International Journal of Engineering Inventions
ISSN: 2278-7461, www.ijeijournal.com
Volume 1, Issue 7 (October2012) PP: 76-81

LATERAL PARAMETER ESTIMATION USING REGRESSION

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Abstract:—The paper presents the estimation of lateral aerodynamic parameters using Regression (Least Squares method) from real flight data of Hansa-3 aircraft. The conventional or output error methods such as Regression, Maximum Likelihood methods require exact model postulation for accurate estimation of parameters. The three sets of lateral-directional flight data were gathered during flight testing of Hansa-3 Aircraft. The processed data was used to carry out data compatibility check. The compatible data sets were used to estimate the lateral (lateral-directional) aerodynamic parameters using Regression. The results have been presented in tabular as well as in graphical form and compared to the wind tunnel values to assess the accuracy level.

Keywords:—Lateral, Aerodynamic, Parameters, Mathematical Model

I. INTRODUCTION

The parameter estimation [1-5] from real flight data is a routine task for many aerospace organizations. Parameter estimation is the process of determining the best possible estimates of the parameters occurring in the model used to represent a system. Although reasonably accurate parameters can be obtained through analytical predictions and wind tunnel testing, the parameter estimation using flight data help to enhance the confidence in the estimates significantly. Designing optimal controls and autopilots, expansion of flight envelopes, updating simulators and verification of overall aircraft performance are some of the uses of parameter estimation. The conventional methods (Output Error method and its variants [1-5] such as Least Squares (LS) & Maximum Likelihood (ML) methods) used for identification assume the model to be exact for parameter estimation. The Least Squares method [5], also known as Regression analysis, is one of the oldest problems in the estimation theory with numerous engineering applications, including flight vehicles. The LS method assumes the independent variables to be error and noise free and dependent variables corrupted by uniformly distributed noise. The major limitation of these methods is that they yield asymptotically biased and inconsistent estimates in the presence of measurement errors and noise in the independent variables. The LS method has been shown to adequately estimate lateral-directional aerodynamic parameters from lateral-directional real flight data in the present work. The paper presents the generation of flight data, data-compatibility check, aerodynamic model, parameter estimation and concluding remark.

II. GENERATION OF FLIGHT DATA

A flight test program using the Hansa-3 [Fig. 1] aircraft, an in-house fully instrumented research aircraft, was conducted at the Flight Laboratory, IIT Kanpur to gather the real flight data with the help of a data acquisition system. An onboard measurement system installed on test aircraft provided the measurements using dedicated sensors for a large number of signals such as aircraft motion variables, atmospheric conditions, control surface deflections etc. The measurements made in flight were recorded on board at a sampling rate of 50 Hz using a suitable interface with a standard Laptop computer. The three sets of lateral-directional flight data were acquired by executing the aileron/rudder control inputs during flight tests. The three lateral-directional flight data sets nomenclatured as HLD1, HLD2 & HLD3 (Where H and LD refer to Hansa-3 and Lateral-Directional respectively) are processed and presented graphically [Figs. 2-4] in terms of the motion variables. These figures present the variation of lateral-directional motion variables such as angle of sideslip (β), roll angle (ϕ), yaw angle (ψ), roll rate (p), yaw rate (r), linear acceleration (a_y) along y-axis and velocity (V) pertaining to doublet aileron and/or rudder (δ_a and/or δ_r) control inputs.

