# Passive Force on Retaining Wall Supporting Φ Backfill Considering Curvilinear Rupture Surface

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Abstract:—The passive resistance of retaining walls denotes the stability of the wall against failure. Most of the available methods for calculating the passive earth pressure are based on linear failure surface. In this paper, the expression of seismic passive earth pressure acting on inclined rigid retaining wall is derived considering the non-linear failure surface. The Horizontal Slices Method with limit equilibrium technique has been adopted to establish the nonlinearity of rupture surface. Detailed discussion of results with variation of several parameters like angle of internal friction ( $\Phi$ ), angle of wall friction ( $\delta$ ), wall inclination angle ( $\alpha$ ) and surcharge loading (q) have been conducted.

**Keywords:**—Active earth pressure, Passive earth pressure,  $\Phi$  backfill, rigid retaining wall, wall inclination, curvilinear rupture surface.

#### I. INTRODUCTION

The lateral earth pressure is generally a function of the soil properties, the wall and the intensity of loadings acting on the wall. Earth pressure theories constitute one of the most important parts in the Geotechnical structures. Coulomb (1776) was the first to establish the formulation for Active and Passive earth pressure for the retaining structures. Rankine's theory (1857) has given a general solution for determination of passive earth pressure. Graphical methods have also been established by Culmann (1865). The analyses have been conducted considering the  $\Phi$  nature of backfill. The calculations are based on Limit equilibrium technique. In most of the cases, the researchers have assumed the failure surface to be linear. In practical condition, the failure surface may not be linear. In this particular analysis, the failure surface has been assumed to be non-linear and Horizontal Slice Method has been taken into consideration to calculate the optimum value of passive earth pressure acting on each slice. Generalized equation has been established to find the solution for 'n' number of slices.

# II. METHOD OF ANALYSIS

The analysis has been made in the same way as in the case of active earth pressure. The line of action of  $P_p$  and R is acting from the upward direction. The slicing and other analysis are the same. Details may be seen in the Figures 1 and 2. The forces acting on the wall has been calculated by considering the following parameters:

 $H_{i-1}$ ,  $H_i$  = Horizontal shear acting on the top and bottom of the i<sup>th</sup> slice.

 $W_i$  = Weight of the failure wedge for i<sup>th</sup> slice.

 $V_{i-1}, V_i = V$ ertical load (UDL) on top and bottom of i<sup>th</sup> slice.

 $\Phi$  = The angle of internal friction of soil.

 $P_{pi} = Passive earth pressure on i<sup>th</sup> slice.$ 

 $R_i^{r}$  = The reaction of the retained soil on i<sup>th</sup> slice.

 $\delta$  = The angle of wall friction.

## III. DERIVATION OF VARIOUS FORMULATIONS DURING THE PASSIVE CASE OF EOUILIBRIUM

Applying the force equilibrium condition, we can solve the equations in the following pattern

$$\sum H = 0$$

$$P_{i}\cos(\delta - \alpha) = R_{i}\cos(\phi - \theta_{1} - (i - 1)\theta_{r})$$

$$+ (\gamma(\Delta H)^{2}\tan\phi \left\{ \left[ \sum_{m=i}^{n-1} \{\tan(\theta_{1} + m\theta_{r})\} \right] + (n - i)\tan\alpha - (i - 1)(\tan(\theta_{1} + (i - 1)\theta_{r}) + \tan\alpha)) \right\}$$

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V = 0

(5)

$$P_{i}\sin(\delta - \alpha) = -R_{i}\sin(\phi - \theta_{1} - (i - 1)\theta_{r}) - (i - \frac{1}{2})\gamma(\Delta H)^{2}(\tan \alpha + \tan(\theta_{1} + (i - 1)\theta_{r}))$$

