

## LATERAL PARAMETER ESTIMATION USING NGN METHOD

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**Abstract:**--The paper presents the estimation of lateral aerodynamic parameters using neural based (Neural-Gauss-Newton) method from real flight data of Hansa-3 aircraft. The conventional methods require exact model postulation whereas the Neural-Gauss-Newton method is an algorithm that utilizes Feed Forward Neural Network and Gauss-Newton optimization to estimate the parameters and does not require a priori postulation of mathematical model or solution of equations of motion. The results obtained in terms of lateral-directional aerodynamic parameters were reasonably accurate to establish neural method with an additional advantage of non-requirement of a priori aerodynamic model.

**Keywords:**--Lateral, Aerodynamic, Parameters, Mathematical Model, NGN

### I. INTRODUCTION

The parameter estimation [1-10] from real flight data is a routine task for many aerospace organizations. Parameter estimation is the process of determining the best possible estimates of the parameters occurring in the model used to represent a system. Although reasonably accurate parameters can be obtained through analytical predictions and wind tunnel testing, the parameter estimation using flight data help to enhance the confidence in the estimates significantly. Designing optimal controls and autopilots, expansion of flight envelopes, updating simulators and verification of overall aircraft performance are some of the uses of parameter estimation. The conventional methods (Output Error method and its variants [2-5] such as Least Squares (LS) & Maximum Likelihood (ML) methods) used for identification assume the model to be exact. Most of the estimation methods find it difficult to handle flight data having reasonable amount of process noise. The problems of handling process noise and requirement of any mathematical model have been successfully dealt with in the present work by using an estimation method that utilizes Feed Forward Neural Network [6-10] and Gauss-Newton optimization that maps the input/output measurements of the system directly and has the capability to model any nonlinear continuous function without going into the physical details of the network. The method named as Neural Gauss-Newton [8-10] method was proposed by Peyada [8]. The neural based method [Neural Gauss-Newton (NGN)] has been shown to adequately estimate lateral-directional aerodynamic parameters from lateral-directional real flight data. The paper presents the generation of flight data, data-compatibility check, aerodynamic model, parameter estimation and concluding remarks.

### II. GENERATION OF FLIGHT DATA

A flight test program using the Hansa-3 [Fig. 1] aircraft, an in-house fully instrumented research aircraft, was conducted at the Flight Laboratory, IIT Kanpur to gather the real flight data with the help of a data acquisition system. An onboard measurement system installed on test aircraft provided the measurements using dedicated sensors for a large number of signals such as aircraft motion variables, atmospheric conditions, control surface deflections etc. The measurements made in flight were recorded on board at a sampling rate of 50 Hz using a suitable interface with a standard Laptop computer. The three sets of lateral-directional flight data were acquired by executing the aileron/rudder control inputs during flight tests. The three lateral-directional flight data sets nomenclatured as HLD1, HLD2 & HLD3 (Where H and LD refer to Hansa-3 and Lateral-Directional respectively) are processed and presented graphically [Figs. 2-4] in terms of the motion variables. These figures present the variation of lateral-directional motion variables such as angle of sideslip ( $\beta$ ), roll angle ( $\phi$ ), yaw angle ( $\psi$ ), roll rate ( $p$ ), yaw rate ( $r$ ), linear acceleration ( $a_y$ ) along y-axis and velocity ( $V$ ) pertaining to doublet aileron and/or rudder ( $\delta_a$  and/or  $\delta_r$ ) control inputs.