Fig. 1 The Hansa-3 research-aircraft

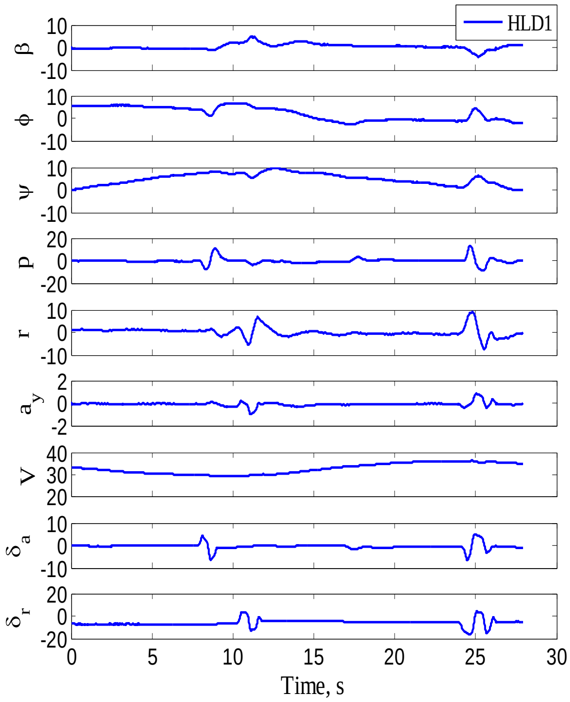

Fig. 2 Lateral-directional flight data: HLD1

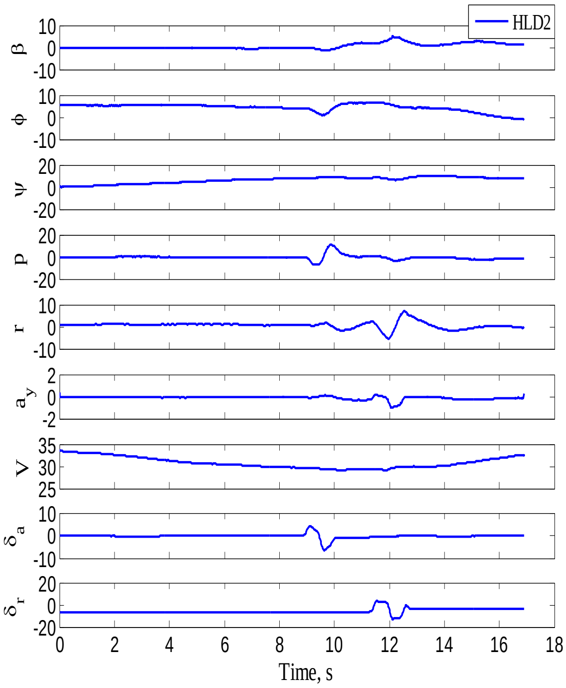

Fig. 3 Lateral-directional flight data: HLD2

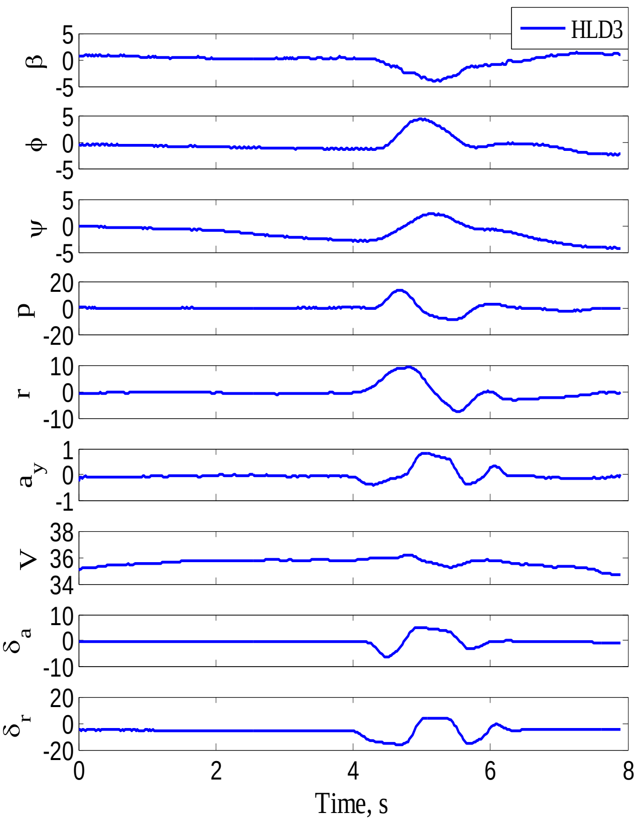

Fig. 4 Lateral-directional flight data: HLD3

It can be seen that the trim angle for aileron and rudder are approximately zero and -7 degrees, respectively. The values of ψ, p, r and a_y corresponding to trim condition can be observed to be zero. The trim value of β can be observed to be approximately zero. The velocity was kept around 32 ms^{-1}. It can also be observed that the lateral variables (ϕ, p) were affected when the ailerons were deflected from trim condition whereas the rudder deflection affects the directional variables (β, r). A positive increase in aileron deflection from trim condition results in negative roll rate and reduction in bank angle whereas a positive increase in rudder deflection results in negative yaw rate and positive angle of sideslip.

III. DATA COMPATIBILITY CHECK

The data compatibility check, which is also known as flight path reconstruction [4], is an integral part of aircraft parameter estimation. The recorded real flight data are mainly corrupted by systematic errors like scale factors, zero shift biases and time shifts. These errors introduce data incompatibility; for example, data incompatibility would exist by way of the measured incidence angles not being in agreement with those reconstructed from the accelerometer and rate gyro measurements. The main aim of a data compatibility check is to ensure that the measurements used for subsequent aerodynamic model identification are consistent and error free (as far as possible).

