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Rational Sequences

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Abstract
Several rational sequences are examined. Integer entries are sought, and it is determined whether 1 is a term.

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Section 1: $S_n = \frac{n}{1}, \frac{n-1}{2}, \frac{n-2}{3}, \ldots$

In this paper, n is a positive integer > 1. Consider the sequence, $S_n = \frac{n}{1}, \frac{n-1}{2}, \frac{n-2}{3}, \ldots$ The sequence decreases, and we only include terms ≥ 1.

Example: (1) $S_{11} = \frac{11}{1}, \frac{10}{2}, \frac{9}{3}, \frac{8}{4}, \frac{7}{5}, \frac{6}{6}$, all integers, except for $\frac{7}{5}$. In contrast, **(2)** $S_{12} = \frac{12}{1}, \frac{11}{2}, \frac{10}{3}, \frac{9}{4}, \frac{8}{5}, \frac{7}{6}$, containing no integers except for $\frac{12}{1}$.

Theorem 1: 1 is a term of S_n if and only if n is odd.

Proof: When n is odd, the difference between the numerator, n, and denominator, 1, of the initial term is *even*. Since these differences decrease by 2 each time we subtract 1 from the numerator and add 1 to the denominator, the difference between the numerator and denominator remains *even* till it reaches 0, at which point the numerator equals the denominator, thereby yielding 1. When n is even, the differences between numerators and denominators are *odd*, in which case no term can equal 1. ■

Question 1: How many terms of S_n are integers?

Question 2: How many terms of S_n are reduced to lowest terms?

Section 2: $L_n = \frac{n}{n}, \frac{n+1}{n-1}, \frac{n+2}{n-2}, \ldots,$

Let n be a positive integer such that $n > 1$. Consider the sequence, $L_n = \frac{n}{n}, \frac{n+1}{n-1}, \frac{n+2}{n-2}, \ldots,$ The sequence increases, and it terminates when the denominator is 1.

Example: **(1)** $L_8 = \frac{8}{8}, \frac{9}{7}, \frac{10}{6}, \frac{11}{5}, \frac{12}{4}, \frac{13}{3}, \frac{14}{2}, \frac{15}{1}$. **(2)** $L_9 = \frac{9}{9}, \frac{10}{8}, \frac{11}{7}, \frac{12}{6}, \frac{13}{5}, \frac{14}{4}, \frac{15}{3}, \frac{16}{2}, \frac{17}{1}$.

Observe that L_8 contains four integers, while L_9 contains five integers. As we did in Section 1, we ask two questions:

Question 3: How many terms of L_n are integers?

Question 4: How many terms of L_n are reduced to lowest terms?

Section 3: $M_n = \frac{n}{1}, \frac{n+1}{2}, \frac{n+2}{3}, \ldots,$

Let $n > 2$, and let $M_n = \frac{n}{1}, \frac{n+1}{2}, \frac{n+2}{3}, \ldots,$

Example: **(1)** $M_8 = \frac{8}{1}, \frac{9}{2}, \frac{10}{3}, \frac{11}{4}, \frac{12}{5}, \frac{13}{6}, \boxed{\frac{14}{7}}, \frac{15}{8}, \ldots$ **(2)** $M_9 = \frac{9}{1}, \frac{10}{2}, \frac{11}{3}, \frac{12}{4}, \frac{13}{5}, \frac{14}{6}, \frac{15}{7}, \boxed{\frac{16}{8}}, \ldots$

Theorem 2: **(a)** The sequence, M_n, is strictly decreasing. **(b)** 2 is a term of M_n. **(c)** 1 is not a term of M_n, for $n > 1$.