Solving these two equations, we find the value of generalized equation for Passive earth pressure for i<sup>th</sup> slice as follows:

$$\frac{\gamma}{2}(\Delta H)^{2} \left| + 2\tan\phi \left\{ \left[ \sum_{m=i}^{n-1} \left\{ \tan(\theta_{1} + m\theta_{r}) \right\} \right] + (n-i)\tan\alpha - (i-1)(\tan(\theta_{1} + (i-1)\theta_{r}) + \tan\alpha)) \right\} \right] \right\}$$

$$P_{p_{i}} = \frac{\sin(\phi - \theta_{1} - (i-1)\theta_{r})}{\sin(\phi + \delta - \alpha - \theta_{1} - (i-1)\theta_{r})}$$
Where,  $\tan(\theta_{1} + m\theta_{r}) = 0$  for  $i = n$  (7)

The passive earth pressure coefficient can be simplified as,

n

$$K_p = \frac{\sum_{i=1}^{i} P_{pi}}{\frac{\gamma}{2} \mathrm{H}^2}$$

(8)

(6)

Optimization of the passive earth pressure coefficient Kip is done for the variables  $\theta 1$  and a satisfying the optimization criteria. The optimum value of Kip is given in Table 1.

## IV. PARAMETRIC STUDY

A detailed Parametric study has been conducted to find the of variations of static passive earth pressure with various other parameters like soil friction angle ( $\Phi$ ), wall inclination ( $\alpha$ ), wall friction angle ( $\delta$ ). For  $\Phi = 10^{0}$ ,  $20^{0}$ ,  $30^{0}$ ,  $40^{0}$  and  $\delta = 0$ ,  $\Phi/2$ ,  $\Phi$  and  $\alpha = +20^{0}$ , 0, -20°

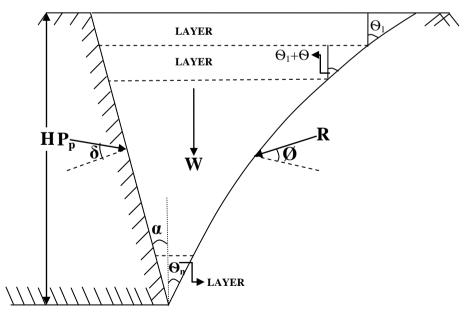
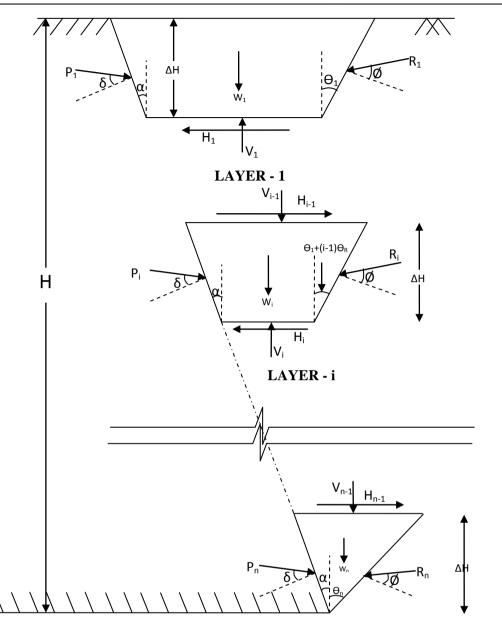


Fig.1 Inclined Retaining Wall (Passive state of equilibrium)



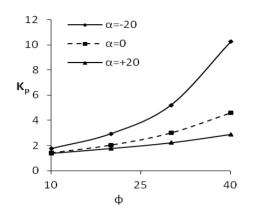
LAYER - n

Fig.2 Detailed drawing showing various components of the retaining wall along with slices (passive state of equilibrium).