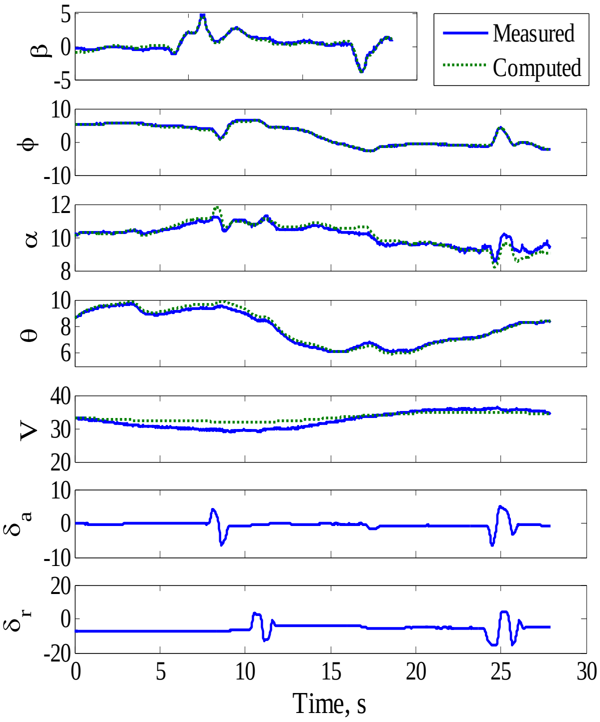

Fig. 5 Data Compatibility Check: HLD1

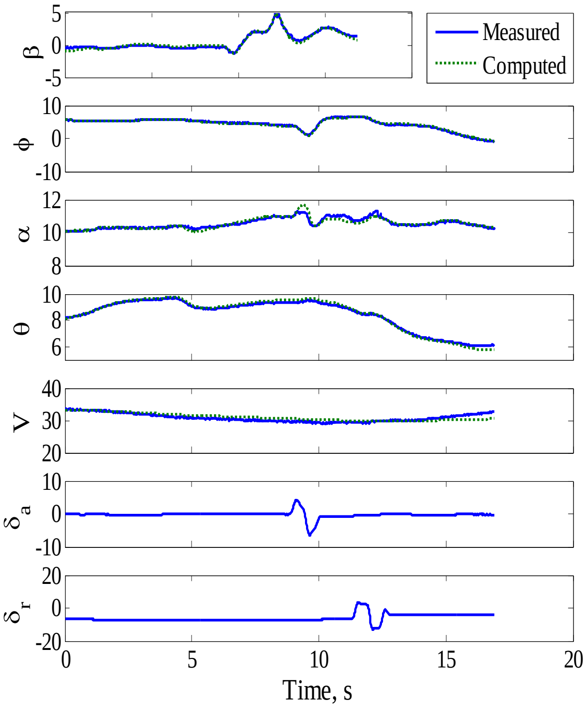

Fig. 6 Data Compatibility Check: HLD2

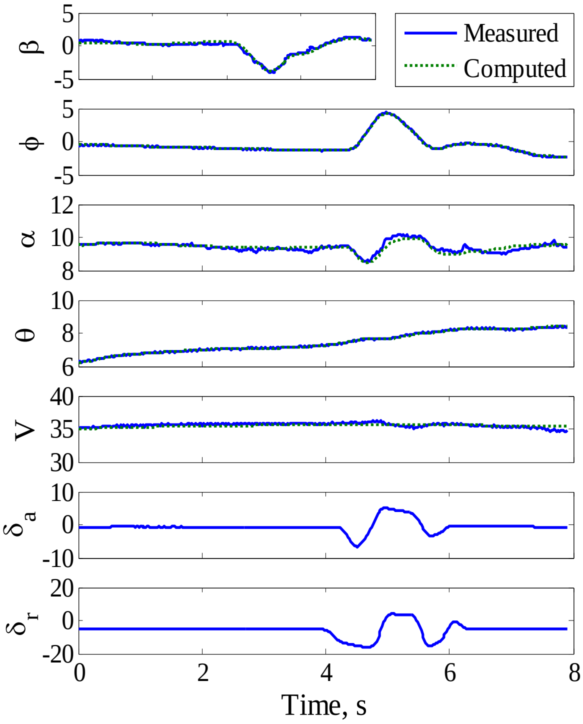

Fig. 7 Data Compatibility Check: HLD3

The data compatibility check was carried out on both flight data sets pertaining to lateral-directional case using observation equations and the ML method. Figs. (5-7) present the measured and computed response of motion variables such as β, ϕ, α, θ and V obtained during the data compatibility check from flight data pertaining to lateral-directional control inputs. It can be observed that the computed response compares well with the measured response for most of the motion variables (α, θ, β and ϕ). A similar trend in the variation of velocity was also observed. The unknown parameter vector [equation (1)], representing scale factor and biases, was considered adequate for reconstructing lateral-directional dynamics of Hansa-3 aircraft.

$$\Theta = \left[\Delta a_x \;\; \Delta a_y \;\; \Delta a_z \;\; \Delta p \; \Delta q \;\; \Delta r \;\; K_\beta \;\; \Delta\beta\right]^T \tag{1}$$

The Maximum Likelihood method was used to estimate the compatibility factors [equation (1)] from flight data pertaining to lateral-directional control inputs. Table 1 presents the values of scale factor (K_β) and biases (Δa_x Δa_y Δa_z Δp Δq Δr $\Delta\beta$) estimated from lateral-directional flight data. The values mentioned in parentheses are the Cramer-Rao bounds suggesting the level of accuracy. It can be observed from Table 1 that the biases are negligible and scale factor is close to unity. Also, the values of Cramer-Rao bounds estimated along with compatibility factors are very low. The scale factor close to unity, negligible biases and very low values of Cramer-Rao bounds establish the high accuracy level of the data gathered during the flight testing.

