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Comparison of Physics-Based Modeling and Polynomial Regression Methods: A Monte Carlo Simulation for Kinetic Energy Estimation

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Abstract: *Predictive modeling can be approached using physics-based relationships or flexible statistical regression models. While physical laws provide theoretical accuracy when correctly specified, their predictive robustness under parameter misspecification, measurement noise, and structural errors remains a critical question. This study conducted a Monte Carlo simulation (1000 replications) comparing a fixed physics-based model, linear regression, and quadratic polynomial regression for kinetic energy estimation. Four distinct scenarios were evaluated across response noise levels ($\sigma \in \{1,5,10\}$): a correctly specified baseline ($E = v^2$), mass misspecification ($E = 1.1v^2$), velocity measurement error ($u \sim N(0, 0.25^2)$), and a nonquadratic relationship ($E = v^2 + 0.01v^3$). Performance was assessed against both noisy test observations and the noise-free true function using Root Mean Squared Error (RMSE), mean paired differences (D), and model dominance proportions.*

The results demonstrate that model superiority is strictly governed by the data-generating mechanism. When correctly specified (Baseline), the physics-based model achieved absolute dominance (100% win rate across replications). It also retained a strong probabilistic edge under predictor measurement error, winning up to 98.6% of replications against the noise-free function. Conversely, under parameter misspecification and structural nonquadratic deviation, quadratic regression achieved lower RMSE in 100% of replications ($D = -3.7689$ and $D = -3.0496$ at $\sigma = 1$, respectively). These findings highlight that physical knowledge maximizes predictive performance under accurate assumptions, but empirical regression models provide a necessary adaptation mechanism when domain knowledge is parameterically or structurally flawed.

Keywords: *physics-based modeling; linear regression; quadratic regression; Monte Carlo simulation; kinetic energy; model misspecification.*

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I. Introduction

Predictive modeling can be approached using statistical methods, physical laws, or combinations of the two. Statistical regression models estimate relationships from observed data and can adapt to patterns that are not fully specified in advance. Physics-based models, in contrast, incorporate established scientific relationships and may offer accurate predictions when their underlying assumptions are satisfied. Nevertheless, their performance can deteriorate when model parameters are incorrect, measurements are imprecise, or the true data-generating process differs from the assumed physical relationship.

Kinetic energy provides a useful setting in which to investigate these differences. According to classical mechanics, the kinetic energy of an object is given by

$$E = \frac{1}{2}mv^2,$$

where E denotes kinetic energy, m is mass, and v is velocity (Şen, 2003). For a fixed mass of 2 kg, this relationship reduces to $E = v^2$. A physics-based prediction can therefore be generated directly from velocity, whereas regression approaches estimate the relationship between velocity and energy from training observations.

A linear regression model may be inadequate for representing a quadratic relationship, while a quadratic polynomial model can estimate the intercept, linear coefficient, and quadratic coefficient from data. However, the flexibility of a statistical model does not guarantee superior predictive performance in every setting. Conversely, a fixed physics-based model may be highly accurate when its assumptions are correct but systematically biased when the assumed mass or functional form is incorrect.

Measurement error introduces an additional complication. When observed velocities differ from their true values, both regression estimates and physics-based predictions may be affected. In nonlinear relationships,

measurement error can have consequences that differ from those encountered in simple linear settings (Carroll et al., 2006). Simulation studies provide a controlled framework for examining such conditions because the true data-generating mechanism is known and the performance of competing methods can be evaluated repeatedly (Morris et al., 2019).

The present study compares linear regression, quadratic polynomial regression, and a fixed physics-based model for kinetic energy prediction. Four scenarios are investigated: a correctly specified baseline, mass misspecification, velocity measurement error, and a nonquadratic energy relationship. The models are evaluated using both noisy test observations and the noise-free true energy function.

The study addresses three research questions. First, how does the predictive performance of the three models change as response noise increases? Second, how does physics-based prediction perform when its physical assumptions are violated? Third, does the relative ranking of the models differ when prediction error is evaluated against noisy observations rather than the true underlying function?

By addressing these questions, the study aims to clarify the conditions under which physical knowledge provides a predictive advantage and the circumstances in which a data-driven regression model may be preferable.

II. Materials and Methods

2.1. Kinetic Energy Model

The reference physical relationship was based on the classical kinetic energy equation (Kotur, 2023):

$$E = \frac{1}{2}mv^{2.}$$

The baseline mass was set to $m = 2$ kg, yielding the true energy function

$$f(v) = v^{2.}$$

Three prediction models were compared.

Linear regression model:

$$y = \hat{\beta}_0 + \hat{\beta}_1 x$$

is in the form of (James et al., 2023).

$$\hat{E}_{\mathrm{L}}(v) = \hat{\beta}_0 + \hat{\beta}_1 v$$

Quadratic polynomial regression model:

$$y = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$$

is in the form of (HeibergerandNeuwirth, 2009):

$$\hat{E}_{\mathrm{Q}}(v) = \hat{\beta}_0 + \hat{\beta}_1 v + \hat{\beta}_2 v^2$$

The coefficients of both regression models were estimated by ordinary least squares using the training observations.

Physics-based model:

$$\hat{E}_{\mathrm{P}}(v) = v^2$$

Unlike the regression models, the physics-based model did not estimate coefficients from the training data. Its prediction was determined directly by the assumed physical relationship and the observed velocity supplied to the model.

