1}{u}} \leq \sigma_H, \quad u = 3.0909 \quad (15)$$

$$Z_E = \sqrt{\frac{\pi \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}{1}} \quad (16)$$

Table 7: Calculation Parameters and Allowable Values for PLA Material

parameter	short-cut process	Basis/Explanation
Tooth shape factor Y_{Fa} / stress correction factor Y_{Sa}	3.3 / 1.7	Conservative value for $z = 11$ standard gear, including undercut correction
Load factor $K = K_A \cdot K_v \cdot K_\beta$	$1.0 \times 1.1 \times 1.1 = 1.21$	Circumferential speed $v = \pi d_1 n_0 / 60 = 0.62 \text{ m/s}$; light load, intermittent, and short-term no-load starting
Elastic coefficient Z_E (PLA—PLA)	$24.2 \text{ MPa}^{1/2}$	In Equation (16), $E = 3.2 \text{ GPa}$ and $\nu = 0.36$ were adopted; applying the same equation with steel – steel parameters yielded $189.8 \text{ MPa}^{1/2}$, which agrees with the values provided in the handbook, thereby verifying the correctness of the formula.
Allowable bending stress $[\sigma_F]$	33 MPa	PLA bending strength: 98 MPa; safety factor: 2.5; printing interlayer reduction factor: 0.85
Permissible contact stress $[\sigma_H]$	65 MPa	Approx. $0.6 \times \text{PLA compressive strength}$: 105 MPa

check result:

$$\sigma_F = 1.21 \times 7.42 \times 3.3 \times \frac{1.7}{(2.2 \times 0.6)} = 38.2 \text{ MPa} > [\sigma_F] = 33 \text{ MPa} \quad (17)$$

$$\sigma_H = 24.2 \times 0.842 = 20.4 \text{ MPa} < [\sigma_H] = 65 \text{ MPa}$$

Under the specified operating conditions, the root bending stress during the starting stall condition is 38.2 MPa, which exceeds the allowable value of 33 MPa derived from assumed PLA material properties. The current limiting target of 0.55 A per unit and the gear width improvement of 2.52 mm are both design recommendations; these must be validated through actual current-limiting circuit testing, current monitoring, and gear performance tests. Existing data, however, are insufficient to determine the gear service life based on these parameters.

3.5 Design and Kinematics of the Crank-Rocker Mechanism

3.5.1 Member Parameters

Table 8: Link lengths of the four-bar mechanism (identical for the front and rear link groups on each side)

symbol	meaning	numeric value /mm	according to
l_1	Crank (eccentricity)	6.49	34T Gear Drawing Annotation and Manual 3.2
l_2	Connecting rod (center-to-center distance between two holes)	19.37	Connecting rod diagram (HD-08: hole ϕ 1.28, width 2.8, thickness 1/1.8)
l_3	Joystick (distance between the two pivot holes)	8.18	Both the front joystick (HD-04) and the rear joystick (HD-07) are rated at 8.18.
l_4	Rack (output shaft center — rocker pivot point)	20.00	Rack hole positions // B1C1 // = 20.001, // B1A1 // = 20.000; and $l_4 = \sqrt{(l_2^2 + l_3^2 - l_1^2)} = 20.00$

3.5.2 Conditions for the existence of the crank and its closed interval

The shortest link is the coupler link, and it satisfies the Grashof criterion:

$$l_{\min} + l_{\max} = 6.49 + 20.00 = 26.49 \leq l_2 + l_3 = 19.37 + 8.18 = 27.55 \quad (18)$$

Based on the geometric closure condition for full revolution of the crank, the feasible range of frame length is

$$\|l_2 - l_3\| + l_1 \leq l_4 \leq l_2 + l_3 - l_1 \Rightarrow 17.68 \leq l_4 \leq 21.06\text{mm} \quad (19)$$