Table 1: Data compatibility check: Lateral-directional flight data

Factors → Input ↓	Δa_x (m/s^2)	Δa_y (m/s^2)	Δa_z (m/s^2)	Δp (rad/s)	Δq (rad/s)	Δr (rad/s)	K_β	$\Delta\beta$ (rad)
HLD1	0.2478	0.1827	0.083	-0.0006	-0.0011	0.0023	1.0167	-0.0122
	(0.0023)	(0.0009)	(0.0005)	(0.0000)	(0.0000)	(0.0000)	(0.0005)	(0.0002)
HLD2	0.4557	0.2303	0.1257	-0.0007	-0.0008	0.0018	1.0804	-0.0141
	(0.0027)	(0.0015)	(0.0006)	(0.0000)	(0.0000)	(0.0000)	(0.0102)	(0.0003)
HLD3	0.1730	0.1649	0.0201	-0.0002	-0.0008	0.0021	0.9816	-0.0041
	(0.0030)	(0.0034)	(0.0011)	(0.0000)	(0.0000)	(0.0000)	(0.0094)	(0.0004)

( ) Cramer-Rao Bounds

IV. AERODYNAMIC MODEL

The following lateral-directional state equations (simplified case) were used to postulate the aerodynamic model for the estimation of lateral-directional parameters.

$$\dot\beta = -r + \frac{g}{V}\sin\phi - \frac{\rho V S_w}{2m} C_Y \tag{2a}$$

$$\dot p = \rho V^2 S_w \bar c \, \frac{(I_Z C_l + I_{XZ} C_n)}{2(I_X I_Z - I_{XZ}^2)} \tag{2b}$$

$$\dot r = \rho V^2 S_w \bar c \, \frac{(I_X C_n + I_{XZ} C_l)}{2(I_X I_Z - I_{XZ}^2)} \tag{2c}$$

$$\dot\phi = p \tag{2d}$$

The side-force, rolling moment and yawing moment coefficient appearing in equation (2) are modeled as per equation (3).

$$C_Y = C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\left(\frac{pb}{2V}\right) + C_{Y_r}\left(\frac{rb}{2V}\right) + C_{Y_{\delta_r}}\delta_r \tag{3a}$$

$$C_l = C_{l_0} + C_{l_\beta}\beta + C_{l_p}\left(\frac{pb}{2V}\right) + C_{l_r}\left(\frac{rb}{2V}\right) + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r \tag{3b}$$

$$C_n = C_{n_0} + C_{n_\beta}\beta + C_{n_p}\left(\frac{pb}{2V}\right) + C_{n_r}\left(\frac{rb}{2V}\right) + C_{n_{\delta_r}}\delta_r \tag{3c}$$

The aim was to estimate the unknown parameter vector, Θ [equation (4)] using Regression methods from the lateral-directional flight data corresponding to the doublet aileron and/or rudder control inputs.