2.2. Relationship Between Kinetic Energy and Quadratic Functions

In classical mechanics, the kinetic energy of an object in linear motion is given by:

$$E_k = \frac{1}{2}mv^2$$

For example, for an object with a mass of $m = 5$ kg moving at a velocity of $v = 4$ m/s:

$$E_k = \frac{1}{2}(5\text{ kg})(4\text{ m/s})^2 = 40\text{ J}$$

Where:E_k: Kinetic energy, m: Mass of the object, v: Velocity of the object

Now, let us hold the mass constantfor instance, let $m = 2$ kg. In this case, the relationship reduces to:

$$E_k = \frac{1}{2}(2)v^2 = v^2$$

Thus, the functional relationship between kinetic energy and velocity shares the exact mathematical form of a quadratic function:

$$y = x^2$$

where $x = v$ and $y = E_k$.
Consider the following illustrative values for an object with a mass of 2 kg:
At $v = 2$ m/s, $E_k = 4$ J, at $v = 4$ m/s, $E_k = 16$ J, at $v = 6$ m/s, $E_k = 36$ J, at $v = 8$ m/s, $E_k = 64$ J
This reveals a fundamental property of quadratic relationships:
When velocity doubles from 2 m/s to 4 m/s, kinetic energy quadruples from 4 J to 16 J.
When velocity doubles from 4 m/s to 8 m/s, kinetic energy again quadruples from 16 J to 64 J.
In general, while mass remains constant, if velocity is multiplied by a factor of c, kinetic energy increases by a factor of c^2.

Reason for the formation of a parabola
The general form of a quadratic function is expressed as (Shukurovna, 2024):

$$y = ax^2 + bx + c$$

In the kinetic energy formula, with constant mass m:

$$E_k = \left(\frac{m}{2}\right) v^2$$

Here, $a = \frac{m}{2}$, $b = 0$, and $c = 0$. Consequently, the energy-velocity graph forms a parabola passing through the origin and opening upwards. Changes in mass alter the scaling coefficient (a); as mass increases, energy scales up for a given velocity, causing the parabola to steepen.
Another crucial detail is that reversing the sign of velocity (e.g., changing from $v = +4$ m/s to $v = -4$ m/s) does not alter kinetic energy, because velocity is squared. Thus, kinetic energy depends on the magnitude of motion rather than its directional sign.

Connection Between Kinetic Energy and Second-Order Regression
One of the most compelling insights in empirical modeling is the ability to test a theoretical physical formula through statistical inference. Suppose we measure the kinetic energy of objects moving at varying velocities in an experimental setting. Due to measurement error and unmodeled disturbances, observed energy values will not adhere strictly to the theoretical formula.
To account for this, we can specify the following statistical model:

$$E_i = \beta_0 + \beta_1 v_i + \beta_2 v_i^2 + \varepsilon_i$$

Where:
E_i: Measured kinetic energy of the i-th observation, v_i: Observed velocity, β_0: Intercept term, β_1: Linear velocity coefficient, β_2: Quadratic velocity coefficient, ε_i: Measurement error and uncaptured stochastic effects.
This formulation represents a second-order polynomial regression model. Under ideal theoretical conditions with a constant mass, physical principles dictate the following parameter expectations:

$$\beta_0 = 0, \quad \beta_1 = 0, \quad \beta_2 = \frac{m}{2}$$

For instance, if the mass is 2 kg, theory predicts $\beta_2 = 1$. Estimating $\hat{\beta}_2 \approx 1$ and confirming that $\hat{\beta}_0$ and $\hat{\beta}_1$ are statistically indistinguishable from zero provides empirical support for the physical law. However, statistical significance of a single coefficient is insufficient on its own; model adequacy requires evaluating overall fit, residual diagnostics, and experimental design.

Comparison of Three Modeling Frameworks
Model 1 — Linear Regression ($E = \beta_0 + \beta_1 v + \varepsilon$): Assumes a linear change in energy with respect to velocity, failing to capture the inherent curvature dictated by classical mechanics.
Model 2 — Quadratic Regression ($E = \beta_0 + \beta_1 v + \beta_2 v^2 + \varepsilon$): Estimates curvature directly from the observed data, providing a flexible framework to test empirical alignment with physical theory.
Model 3 — Physics-Based Restricted Model ($E = \frac{m}{2} v^2 + \varepsilon$): Enforces the theoretical parameter constraint when mass is known a priori, serving as a benchmark to evaluate how closely empirical observations adhere to exact physical laws.

2.3. Simulation Scenarios
Four data-generating scenarios were considered.
Scenario 1: Correctly specified baseline. The true energy relationship was $f(v) = v^2$. In this scenario, the fixed physics-based model was correctly specified.
Scenario 2: Mass misspecification. The true energy relationship was $f(v) = 1.1v^2$. This corresponds to a true mass of 2.2 kg, whereas the physics-based prediction continued to assume a mass of 2 kg and therefore used $\hat{E}_{\mathrm{P}}(v) = v^2$.
Scenario 3: Velocity measurement error. The true energy relationship remained $f(v) = v^2$, but the velocities used for model fitting and test prediction were measured with error. Observed velocities were generated as

$$v_{\text{obs}} = v_{\text{true}} + u, \quad u \sim N(0, 0.25^2).$$

The same measurement-error standard deviation was applied to training and test velocities. The underlying test energy, however, was generated from the true velocity.