3.5.3 Swing Angle, Transmission Angle, and Emergency Return Characteristics

The two positions where the crank and the connecting rod are collinear correspond to the two extreme positions of the rocker:

$$\begin{aligned} \cos\psi_1 &= \frac{[l_3^2 + l_4^2 - (l_1 + l_2)^2]}{(2l_3l_4)} \Rightarrow \psi_1 = 128.1^\circ \\ \cos\psi_2 &= \frac{[l_3^2 + l_4^2 - (l_2 - l_1)^2]}{(2l_3l_4)} \Rightarrow \psi_2 = 23.1^\circ \end{aligned} \quad (20)$$

$$\begin{aligned} \psi &= \psi_1 - \psi_2 = 105.0^\circ \text{ (半幅 } \pm 52.5^\circ \text{)} \\ \gamma_{\min} &= \arccos \left[\frac{[l_2^2 + l_3^2 - (l_4 + l_1)^2]}{2l_2l_3} \right] = 35.0^\circ \text{ (另一极限 } 145.0^\circ \text{)} \end{aligned} \quad (21)$$

The minimum transmission angle calculated for the four-bar mechanism based on the specified dimensions is 35.0° . The dwell-return characteristic and the actual oscillation sequence must be further verified using the assembly geometry and high-speed video; in this study, asymmetric oscillation is not considered as a realized performance feature.

3.5.4 Synchronization of the two wings and phase relationship between the front and rear wings

The closed gear chain ensures that the left and right driven gears rotate at strictly equal speeds in opposite directions. In conjunction with the left and right crankshafts installed in mirror symmetry, the two wings operate at the same frequency and amplitude, with their phase angles uniquely determined by the assembly installation angle. Based on the coordinate positions of the frame holes, the frame line orientations for the two chain sets on the same side can be calculated as $B1 \rightarrow C1 = 89.43^\circ$ and $B1 \rightarrow A1 = 109.27^\circ$, indicating a spatial offset of $\Delta f = 19.84^\circ$. The temporal phase difference between the wings equals the angular difference between the two crankshaft pins relative to their respective frame lines; if the angular offset is defined using the gear end face as a common reference, it is denoted as Δ .

$$\Delta\varphi_{\text{wing}} \approx \Delta - \Delta f \text{ (}\Delta f = 19.84^\circ\text{)} \quad (22)$$

To achieve a wing surface phase difference of 90° between the hind wing and the fore wing, one should set $\Delta = 70.2^\circ$ (or 109.8° , with the sign depending on the direction of rotation). Expressed in terms of time, this 90° phase difference corresponds to a time lag.

$$\Delta t = T \cdot \left(\frac{90^\circ}{360^\circ} \right) = 1/(4f) = 26.5\text{ms} \text{ (}f = 9.43\text{Hz}\text{)} \quad (23)$$

The current design aims for a 90° phase difference between the front and rear wings during assembly; the correspondence between the crank pin angle and the wing surface phase must be verified frame-by-frame using video footage.

3.6 Output Torque Capability and Matching Conclusions

$$T_{\text{crank,max}} = T_{\text{st}} \cdot i \cdot \eta_{\text{ext}} = 250 \times 3.0909 \times 0.95 \approx 734 \text{ g} \cdot \text{cm/侧} \quad (24)$$

Based on the given motor and transmission parameters, the peak torque at each crank end is approximately 734 g • cm; this value represents the theoretical upper limit near the locked-rotor condition and does not indicate the continuously usable torque.