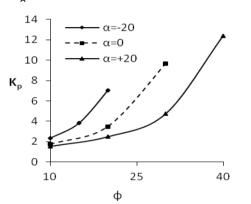
# The details of these studies are presented below:

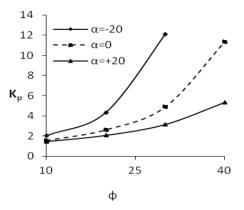
#### 4.1 Effect of Inclination of the wall (α)

Fig.3 to 5 represents the effect of inclination of the wall on the passive earth pressure for different value of  $\delta$ . From these Figures, it is seen that due to increase in  $\alpha$ , passive earth pressure is going to be decreased. The reason behind it is that when the inclination of the wall is positive with the vertical then the passive resistance acting on the wall is less compared to the wall inclined negative with the vertical. For example at  $\Phi = 10^{\circ}$  and  $\delta = 0^{\circ}$ , the decrease in  $K_p$  is 4.4% for  $\alpha = +20^{\circ}$  over  $\alpha = 0$  value, where as the increase in  $K_p$  is 22.8% over  $\alpha = 0$  value for  $\alpha = -20$ . Again at  $\Phi = 20^{\circ}$  and  $\delta = \Phi /2$ , the decrease in  $K_p$  is 21.5% for  $\alpha = +20^{\circ}$  over  $\alpha = 0$  value, where as the increase in  $K_p$  is 28.3% for  $\alpha = +20^{\circ}$  over  $\alpha = 0$  value, where as the decrease in  $K_p$  is 102.4% over  $\alpha = 0$  value for  $\alpha = -20$ . It is also observed that the value of  $K_p$  for  $\alpha = -20^{\circ}$  and  $\delta = \Phi$ , increases with the increase in the value of  $\Phi$  upto  $\Phi = 20$ .

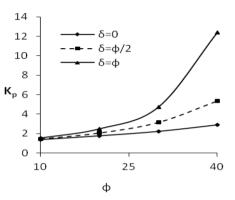


**Fig.3.** Shows the variation of Passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at different Wall inclination angles ( $\alpha$ = -20, 0, 20) for  $\delta$  =





**Fig.4.** Shows the variation of Passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at different Wall inclination angles ( $\alpha$ = -20, 0, 20) for  $\delta = \Phi/2$ .

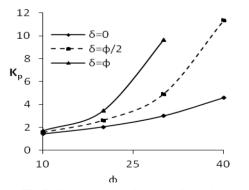


**Fig.5** Shows the variation of Passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at different Wall inclination angles ( $\alpha$ = -20, 0, 20) for  $\delta$  =  $\Phi$ .

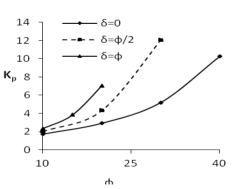
**Fig.6** Shows the variations of passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at different Wall friction angles ( $\delta$ = 0,  $\Phi/2$ ,  $\Phi$ ) for  $\alpha = +20^{0}$ .

#### **1.2** Effect of Wall Friction Angle $(\delta)$

Fig.6 to 8 represents the effect of wall friction angle on the passive earth pressure for different value of  $\alpha$ . From these Figures, it is seen that due to increase in  $\delta$ , passive pressure is going to be increased. The reason behind it is that the frictional resistance between wall and soil is increasing with the increase in the value of  $\delta$ . For example at  $\Phi = 10^{\circ}$  and  $\alpha = 20^{\circ}$ , the increase in  $K_p$  is 6.3% for  $\delta = \Phi/2$  over  $\delta = 0$  value, where as the increase in  $K_p$  is 13.6% for  $\delta = \Phi$  over  $\delta = 0$ . Again at  $\Phi = 20^{\circ}$  and  $\alpha = -20^{\circ}$ , the increase in  $K_p$  is 28.7% for  $\delta = \Phi/2$  over  $\delta = 0$  value, where as the increase in  $K_p$  is 70.7% for  $\delta = \Phi$  over  $\delta = 0$ . Again at  $\Phi = 20^{\circ}$  and  $\alpha = -20^{\circ}$ , the increase in  $K_p$  is 47.7% for  $\delta = \Phi/2$  over  $\delta = 0$  value, where as the increase in  $K_p$  is 140.2% for  $\delta = \Phi$  over  $\delta = 0$ .