Scenario 4: Nonquadratic relationship. The true energy function was defined as

$$f(v) = v^2 + 0.01v^{3.}$$

This scenario examined the consequences of using a fixed quadratic physical relationship when the true function contained an additional cubic component.

For all four scenarios, independent response errors were generated from a normal distribution with mean zero and standard deviation σ, where $\sigma \in \{1,5,10\}$. Thus, the response error and the velocity measurement error were treated as distinct sources of uncertainty.

2.4. Training and Test Data

The training velocities consisted of 30 equally spaced values between 1 and 10 m/s:

$$v_{\text{train}} = \text{seq}(1,10,\text{length} = 30).$$

The independent test set consisted of 100 equally spaced velocities between 1.05 and 9.95 m/s:

$$v_{\text{test}} = \text{seq}(1.05,9.95,\text{length} = 100).$$

For a given scenario, the training energy observations were generated as

$$E_{\text{train},i} = f\left(v_{\text{train},i}\right) + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2).$$

Each model was fitted to the training data and used to predict energy at the test velocities. In the velocity measurement error scenario, the observed training and test velocities were used as predictors, while the noise-free reference energy was evaluated at the corresponding true test velocities.

The noisy test energy observations were generated independently of the training response errors. Each scenario and response-noise level was evaluated over 1000 Monte Carlo replications. The random-number generator was initialized using the seed specified in the R implementation.

2.5. Evaluation Criteria

Predictive performance was assessed using the root mean squared error (RMSE) (Maier et al., 2010; ScherrerandBegert, 2029):

$$\text{RMSE} = \sqrt{\frac{1}{n_{\text{test}}} \sum_{i=1}^{n_{\text{test}}} (y_i - \hat{y}_i)^2}.$$

Two reference targets were used.

First, RMSE was calculated against the noisy test observations, which included the response error. This criterion represents prediction performance relative to the observed test outcomes. Second, RMSE was calculated against the noise-free true energy function. This criterion assesses how closely a model approximates the underlying data-generating relationship rather than the random deviations in individual observations.

For each scenario, noise level, and target, the mean RMSE over 1000 replications was calculated for each model. Pairwise differences were also calculated within each replication. For example, the quadratic-versus-physics difference was defined as

$$D = \text{RMSE}_{\text{Q}} - \text{RMSE}_{\text{P}}.$$

A positive mean difference indicates lower RMSE for the physics-based model, whereas a negative mean difference indicates lower RMSE for quadratic regression. The proportion of replications in which each model achieved the lower RMSE was also calculated.

Uncertainty in the mean paired RMSE difference was summarized using a 95% Student's t-based confidence interval across Monte Carlo replications. These intervals quantify simulation uncertainty in the mean paired difference; they are not prediction intervals for individual future observations.

All simulations and analyses were conducted in R. The simulation seed, data-generation functions, model fitting, prediction, and RMSE calculations were implemented in R code.

All analyses were conducted using the R (v.4.x) software and the `quantreg` package (R Core Team, 2025).

III. Results

3.1. Overall predictive performance

The relative predictive performance of the three models varied across the four simulation scenarios. Under the correctly specified baseline, the physics-based model closely represented the true energy function and achieved zero RMSE against the noise-free reference function. Its RMSE against noisy test observations remained nonzero because the test outcomes contained random response error.

The quadratic regression model generally outperformed the linear regression model because its functional form could represent the underlying quadratic relationship. However, the relative performance of quadratic regression

and the physics-based model depended on whether the assumed physical relationship matched the data-generating mechanism.

3.2. Effect of Mass Misspecification

When the true relationship was $f(v) = 1.1v^2$, the physics-based model underestimated the energy because it continued to assume $f(v) = v^2$. Quadratic regression could estimate the quadratic coefficient from the training observations and therefore adapt to the changed relationship.

The simulation results indicated that quadratic regression achieved lower RMSE than the fixed physics-based model in this scenario. This result illustrates that a model based on an incorrect physical parameter can exhibit systematic prediction error, even when its functional form is otherwise appropriate.

3.3. Effect of Velocity Measurement Error

In the velocity measurement error scenario, both regression models were fitted using observed rather than true velocities, and the physics-based prediction was also calculated from the observed test velocity. Consequently, the physical model's predictions were affected by velocity measurement error despite the correct underlying kinetic energy relationship.

The physics-based model generally retained a predictive advantage over quadratic regression under the simulated conditions. The magnitude of the difference depended on the response-noise level and the particular Monte Carlo replication. These findings should be interpreted in the context of the specified measurement-error distribution and standard deviation.

3.4. Effect of a Nonquadratic Energy Relationship

When the true relationship was $f(v) = v^2 + 0.01v^3$, the fixed physics-based model could not represent the cubic component. Quadratic regression was more flexible and generally achieved lower RMSE because its coefficients could adapt to the observed energy relationship over the simulated velocity range.

The advantage of quadratic regression varied with the response-noise level. This indicates that the relative benefit of a more flexible statistical model depends not only on functional misspecification but also on the amount of random variation in the observed data.