3.7 Shaft Assembly, Tolerances, and Assembly

Table 9: Shaft Assembly, Supports, and Key Tolerances

project	Design values
Motor Support	Φ 6 mm motor housing is pressed into the Ø6.1 through-hole of the frame (D1/D2), with a single-sided clearance of 0.05 mm + adhesive application for anti-rotation.
output axis	Φ 3 mm SUS304 hardened shaft, transitioning via a printed bushing to the Φ 6 hole of the 34T gear, with interference ranging from 0.02 to 0.05 mm.
Support Type	Printed hole + PTFE film gasket sliding support (low speed, light load; no micro-bearing required)
Axial positioning	Gear clamp plate HD-05 + M3 round-head/hexagonal self-locking nut, nylon ring for anti-loosening
Critical distance tolerance	For 6,600 / 13,500 / 20,400 / 20,000 mm, a tolerance of ±0.05 mm shall be applied (as required for closed chains).
Recommended side clearance	j _n = 0.10 – 0.15 mm, achieved via the negative tooth thickness deviation plus printing shrinkage compensation (+0.2%).
lubricating	PTFE dry-film lubricant; use of oil-based greases is prohibited (risk of PLA swelling)
Assembly phase	First install the rear chain and calibrate the crank pin orientation, then install the front chain; for both pins, set the angle to 70.2° according to Equation (22).
Printing Process	FDM, 0.4 mm nozzle, layer height 0.12 mm, 45% honeycomb fill, tooth length direction parallel to the print layer, no annealing

IV. Aircraft Structural Design

4.1 Wing Geometry and Aerodynamic Parameters

Single-sided wing membrane mass: 0.5 g; PET density: 1.40 g/cm³ ; thickness: 0.03 mm; wing membrane cutting area:

$$S_c = \frac{m_{\text{film}}}{(\rho t)} = \frac{0.5}{(1.40 \times 0.003)} = 119 \text{ cm}^2 \quad (25)$$

After deducting approximately 6% for overlapping and bonded edges, the effective projected area on one side is $S = 112 \text{ cm}^2$, and the total area is $2S = 224 \text{ cm}^2$, of which the forewing accounts for 71 cm^2 and the hindwing accounts for 41 cm^2 (consistent with the segmented butterfly-wing layout). From this, it follows that

$$\begin{aligned} \bar{c} &= \frac{2S}{b} = \frac{224}{380} = 59 \text{ mm} \\ \text{AR} &= \frac{b^2}{2S} = 6.4 \\ \frac{W}{S} &= \frac{0.1911}{0.0224} = 8.5 \text{ N/m}^2 \end{aligned} \quad (26)$$

Here, $W = mg = 0.01949 \times 9.807 = 0.1911 \text{ N}$. The airfoil of this design is slender ($\text{AR} \approx 6.4$), which exceeds the 2 – 4 range observed in natural butterfly species; this configuration achieves structural stiffness and low wing loading at the expense of a narrow, elongated forewing and a small hindwing.

4.2 Wing Veins and Wing Membrane

The wing veins employ carbon fiber rods with a diameter of Φ 0.8 mm and a single-rod mass of 0.25 g; the length of each rod was calculated by back-calculating from a density of 1.6 g/cm³.

$$L = \frac{m}{\rho} \cdot \frac{\pi d^2}{4} = \frac{0.25}{1.6} \cdot \pi \cdot 0.4^2 \approx 311 \text{ mm} \quad (27)$$

The wing veins utilize Φ 0.8 mm carbon fiber rods, the wing membranes utilize 0.03 mm PET film, and the frame utilizes a PLA-printed part. The frame and hole dimensions adhere to the design drawings.

component	quality/g	proportion	component	quality/g	proportion
Battery (2S 7.4 V 60 mAh)	3.39	17.4%	Wing membrane (bilateral PET)	1.00	5.1%
Motor $\times 2$	2.60	13.3%	Receiver MXR62 V3	1.60	8.2%
Gear set ($2 \times 11T + 2 \times 34T +$ shaft + clamping plate)	2.70	13.9%	Switch, wiring, and sheath	0.55	2.8%
Connecting rod + rocker $\times 4$	0.86	4.4%	Fasteners ($M3 \times 6 \times 20$, nut)	0.60	3.1%

component	quality/g	proportion	component	quality/g	proportion
Rack and printing structural components	3.20	16.4%	Adhesive layer and counterweight	0.99	5.1%
Carbon fiber ribs ×8	2.00	10.3%	amount to	19.49	100%

4.4 Lift – Power Range Verification

The swept area of the actuator disk is $A = \pi (b/2)^2 = \pi (0.190)^2 = 0.1134 \text{ m}^2$, and the air density is $\rho = 1.20 \text{ kg/m}^3$ (at an indoor temperature of 20° C); calculations are based on the momentum theory.

$$P_i = \frac{W^3}{2\rho A} = \frac{0.19^{11.5}}{0.5^{215}} = 0.160 \text{ W}$$

$$P_{\text{req}} = \frac{P_i}{\text{FM}}, \text{ FM} = 0.50 - 0.65 \quad (28)$$

The required flight power is $P_{\text{req}} = 0.25 - 0.32 \text{ W}$ (including type resistance loss in FM).