*Fig. 1 The Hansa-3 research-aircraft*

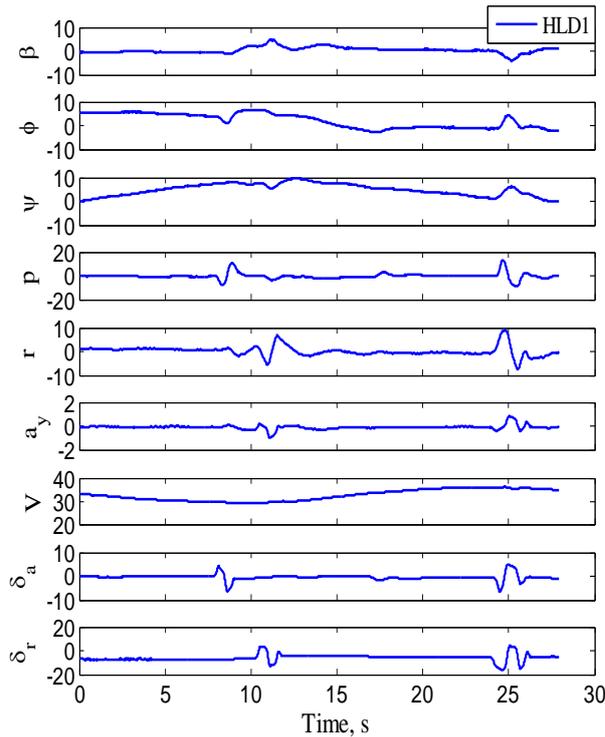


Fig. 2 Lateral-directional flight data: HLD1

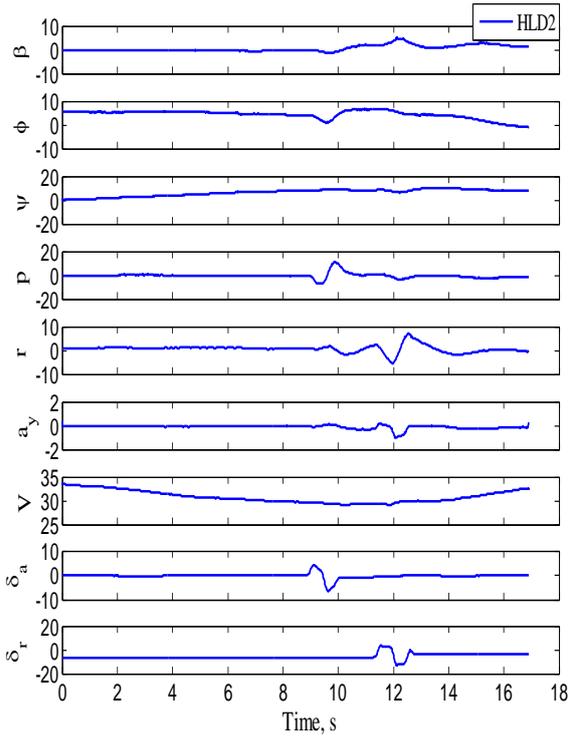


Fig. 3 Lateral-directional flight data: HLD2

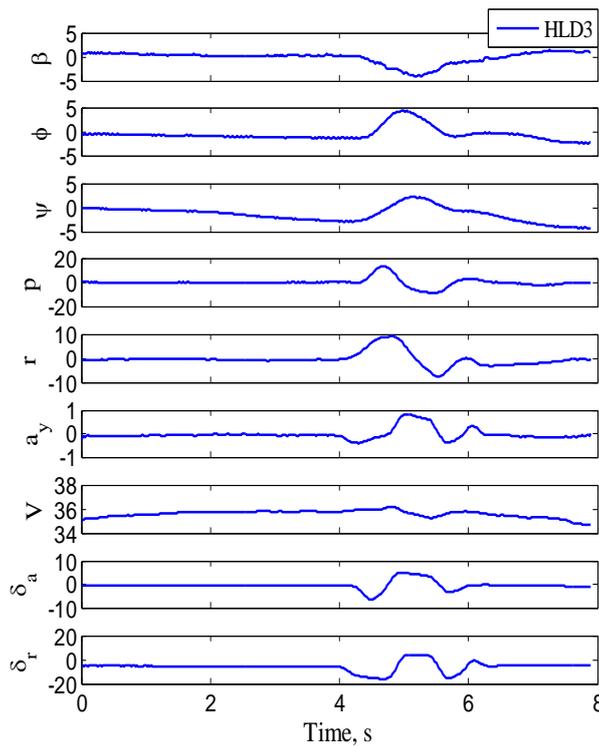


Fig. 4 Lateral-directional flight data: HLD3

It can be seen that the trim angle for aileron and rudder are approximately zero and -7 degrees, respectively. The values of  $\psi$ ,  $p$ ,  $r$  and  $a_y$  corresponding to trim condition can be observed to be zero. The trim value of  $\beta$  can be observed to be approximately zero. The velocity was kept around  $32 \text{ ms}^{-1}$ . It can also be observed that the lateral variables ( $\phi$ ,  $p$ ) were affected when the ailerons were deflected from trim condition whereas the rudder deflection affects the directional variables ( $\beta$ ,  $r$ ). A positive increase in aileron deflection from trim condition results in negative roll rate and reduction in bank angle whereas a positive increase in rudder deflection results in negative yaw rate and positive angle of sideslip

### III. DATA COMPATIBILITY CHECK

The data compatibility check, which is also known as flight path reconstruction [4], is an integral part of aircraft parameter estimation. The recorded real flight data are mainly corrupted by systematic errors like scale factors, zero shift biases and time shifts. These errors introduce data incompatibility; for example, data incompatibility would exist by way of the measured incidence angles not being in agreement with those reconstructed from the accelerometer and rate gyro measurements. The main aim of a data compatibility check is to ensure that the measurements used for subsequent aerodynamic model identification are consistent and error free (as far as possible).