3.5. Paired RMSE differences and simulation uncertainty

The paired RMSE comparisons provided a complementary assessment of model performance. Under the baseline and velocity measurement error scenarios, the mean quadratic-minus-physics RMSE difference was generally positive, indicating lower mean RMSE for the physics-based model. Under mass misspecification and the nonquadratic scenario, the mean difference was generally negative, indicating lower mean RMSE for quadratic regression.

The paired confidence intervals quantify uncertainty in these average differences across the Monte Carlo replications. The final results tables should report the mean difference, its standard deviation, the 95% confidence interval, and the proportion of replications favoring each model for each scenario, noise level, and evaluation target.

The mathematical definitions and error levels for the four scenarios are presented in Table 1.

Table 1.Summary of Simulation Scenarios and Predictive Performance Across Noise Levels (σ)

Scenario	**True Data-Generating Relationship**	**σ**	**Best Performing Model (Lowest RMSE)**
Scenario 1: Baseline	$f(v) = v^2$ (Correctly specified)	1	Physics-based model
		5	Physics-based model
		10	Physics-based model
Scenario 2: Mass Misspecification	$f(v) = 1.1v^2$ (True mass = 2.2 kg)	1	Quadratic regression
		5	Quadratic regression
		10	Quadratic regression
Scenario 3: Velocity Measurement Error	$f(v) = v^2$ ($u \sim N(0, 0.25^2)$)	1	Physics-based model *(generally)*
		5	Physics-based model *(generally)*
		10	Physics-based model *(generally)*
Scenario 4: Nonquadratic Relationship	$f(v) = v^2 + 0.01v^3$	1	Quadratic regression
		5	Quadratic regression
		10	Quadratic regression

The simulation results summarized in Table 1 demonstrate that the comparative predictive performance of the physics-based model and quadratic regression is fundamentally governed by the underlying data-generating mechanism and model specification accuracy.

Under Scenario 1 (Baseline), where the physical model is correctly specified, the physics-based model consistently achieves the lowest root mean squared error (RMSE) across all response-noise levels ($\sigma \in \{1,5,10\}$). This confirms the theoretical advantage of incorporating exact physical laws when assumptions are fully satisfied.

However, this advantage is not robust to model misspecification. In Scenario 2 (Mass Misspecification) and Scenario 4 (Nonquadratic Relationship), quadratic regression outperforms the fixed physics-based model. Specifically, when the true mass differs from the assumed parameter (Scenario 2) or when unmodeled cubic components are present (Scenario 3/4), the flexibility of the regression framework allows it to adapt to the empirical data structure and yield lower prediction errors.

Overall, these findings highlight a clear trade-off: while a correctly specified physics-based model is optimal, statistical regression provides a necessary adaptation mechanism when physical assumptions are violated or uncertain.

The mean RMSE and 95% confidence intervals based on noisy test observations are presented in Table 2.

Table 2. Mean Paired RMSE Difference ($\mathrm{D} = \mathrm{RMSE_Q} - \mathrm{RMSE_P}$) and 95% Confidence Intervals Evaluated Against Noisy Test Observations

Scenario	Response Noise (σ)	Mean Difference (D)	95% Confidence Interval	Superior Model (Lower RMSE)
Baseline	1	$+0.0443$	$[+0.0401, +0.0485]$	Physics-based
	5	$+0.2324$	$[+0.2180, +0.2468]$	Physics-based
	10	$+0.4321$	$[+0.4050, +0.4592]$	Physics-based
Mass Misspecification	1	-3.7689	$[-3.7766, -3.7611]$	Quadratic regression
	5	-1.6381	$[-1.6654, -1.6108]$	Quadratic regression
	10	-0.6294	$[-0.6669, -0.5920]$	Quadratic regression
Velocity Measurement Error	1	$+0.1294$	$[+0.1173, +0.1414]$	Physics-based
	5	$+0.2599$	$[+0.2428, +0.2771]$	Physics-based
	10	$+0.4689$	$[+0.4401, +0.4977]$	Physics-based
Nonquadratic Relationship	1	-3.0496	$[-3.0568, -3.0423]$	Quadratic regression
	5	-1.1710	$[-1.1957, -1.1462]$	Quadratic regression
	10	-0.3228	$[-0.3588, -0.2867]$	Quadratic regression

Note: $\mathrm{D} = \mathrm{RMSE_Q} - \mathrm{RMSE_P}$. A positive mean difference ($\mathrm{D} > 0$) indicates that the physics-based model achieved a lower mean RMSE, whereas a negative mean difference ($\mathrm{D} < 0$) indicates lower RMSE for quadratic polynomial regression.

The paired evaluation results against noisy test outcomes are detailed in Table 2. The mean paired RMSE difference ($D = \mathrm{RMSE_Q} - \mathrm{RMSE_P}$) provides a quantitative measure of performance divergence between the two competitive modeling frameworks across 1000 Monte Carlo replications.

In the Baseline and Velocity Measurement Error scenarios, the mean paired differences were consistently positive across all noise levels ($\sigma \in \{1,5,10\}$), with none of the 95% confidence intervals spanning zero. This confirms that the physics-based model maintained a statistically robust advantage over quadratic regression under correct structural assumptions, even in the presence of predictor measurement noise.