The aforementioned power results were derived from an idealized momentum model and assumed power factors; they are therefore applicable only for preliminary verification at early design stages, and cannot be used to substantiate stable hover performance, flight endurance, or aerodynamic efficiency.

V. Control and Power Supply Links

5.1 System Architecture and Degrees of Freedom

The system consists of a FlySky FS-i6X transmitter (10 channels, 2.4 GHz) → an MXR62 V3 receiver → and two 615 motors. There is no electronic speed control, flight control, or IMU between the receiver and the motors; the two motors are connected in parallel to the same channel of the receiver, and the motor terminal voltage (PWM) is adjusted via throttle and rudder inputs to vary the motor rotation frequency. As described in Section 3.5.4, the closed-loop motion chain mechanically locks the rotational speeds of both sides; therefore, both motors must receive the same signal, otherwise they will exert mutual load and generate circulating currents.

Table 11: Channel Allocation and Out-of-Control Protection

channel	Transmitter controls	Target of Execution	Functions and Mapping
Ch3	Move the right joystick up and down	Two motors (in parallel)	Frequency adjustment; initial throttle set to 40%;
Ch1/Ch2/Ch4—Ch6	—	unused	No gimbal, no pan-tilt mechanism
Overrun Protection	—	any power-generating or power-driven machine	Set the throttle to zero, stop the motor, and glide down.

5.2 Power Supply, Protection, and Operating Specifications

The power supply chain is planned to utilize a 2S 7.4 V/60 mAh battery. The receiver's input voltage, the continuous and peak currents of the output stage, and the overcurrent protection behavior shall be further verified using the product data sheet and bench tests. Reducing the initial throttle input cannot substitute for a validated current-limiting circuit.

VI. Limitations of Design Verification

The current analysis in this study is primarily based on structural parameter calculations and the derivation of theoretical models; however, full-system assembly and flight testing have not yet been completed. Consequently, the following conclusions remain subject to the limitation that they require validation through experimental measurements.

(1) Gear strength and root cutting verification. The calculations in Section 3.4 indicate that under the starting stall condition, the root bending stress of the gear is 38.2 MPa, which exceeds the allowable value of 33 MPa derived from the PLA material properties. This result is contingent upon the assumptions underlying the allowable value (PLA bending strength: 98 MPa; safety factor: 2.5; inter-layer reduction factor for printing: 0.85); the actual allowable value may deviate due to the influence of FDM printing process parameters (print layer height, fill ratio, printing direction). The flow-limiting target of 0.55 A per unit and the gear width improvement target of 2.52 mm are both design recommendations, which must be validated through actual flow-limiting circuit testing, current monitoring, and gear bench tests; existing data alone cannot be used to determine the gear service life. Furthermore, Section 3.3 notes that the 11-tooth driving gear does not meet the minimum tooth count requirement for non-root-cutting, posing a risk of root cutting; the shift amount and the actual root profile must be verified

against the original gear profile diagram or via physical measurements. Subsequently, positive and negative height shift schemes can be compared, and the center distance, interference, and gear root strength should be re-evaluated.

(2) Kinematics and Phase Relationship Verification. The minimum transmission angle calculated in Section 3.5.3 is 35.0° . The emergency return characteristic and the actual pulsation timing require further verification based on assembly geometry and high-speed video footage; therefore, asymmetric pulsation is not listed as a realized performance feature in this study. Section 3.5.4 indicates that a 90° phase difference between the front and rear wings corresponds to a crank pin angle of 70.2° ; however, the correspondence between the crank pin angle and the wing phase angles still requires frame-by-frame video verification. Although a closed gear train can theoretically achieve forced constant-speed reverse synchronization, in practical assembly, all four center distances must simultaneously be satisfied; otherwise, gear preloading-induced jamming may compromise synchronization accuracy. This assembly constraint must be verified individually after physical assembly.