Proof: **(a)** Let $\frac{a}{b}$ be a term of M_n. Then $a > b$. We must show that $\frac{a}{b} > \frac{a+1}{b+1}$. Multiplying both sides by $b(b + 1)$, our inequality becomes $a(b + 1) > b(a + 1)$, or $ab + a > ab + b$, which becomes $a > b$. Reversing our steps yields $\frac{a}{b} > \frac{a+1}{b+1}$. **(b)** $\frac{n}{1}$ is the first term of M_n, so the general term is $\frac{n+k}{1+k}$. (For the first term, $k = 0$.) We will find a value of k, such that $\frac{n+k}{1+k} = 2$ or, equivalently, $n + k = 2 + 2k$, which becomes $k = n - 2$. **(c)** The difference between the numerator and denominator of each term is the constant, $n - 1$, so 1 is never achieved. ■

Section 4: $P_n = \frac{n}{1}, \frac{n-2}{3}, \frac{n-4}{5}, \ldots,$

Let $P_n = \frac{n}{1}, \frac{n-2}{3}, \frac{n-4}{5}, \ldots,$ The sequence decreases, and we only include terms ≥ 1.

Example: **(1)** $P_{13} = \frac{13}{1}, \frac{11}{3}, \frac{9}{5}, \boxed{\frac{7}{7}}$ **(2)** $P_{15} = \frac{15}{1}, \frac{13}{3}, \frac{11}{5}, \frac{9}{7}.$

Theorem 3: 1 is a term of P_n if and only if $4 \mid n - 1$.

Proof: The general term is $\frac{n-2k}{1+2k}$. (For the first term, $k = 0$.) We will find a value of k, such that $\frac{n-2k}{1+2k} = 1$ or, equivalently, $n - 2k = 2 + 4k$, which becomes $k = n - 2$. ■

Section 5: $B_n = \frac{n}{1}, \frac{n+2}{3}, \frac{n+4}{5}, \ldots,$

Let $B_n = \frac{n}{1}, \frac{n+2}{3}, \frac{n+4}{5}, \ldots,$

Example: **(1)** $B_{13} = \frac{13}{1}, \frac{15}{3}, \frac{17}{5}, \frac{19}{7}, \cdots$ **(2)** $B_{15} = \frac{15}{1}, \frac{17}{3}, \frac{19}{5}, \frac{21}{7}, \cdots$

Theorem 4: The sequence decreases, and approaches 1, but 1 is not a term of B_n.

Proof: The term corresponding to k is $\frac{n+2k}{2k+1}$. Then we wish to find a value of k for which this expression equals 1. That is, we must solve $\frac{n+2k}{2k+1} = 1$, or $n + 2k = 2k + 1$. Then $n = 1$, which we reject. Also, since n is constant, $\lim_{k \to \infty} \frac{n+2k}{2k+1} = 1$. The proof that the sequence decreases is left for the reader. ■

Section 6: $V_n = \frac{n}{1}, \frac{n-3}{4}, \frac{n-6}{7}, \frac{n-9}{10}, \ldots,$

Let $V_n = \frac{n}{1}, \frac{n-3}{4}, \frac{n-6}{7}, \frac{n-9}{10}, \ldots,$

Example: **(1)** $V_{31} = \frac{31}{1}, \frac{28}{4}, \frac{25}{7}, \frac{22}{10}, \frac{19}{13}, \boxed{\frac{16}{16}}$. **(2)** $V_{21} = \frac{21}{1}, \frac{18}{4}, \frac{15}{7}, \frac{12}{10}$.

Since the numerators are decreasing and the denominators are increasing, V_n is a decreasing sequence.
Theorem 5: 1 is a term of V_n if and only if $6 \mid n-1$.

Proof: The term corresponding to k is $\frac{n-3k}{3k+1}$. Then we wish to find a value of k for which this expression equals 1. That is, we must solve $\frac{n-3k}{3k+1}$, or $n - 3k = 3k + 1$, which becomes $6k = n - 1$. Then $\boxed{k = \frac{n-1}{6}}$. Note that this yields a natural number if and only if $6 \mid n - 1$, implying that n is one more than a multiple of 6. ■

Bibliography

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