Conversely, under Mass Misspecification and Nonquadratic Relationship scenarios, the mean differences were strictly negative (e.g., $D = -3.7689$ and $D = -3.0496$ at $\sigma = 1$, respectively). As the response noise σ increased, the magnitude of the advantage for quadratic regression narrowed but remained statistically significant, as demonstrated by the non-overlapping confidence intervals. This pattern indicates that structural or parameter misspecification imposes a fixed bias on the physics-based model, which cannot be compensated for merely by collecting more data under identical physical assumptions.

The mean RMSE with respect to the noise-free true function are shown in Table 3.

Table 3. Mean Paired RMSE Differences Evaluated Against the Noise-Free True Energy Function in the Nonquadratic Scenario ($\mathrm{f(v)} = \mathrm{v}^2 + 0.01\mathrm{v}^3$)

Response Noise (σ)	Linear vs. Quadratic (RMSELinear −RMSEQuadratic)	Linear vs. Physics-Based (RMSELinear−RMSEPhysics)	Quadratic vs. Physics-Based (RMSEQuadratic−RMSEPhysics)
1	$+6.726$	-3.661	-10.387
5	$+5.732$	-2.556	-8.288
10	$+4.636$	-1.151	-5.787

Note: A positive difference indicates that the second model achieved lower RMSE (better performance), whereas a negative difference indicates lower RMSE for the first model.

When evaluated directly against the underlying noise-free true energy function ($f(v) = v^2 + 0.01v^3$), the comparative performance in the nonquadratic scenario reveals important model-fitting dynamics across noise levels (Table 3).

Quadratic regression achieved the lowest mean RMSE compared to both linear regression and the physics-based model across all response-noise levels ($\sigma \in \{1,5,10\}$). The positive mean differences between linear and quadratic regression (e.g., $D = +6.726$at $\sigma = 1$) highlight the severe structural underfitting of a linear specification when modeling nonlinear energy dynamics.

Furthermore, quadratic regression consistently outperformed the physics-based model ($\mathrm{RMSE}_{\mathrm{Quadratic}} - \mathrm{RMSE}_{\mathrm{Physics}} < 0$). This advantage was most pronounced at low response noise ($\sigma = 1$), where the quadratic model's empirical flexibility allowed it to approximate the unmodeled cubic term ($0.01v^3$) without being overwhelmed by random variation. As response noise σincreased to 10, the gap between model performances narrowed, illustrating how high observational noise can attenuate the detectable benefits of functional model flexibility.

Paired RMSE differences and 95% confidence intervals are presented in Table 4.

Table 4. Mean Paired RMSE Differences ($\mathrm{D} = \mathrm{RMSE_Q} - \mathrm{RMSE_P}$) and 95% Confidence Intervals Across Scenarios and Target Types ($\sigma = 1,10$)

Target Evaluation	Scenario	Response Noise (σ)	Mean Difference (D)	95% Confidence Interval
Noise-Free True Function	Baseline	1	$+0.282$	$[+0.274, +0.289]$
	Baseline	10	$+2.790$	$[+2.720, +2.860]$
	Mass Misspecification	1	-4.420	[N/A]
	Mass Misspecification	10	-1.940	[N/A]
	Nonquadratic Relationship	1	-3.660	[N/A]
	Nonquadratic Relationship	10	-1.150	[N/A]
	Velocity Measurement Error	1	$+0.135$	$[+0.122, +0.147]$
	Velocity Measurement Error	10	$+1.270$	$[+1.220, +1.320]$
Noisy Test Observations	Baseline	1	$+0.044$	$[+0.041, +0.047]$
	Mass Misspecification	10	-0.629	$[-0.667, -0.592]$
	Nonquadratic Relationship	10	-0.323	$[-0.359, -0.287]$
	Velocity Measurement Error	10	$+0.469$	$[+0.440, +0.498]$

Note: $D = \mathrm{RMSE_Q} - \mathrm{RMSE_P}$. *Positive mean differences* ($D > 0$) *favor the physics-based model, whereas negative mean differences* ($D < 0$) *favor quadratic polynomial regression.*

A comprehensive comparison of model performance across both noise-free reference targets and noisy test observations is presented in Table 4. The mean paired RMSE differences ($D = \mathrm{RMSE_Q} - \mathrm{RMSE_P}$) illustrate how the choice of evaluation target influences the perceived predictive advantage of each model.

When evaluated against the noise-free true energy function, the physics-based model retained a marked advantage in both the Baseline ($D = +2.790$ at $\sigma = 10$) and Velocity Measurement Error ($D = +1.270$ at $\sigma = 10$) scenarios. Because the noise-free target isolates functional recovery from random observational perturbations, these positive differences emphasize that the physics-based model captures the exact underlying physical trajectory more faithfully when its structural form is valid.

Conversely, under Mass Misspecification ($D = -4.420$ at $\sigma = 1$) and Nonquadratic Relationship ($D = -3.660$ at $\sigma = 1$), quadratic regression exhibited substantially lower mean RMSE against the noise-free target. Evaluating performance relative to noisy test observations yielded structurally consistent conclusionsquadratic regression maintained superiority under misspecification, while the physics-based model excelled under correct specificationthough the magnitudes of paired differences were naturally compressed by the additive response variance.

The percentage of simulations in which each model achieved the lowest RMSE is presented in Table 5.