$$\Theta = \left[C_{Y_0}\, C_{Y_\beta}\, C_{Y_p}\, C_{Y_r}\, C_{Y_{\delta_r}}\, C_{l_0}\, C_{l_\beta}\, C_{l_p}\, C_{l_r}\, C_{l_{\delta_a}}\, C_{l_{\delta_r}}\, C_{n_0}\, C_{n_\beta}\, C_{n_p}\, C_{n_r}\, C_{n_{\delta_r}} C_{n_{\dot\beta}}\right]^T \tag{4}$$

V. PARAMETER ESTIMATION USING REGRESSION

A Least Squares method [4], also known as Regression analysis, is one of the oldest problems in the estimation theory with numerous engineering applications, including flight vehicles. The LS method assumes the independent variables to be error and noise free and dependent variables corrupted by uniformly distributed noise. The major limitation of these methods is that they yield asymptotically biased and inconsistent estimates in the presence of measurement errors and noise in the independent variables. The least squares of unknown parameters (Θ) are obtained by minimizing the sum of squares of the residuals, or alternatively the weighted sum of squares of the residuals. According to definition, the least-square cost function is defined as:

$$J(\Theta) = \frac{1}{2}\varepsilon^T\varepsilon = \frac{1}{2}\left[Y^T - \Theta^T X^T\right][Y - X\Theta] \tag{5}$$

where X and Y are independent and measured dependent variables, respectively and ε is the equation error representing modeling discrepancies and/or noise in the dependent variable. The LS method was applied to the lateral-directional flight data sets for the estimation of the lateral-directional aerodynamic parameter. Figs. (8-10) present the

measured and the estimated response of coefficients of aerodynamic force (C_Y) and moments (C_l and C_n) obtained during the process of the parameter estimation using the LS method using lateral-directional flight data. It can be observed that the measured response of aerodynamic coefficients matches well with the model estimated response for all the flight data sets.
The parameter vector (Θ) given in Eqn 4 was estimated from the compatible real flight data by minimizing the cost function $[J(\Theta, R)]$ using LS method. Table 2 presents the estimated lateral-directional aerodynamic parameters along with their Cramer-Rao bounds. The estimated parameters are compared to the wind tunnel estimates (col. 2).