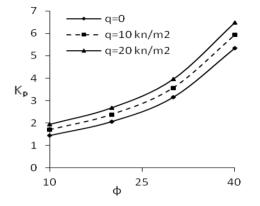
**Fig.7** Shows the variations of passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at different Wall friction angles ( $\delta$ = 0,  $\Phi$  /2,  $\Phi$ ) for  $\alpha = 0^0$ .



**Fig.8** Shows the variations of passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at different Wall friction angles ( $\delta$ = 0,  $\Phi/2$ ,  $\Phi$ ) for  $\alpha = -20^{\circ}$ .

## **1.3** Effect of Surcharge (q)

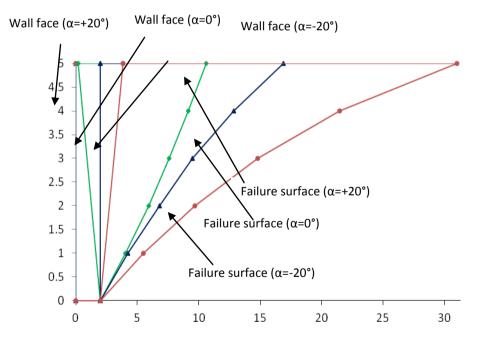
Fig. 9 shows the variations of passive earth pressure for inclusion of surcharge. It is seen that the value of passive earth pressure increases gradually with the increase of surcharge. For  $\Phi = 30^{0}$ ,  $\alpha = 20^{0}$  and  $\delta = \Phi /2$ , the increment in K<sub>p</sub> is 18% and 36% for q=10KN/m<sup>2</sup> and 20KN/m<sup>2</sup> respectively compared to q = 0 for constant height.



**Fig.9** Shows the variations of passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) for different surcharge loads ( $\alpha = 20^{0}$ ,  $\delta = \Phi/2$ ).

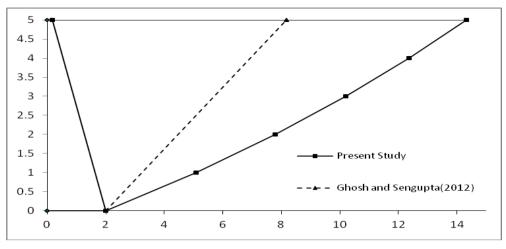
#### 4.3 Wall Inclination and Nonlinearity of Failure Surface

Fig. 10 shows the nonlinearity of failure surface of backfill (passive case) for different values of wall inclinations  $(\alpha = -20^{\circ}, 0^{\circ}, +20^{\circ})$ . It is seen that the failure angle increases with the increase in the wall inclination angles. For example, at  $\Phi=30^{\circ}, \delta=\Phi/2$  and  $\alpha = +20^{\circ}$ , the value of failure angle at bottom is 64° whereas the value of failure angle at top is 55°. The comparison shows that the value of failure angle is 41.5° in case of Ghosh and Sengupta (2012) for the aforesaid conditions. Also Fig. 11 -12 shows that the failure wedge is quite different as compared to the failure surface of the Ghosh and Sengupta (2012) analysis. The surface shows curvilinear shape as it progresses upward.



**Fig.10** Shows the nonlinearity of failure surface of backfill (passive case) for different values of wall inclinations,  $\alpha = -20^{\circ}$ ,  $0^{\circ}$ ,  $+20^{\circ}$  at  $\Phi = 30^{\circ}$ ,  $\delta = \Phi/2$ .

Comparison of Failure Surface -



**Fig.11.** Shows the comparison between failure surface of backfill (passive case) for wall inclination,  $\alpha = +20^{\circ}$  at  $\Phi = 30^{\circ}$ ,  $\delta = \Phi/2$ .

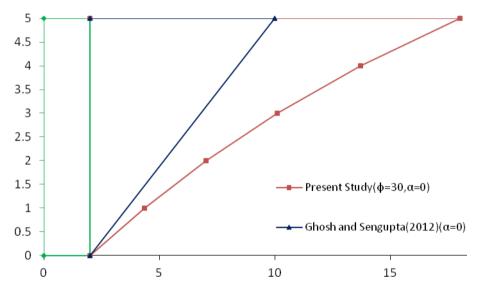
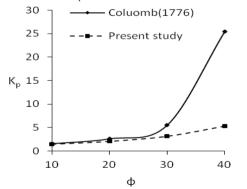


Fig.12 Shows the comparison between failure surface of backfill (passive case) for wall inclination.  $\alpha = 0^0$  at  $\Phi = 30^0$ .  $\delta = \Phi/2$ .