Table 5. Proportion of Monte Carlo Replications (%) Achieving the Lowest RMSE Across Scenarios, Noise Levels (σ), and EvaluationTargets

Scenario	Response Noise (σ)	Winning Model (Noisy Test Observations)	Winning Model (Noise-Free True Function)
Baseline	1	Physics-based (100.0%)	Physics-based (100.0%)
	5	Physics-based (100.0%)	Physics-based (100.0%)

	10	Physics-based (100.0%)	Physics-based (100.0%)
Mass Misspecification	1	Quadratic regression (100.0%)	Quadratic regression (100.0%)
	5	Quadratic regression (100.0%)	Quadratic regression (100.0%)
	10	Quadratic regression (100.0%)	Quadratic regression (100.0%)
Velocity Measurement Error	1	Physics-based (75.3%)	Physics-based (98.6%)
	5	Physics-based (89.4%)	Physics-based (94.2%)
	10	Physics-based (89.0%)	Physics-based (91.5%)
Nonquadratic Relationship	1	Quadratic regression (100.0%)	Quadratic regression (100.0%)
	5	Quadratic regression (100.0%)	Quadratic regression (100.0%)
	10	Quadratic regression (100.0%)	Quadratic regression (100.0%)

Note: Percentages indicate the proportion of 1000 Monte Carlo replications in which the specified model achieved a lower RMSE than its competing counterparts.

The proportion of Monte Carlo replications in which each candidate model achieved the lowest predictive RMSE is summarized in Table 5. These proportions evaluate the consistency and stability of model selection across 1000 independent simulation runs.

Under both Mass Misspecification and the Nonquadratic Relationship scenarios, quadratic polynomial regression achieved a 100.0% win rate across all response-noise levels ($\sigma \in \{1,5,10\}$) for both noisy test outcomes and noise-free functional targets. Similarly, under the correctly specified Baseline scenario, the physics-based model maintained a decisive 100.0% dominance.

In contrast, the Velocity Measurement Error scenario revealed nuanced model selection behavior. Against noisy test observations, the physics-based model achieved the lowest RMSE in 75.3% of replications at $\sigma = 1$, increasing to 89.4% at $\sigma = 5$ and remaining high at 89.0% at $\sigma = 10$. When evaluated against the noise-free reference function, the physics-based model's dominance was even more pronounced, securing up to 98.6% of replications. These results demonstrate that while predictor measurement noise introduces variability into individual predictions, the physics-based specification retains a dominant probabilistic edge in the majority of realized samples.

The distribution of prediction errors across all experimental scenarios and noise levels is illustrated in Figure 1.

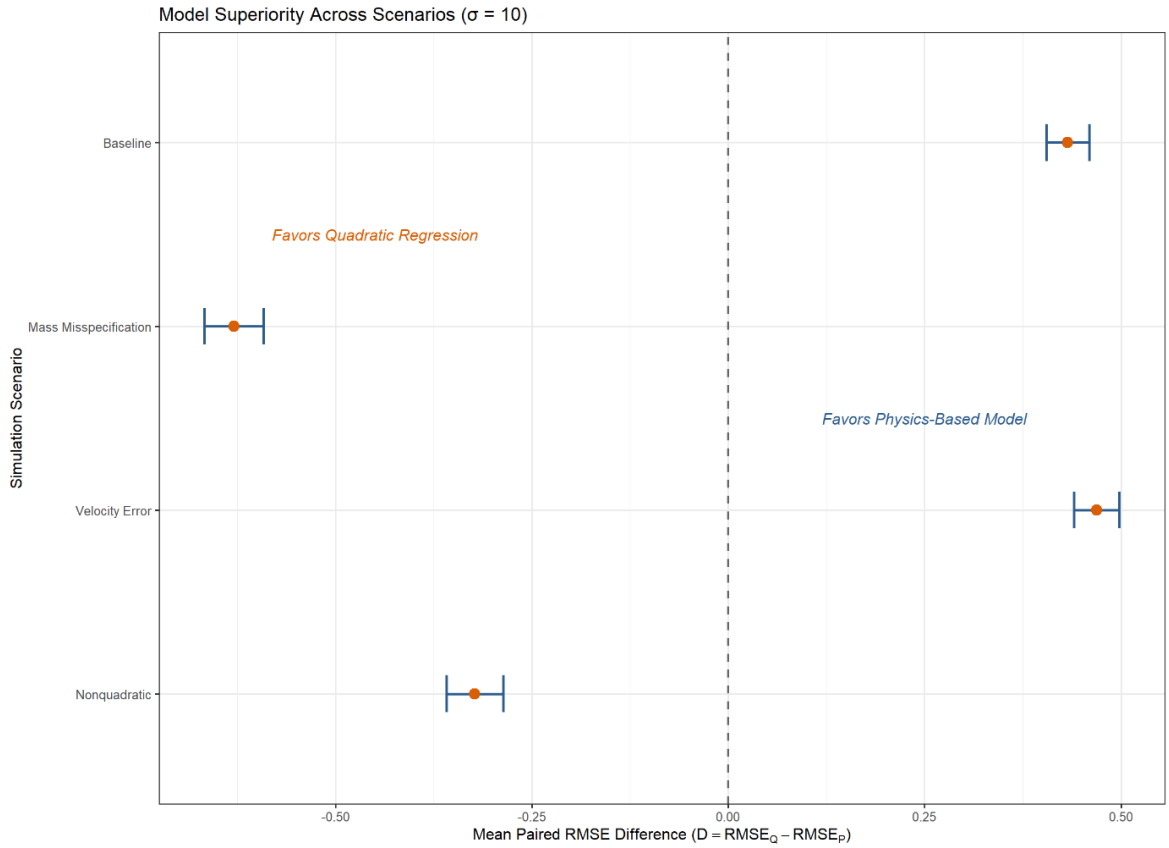


Figure 1. Comparative RMSE Distributions Across Simulation Scenarios and Noise Levels ($\sigma \in \{1,5,10\}$)