(3) Power and Flight Performance Verification. In Section 4.4, the flight requirement power estimated based on the momentum theory ranges from 0.25 to 0.32 W; this result is derived from an idealized momentum model and assumed power factors, and is therefore only suitable for preliminary verification at early design stages. It cannot be used to substantiate stable hovering, endurance, or aerodynamic efficiency. The design mass of 19.49 g and wingspan of 380 mm satisfy the design specifications outlined in Table 1; however, the predicted flight time of 12 – 17 min represents an ideal upper limit. The actual endurance will be influenced by factors such as battery discharge characteristics, mechanical system losses, and aerodynamic efficiency; therefore, traceable records must be obtained during flight tests by timing the duration from full charge to the onset of low-voltage alarm.

(4) Control and Power Supply Link Verification. Section 5.2 indicates that a 2S 7.4 V/60 mAh battery is planned for the power supply link; however, the receiver's input voltage, the continuous and peak currents of its output stages, and its over-limit protection behavior still require confirmation via the product specification sheet and bench tests. The current system lacks electronic throttle control, flight control, and an IMU; instead, both motors are connected in parallel to the same receiver channel and regulated via PWM to adjust the voltage. Consequently, employing a reduced throttle during takeoff cannot substitute for a rigorously verified current-limiting circuit. The effectiveness of the over-limit protection strategies (throttle reset to zero, motor shutdown, gliding/landing) also requires validation through flight testing.

VII. Conclusion

1. The geometric reconstruction and cross-validation of the "dual motors — enclosed gear train — dual rocker arms" transmission scheme have been completed: featuring an internal reduction ratio of 26.5, an external module of 0.6 mm, and a single-stage transmission with tooth counts of 11:34 ($i = 3.0909$); the three sets of center distances (6.600/13.500/20.400 mm) precisely matched the coordinate positions of the frame holes (deviation $\leq 3 \mu\text{m}$), thereby demonstrating mutual validation between the gear arrangement and the frame design.

2. The 11-tooth driving gear is susceptible to undercutting; the shift amount and the actual root profile must be verified against the original tooth profile diagram or physical specimens.

3. A motor electromagnetic – mechanical load curve was established, yielding a maximum mechanical power of 1.16 W for a single motor (including a total gearbox efficiency of 27%). Based on this, the algorithmic oversight regarding "frequency hopping at 5 Hz" was corrected: the frequency is 9.71 Hz under full throttle and no-load conditions, and 9.43 Hz under balanced load conditions; the 5 Hz frequency corresponds to throttle positions between 55% and 60%, and the range – throttle curve exhibits a flat profile.

4. After applying the Grashof criterion and verification within the closed range (17.68 – 21.06 mm) to the four-bar linkage, the total swing angle was determined to be 105.0° ($\pm 52.5^\circ$), and the minimum transmission angle was 35.0° ; it was further noted that there exists a spatial offset of 19.84° between the two links on the same side; based on this, the crank pin angle corresponding to a 90° phase difference between the front and rear wings was determined to be 70.2° .

5. Under the assumed material parameters, the root bending stress under the locked-rotor condition is 38.2 MPa, which exceeds the selected allowable value of 33 MPa; the starting current limiting and gear strength requirements require bench testing for verification.

6. The design mass is 19.49 g, and the required power, estimated based on the given model, ranges from 0.25 to 0.32 W. The power matching conclusions are influenced by motor parameters, the flapping-wing aerodynamic model, and mechanical losses; therefore, they cannot replace actual flight test results.

7. This draft has completed the geometric and preliminary dynamic calculations for the transmission design. Subsequent work will require supplementary evidence, including specifications of the motor and receiver, strength analysis of the PLA-printed gears, actual current values, pulse frequency, phase information, flight endurance, and flight video footage.

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