#### 4.4 Comparison of Results

Fig. 13 shows the variations of passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at Wall friction angles  $\delta = \Phi/2$  for  $\alpha = 20^{\circ}$ . K<sub>p</sub> increases uniformly with the increase in the value of soil friction angle ( $\Phi$ ). It can also be observed from Table 2 that the value of K<sub>p</sub> around 15% lesser than the values of Classical Coulomb (1776) theory.



**Fig.13** Shows the variations of passive earth pressure coefficient with respect to soil friction angle ( $\Phi$ ) at Wall friction angles  $\delta = \Phi/2$  for  $\alpha = 20^{\circ}$ .

Ø	Table 1: Passive Earth Pressure Coefficients (Static Case)         Co-efficient of Passive earth pressure, K <sub>p</sub>				
	δ	α=-20	α=0	α=+20	
10	0	1.745	1.420	1.358	
	Ø/2	2.023	1.568	1.444	
	Ø	2.351	1.727	1.543	
20	0	2.933	2.039	1.754	
	Ø/2	4.331	2.625	2.061	
	Ø	7.044	3.481	2.493	
30	0	5.204	3.00	2.216	
	Ø/2	12.096	4.909	3.146	
	Ø		9.643	4.732	
40	0	10.246	4.599	2.898	
	Ø/2		11.348	5.338	
	Ø			12.388	

 Table 1: Passive Earth Pressure Coefficients (Static Case)

**Table 2:** Comparison of Results (Passive Case)

φ	δ	α	K <sub>p</sub> , Present study	K <sub>p</sub> , Coulomb (1776)		
10	5	20	1.444	1.579		
20	10	20	2.061	2.616		
30	15	20	3.146	5.501		
40	20	20	5.338	25.412		

# NOTATIONS

 $\theta_1$  =Failure surface angle with vertical for top slice.

 $\theta_n$  =Failure surface angle with vertical for bottom slice.

 $\theta_R$  = Rate of change of failure surface angle.

 $\Phi$  = Soil friction angle.

 $\delta$  = Wall friction angle.

 $\alpha$  = Wall inclination angle with the vertical.

 $P_p$  = passive earth pressure.

 $\hat{H_{1}}$ ,  $\hat{H_{2}}$  = horizontal shear.

 $\Delta H$  = height of each slice.

 $W_i$  = weight of i<sup>th</sup> slice.

R = soil reaction force.

 $V_1$  = vertical load (UDL) acting on the bottom surface of the 1<sup>st</sup> layer.

 $V_2$  = vertical load (UDL) acting on the top surface of the 1<sup>st</sup> layer.

 $\gamma$  = unit weight of soil.

# V. CONCLUSIONS

The present study provides an analytical model for the solution of passive earth pressure on the back of a battered face retaining wall supporting  $\phi$  backfill. Using horizontal slice method, the solution of this model generates a non-linear failure surface and the value of passive earth pressure coefficients as obtained from this solution are of lesser magnitude in comparison to other available solutions like Coulomb (1776). The nature of the failure surface may be sagging or hogging in nature depending upon the soil-wall parameters. The results of the analysis are shown in tabular form and a detailed parametric study is done for the variation of different parameters. The present study shows that the seismic passive earth pressure co-efficient (K<sub>p</sub>) increases due to the increase in wall friction angle ( $\delta$ ), soil friction angle ( $\Phi$ ) and surcharge (q); at the same time the value of K<sub>p</sub> decreases with the increase in wall inclinations ( $\alpha$ ).

The model as suggested here can be used for seismic analysis of retaining wall earth pressure problems using both pseudodynamic and pseudo-static methods.

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