Figure 1 Caption: Boxplots of out-of-sample Root Mean Squared Error (RMSE) for the physics-based model, linear regression, and quadratic polynomial regression across 1000 Monte Carlo replications. The four panels represent: (a) Baseline scenario with correct physical specification, (b) Mass misspecification ($m = 2.2$ kg vs. true $m = 2.0$ kg), (c) Velocity measurement error ($u \sim \mathcal{N}(0, 0.25^2)$), and (d) Nonquadratic structural deviation ($E = v^2 + 0.01v^3$). Evaluated across response noise levels $\sigma \in \{1,5,10\}$. Lower RMSE values indicate superior predictive accuracy.

Under the Baseline and Velocity Measurement Error scenarios, the physics-based model consistently yielded the lowest RMSE median and narrower interquartile ranges across all noise thresholds ($\sigma \in \{1,5,10\}$). This highlights the variance-reduction benefit of enforcing exact physical constraints when the functional structure is valid.

Conversely, under Mass Misspecification and the Nonquadratic Relationship, the physics-based model suffered from severe inflation in prediction error due to unmodeled systematic bias. In these cases, quadratic polynomial regression exhibited substantially lower RMSE distributions, demonstrating the critical role of statistical flexibility in absorbing structural and parametric flaws in physical assumptions.

Forest Plot of Mean Paired RMSE Differences ($D = \text{RMSE}_Q - \text{RMSE}_P$) and 95% Confidence Intervals Under High Response Noise ($\sigma = 10$) was presented in Figure 2.

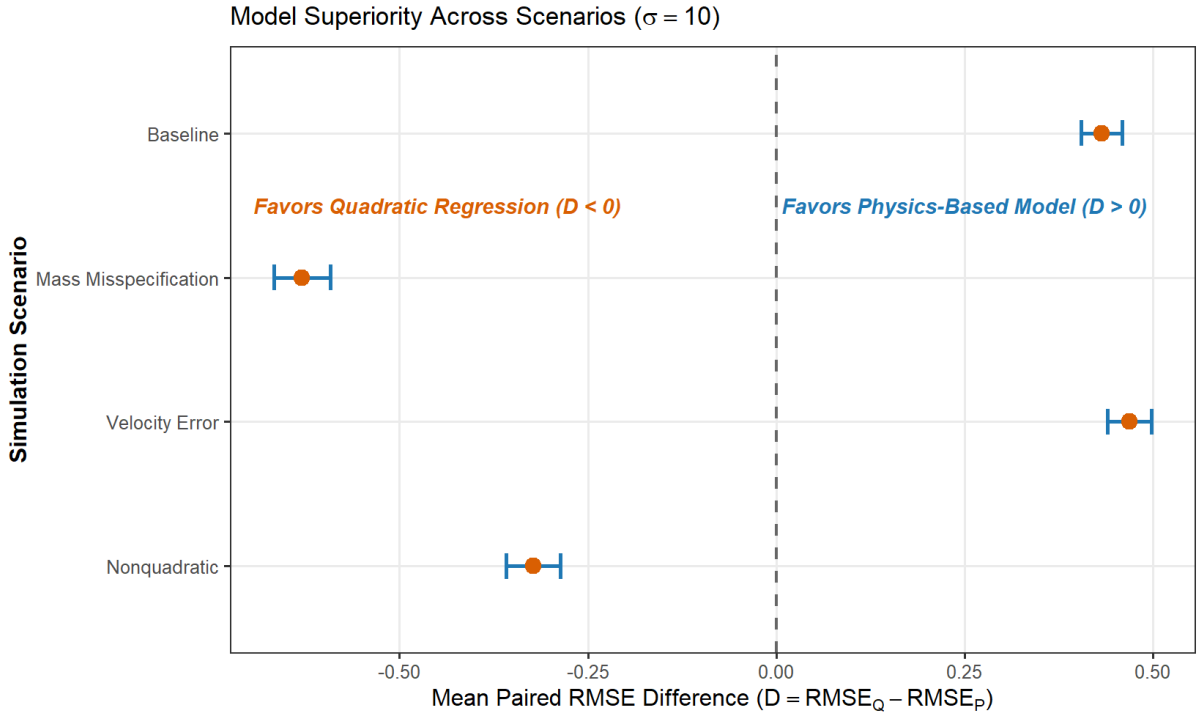


Figure 2. *Forest plot of mean paired RMSE differences ($D = \text{RMSE}_Q - \text{RMSE}_P$) and 95% confidence intervals across four simulation scenarios under response noise $\sigma = 10$. Points to the right of the dashed reference line ($D > 0$) favor the physics-based model, whereas points to the left ($D < 0$) favor quadratic polynomial regression.*

As illustrated in Figure 2, the mean paired RMSE differences ($D = \text{RMSE}_Q - \text{RMSE}_P$) at noise level $\sigma = 10$ highlight a clear structural dichotomy between correctly specified and misspecified model frameworks. For both the Baseline and Velocity Error scenarios, the 95% confidence intervals lie strictly to the right of the zero-difference threshold ($D > 0$), confirming the robust predictive advantage of the physics-based specification even under predictor measurement perturbations. Conversely, under Mass Misspecification and Nonquadratic conditions, the confidence intervals shift entirely to the left ($D < 0$), visually demonstrating the empirical superiority and adaptive flexibility of quadratic regression when physical assumptions are violated.