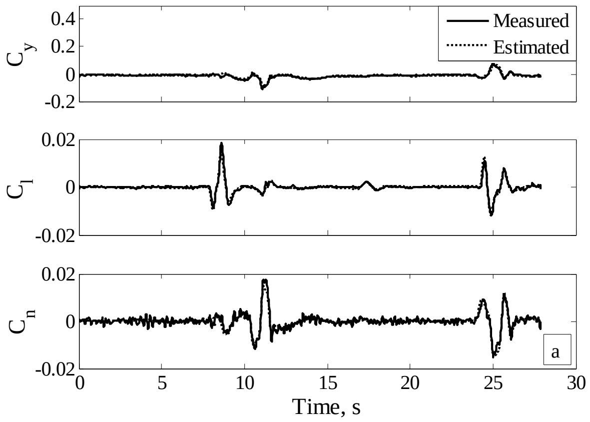

Fig. 8 Measured and estimated response: HLD1

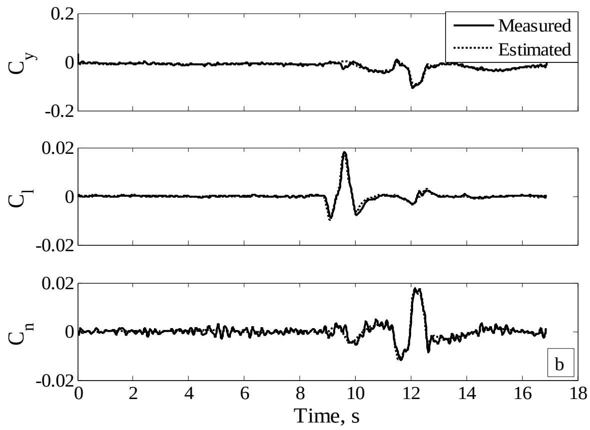

Fig. 9 Measured and estimated response: HLD2

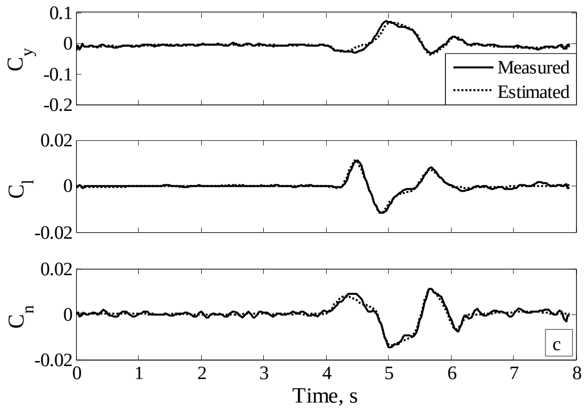

Fig. 10 Measured and estimated response: HLD3

It can be observed that the estimated aerodynamic parameters such as C_{Y_β}, $C_{Y_{\delta_r}}$, C_{l_β}, $C_{l_{\delta_a}}$, C_{n_β} and $C_{n_{\delta_r}}$ are consistent and in close agreement with the wind tunnel estimates. The flight data sets gave consistent values of the estimated damping (C_{l_p} and C_{n_r}) and the cross (C_{l_r} and C_{n_p}) derivatives (parameters). The obtained values of aerodynamic parameters such as C_{Y_p} and C_{Y_r} were also consistent. However, the values of the estimated parameters such as C_{l_0} and C_{n_0} are having opposite sign in contrast to the wind tunnel estimates but their value is quite small or negligible as desired for most of flight data sets. The aerodynamic parameter C_{Y_0} could not be estimated correctly. The estimated value of the parameter $C_{Y_{\delta_r}}$ was consistent but on a higher side for most of the flight data sets. The estimated value of the parameter $C_{l_{\delta_r}}$ could be estimated correctly for HLD2 only.

VI. CONCLUSION

The Least Squares method (Regression) was used to model the lateral-directional aerodynamics using flight data of Hansa-3 aircraft. The following points were observed during the aerodynamic modeling in time domain.

1. The correct calibration is necessary before the flight test program to acquire flight data of good quality.
2. Any estimation method (output error methods) will give reasonably good estimates of parameters, if real flight data is of good quality.
3. The estimated compatibility factors established the high quality of flight data gathered during flight testing.
4. The Regression method resulted in satisfactory estimation of lateral-directional aerodynamic parameters.
5. The reason for the deviation of some parameters being the insufficient information content in the data generated.

Weak parameters are difficult to estimate. Therefore, this could be the second reason for non-estimation weak parameters.

Table 2: Parameter Estimation using Regression

Parameters	WT Value	HLD1	HLD2	HLD3
C_{Y_0}	-0.013	0.0127 (0.0004)	0.0194 (0.0011)	0.0139 (0.0006)
C_{Y_β}	-0.531	-0.6491 (0.0076)	-0.6185 (0.0075)	-0.7207 (0.0099)
C_{Y_p}	-	0.1102 (0.0257)	0.0239 (0.0423)	-0.1901 (0.0320)
C_{Y_r}	-	0.6043 (0.0316)	0.7021 (0.0312)	0.7745 (0.0429)
$C_{Y_{\delta_r}}$	0.150	0.2030 (0.0038)	0.2242 (0.0076)	0.2374 (0.0056)
C_{l_0}	0.0015	0.0000 (0.0000)	0.0010 (0.0001)	-0.0002 (0.0)
C_{l_β}	-0.031	-0.0371 (0.0008)	-0.0218 (0.0010)	-0.0384 (0.0010)
C_{l_p}	-	-0.2730 (0.0032)	-0.2467 (0.0062)	-0.2863 0.0041
C_{l_r}	-	0.0137 (0.0035)	0.0870 (0.0040)	0.1398 (0.0043)
$C_{l_{\delta_a}}$	-0.153	-0.1493 (0.0011)	-0.1545 (0.0022)	-0.1627 0.0014
$C_{l_{\delta_r}}$	0.005	0.0107 (0.0004)	0.0080 (0.0010)	0.0085 (0.0005)
C_{n_0}	0.001	-0.0074 (0.0000)	-0.0090 (0.0002)	-0.0093 (0.0001)
C_{n_β}	0.061	0.0635 0.0015	0.0590 0.0015	0.0853 (0.0020)
C_{n_p}	-	-0.1139 (0.0052)	-0.1134 (0.0086)	-0.0964 (0.0066)
C_{n_r}	-	-0.1110 (0.0064)	-0.1215 (0.0063)	-0.0902 (0.0089)
$C_{n_{\delta_r}}$	-0.05	-0.0685 (0.0007)	-0.0774 (0.0015)	-0.0814 (0.0011)

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