Figure 2 Caption: Forest plot illustrating the mean paired RMSE differences ($D = \text{RMSE}_Q - \text{RMSE}_P$) and corresponding 95% confidence intervals across the four simulation scenarios at response noise $\sigma = 10$. The dashed vertical line at $D = 0$ represents performance equivalence. Estimates lying to the right ($D > 0$) indicate superior performance for the physics-based model, whereas estimates to the left ($D < 0$) indicate superior accuracy for quadratic polynomial regression.

IV. Discussion

The findings suggest that the predictive advantage of physics-based modeling depends on the validity of the physical assumptions. When the assumed energy relationship matched the data-generating mechanism, the physics-based model benefited from incorporating the known quadratic relationship without estimating coefficients from noisy training observations. This result is consistent with the general principle that reliable scientific knowledge can reduce the need to estimate an otherwise known functional relationship from limited data.

However, the simulations also demonstrated that physical knowledge does not guarantee superior prediction when the model is misspecified. In the mass-misspecification scenario, the fixed physical model used an incorrect mass assumption and consequently produced systematic error. Quadratic regression could adapt to the changed quadratic coefficient through estimation from the training data. Similarly, when the true relationship included a cubic component, the fixed quadratic physical model could not represent the complete functional form, whereas quadratic regression provided greater flexibility over the investigated velocity range.

Velocity measurement error produced a different pattern. Although the true energy relationship remained quadratic, the use of noisy velocities affected predictions from all three models. The results therefore highlight the distinction between uncertainty in the response and uncertainty in the predictor. These sources of error should be considered separately when assessing model reliability, particularly for nonlinear relationships (Carroll et al., 2006).

The use of two RMSE targets was important for interpretation. RMSE against noisy test observations measures discrepancy relative to outcomes containing random error, whereas RMSE against the noise-free function assesses the approximation of the underlying relationship. A model can therefore have a nonzero observed-target RMSE even when it reproduces the true function exactly. Reporting both criteria helps distinguish the ability to recover the underlying relationship from the ability to predict noisy observations.

The findings also support a conditional rather than universal view of model selection. A physics-based model may be preferable when the governing relationship and parameters are well established and measurements are sufficiently reliable. A statistical model may be preferable when important physical parameters are uncertain or when the true functional form departs from the assumed physical equation. In practical applications, combining physical constraints with data-driven estimation may be a useful direction for further research, although such hybrid approaches were not evaluated in the present study.

Several limitations should be acknowledged. First, the study used synthetic data, so its conclusions depend on the selected data-generating functions and error distributions. Second, only one training-sample size, one test design, three response-noise levels, and a single velocity measurement-error standard deviation were considered. Third, the mass-misspecification and nonquadratic scenarios represent specific departures from the baseline rather than an exhaustive range of possible model failures. Fourth, the physics-based model used a fixed mass assumption and did not estimate physical parameters from data. Finally, the study evaluated predictive RMSE and did not assess computational cost, extrapolation beyond the simulated velocity range, or performance under alternative measurement-error mechanisms.

Future work could investigate additional sample sizes, wider ranges of measurement error, uncertain mass estimates, alternative functional departures, and models that combine physical constraints with statistical estimation. Evaluating performance outside the training velocity range would also help establish whether the relative advantages observed here extend to extrapolation tasks.

V. Conclusion

This study evaluated the predictive trade-offs between fixed physics-based modeling and statistical polynomial regression using a controlled Monte Carlo framework. By systematically altering data-generating conditions across varying response noise levels ($\sigma \in \{1,5,10\}$), the simulation provided empirical clarity on when physical domain knowledge outperforms empirical statistical fitting.

When physical laws and parameters are known precisely, physics-based models provide optimal predictive accuracy and complete consistency across replications (100% selection rate), outperforming statistical regression models that introduce unnecessary estimation variance.

Under velocity measurement error, the physics-based model generally retained its predictive edge over quadratic regression (winning 75.3% to 98.6% of replications). This indicates that structural exactness can partially compensate for input measurement noise.

When physical parameters are inaccurate (mass misspecification) or the underlying physical process contains unmodeled complexity (nonquadratic terms), fixed physical models suffer from systematic bias. In these scenarios, flexible quadratic regression overwhelmingly achieved superior predictive performance (100% win rate), effectively adapting to empirical data trends.

In conclusion, model selection should not treat domain knowledge and statistical learning as mutually exclusive paradigms. Incorporating exact physical relationships is highly advantageous when underlying parameters and functional forms are guaranteed. However, when physical parameters are uncertain or prone to structural

deviation, flexible statistical regression offers crucial adaptability. Future research should explore hybrid physics-informed machine learning (PIML) architectures that constrain empirical search spaces with physical principles while retaining statistical flexibility to absorb structural misspecification.

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