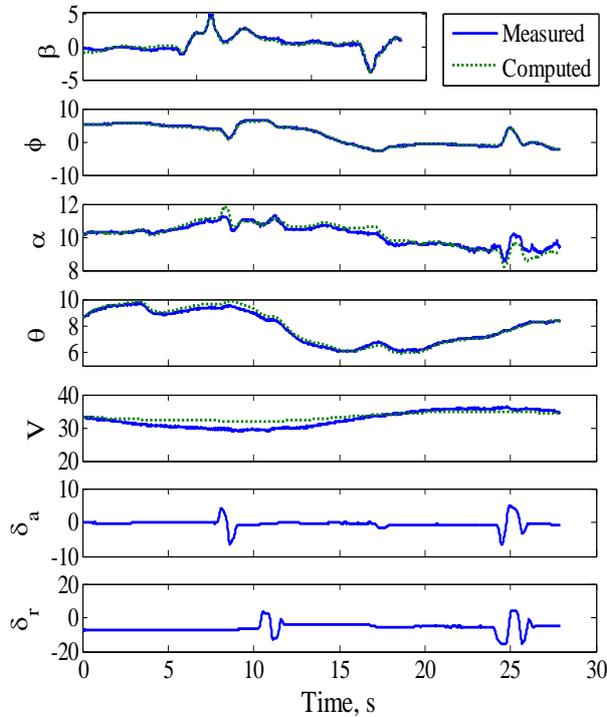


Fig. 5 Data Compatibility Check: HLD1

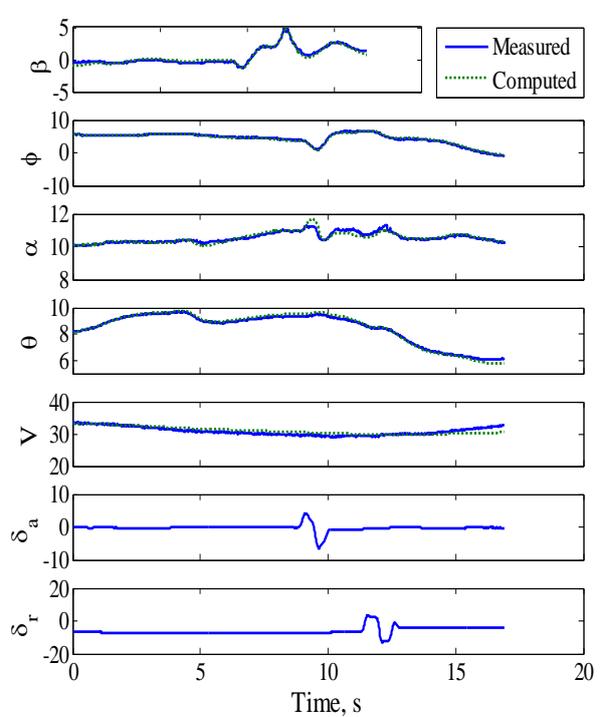


Fig. 6 Data Compatibility Check: HLD2

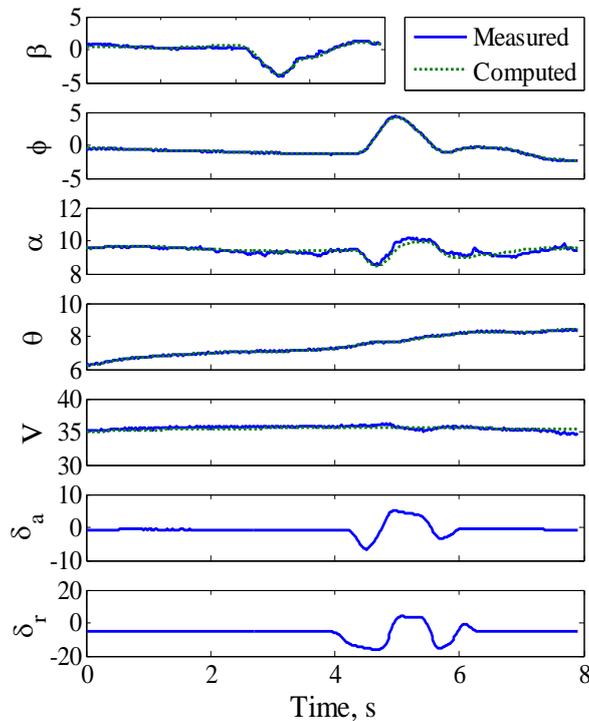


Fig. 7 Data Compatibility Check: HLD3

The data compatibility check was carried out on both flight data sets pertaining to lateral-directional case using observation equations and the ML method. Figs. (5-7) present the measured and computed response of motion variables such as  $\beta$ ,  $\phi$ ,  $\alpha$ ,  $\theta$  and  $V$  obtained during the data compatibility check from flight data pertaining to lateral-directional control inputs. It can be observed that the computed response compares well with the measured response for most of the motion variables ( $\alpha$ ,  $\theta$ ,  $\beta$  and  $\phi$ ). A similar trend in the variation of velocity was also observed. The unknown parameter vector [equation (1)], representing scale factor and biases, was considered adequate for reconstructing lateral-directional dynamics of Hansa-3 aircraft.

$$\Theta = [\Delta a_x \ \Delta a_y \ \Delta a_z \ \Delta p \ \Delta q \ \Delta r \ K_\beta \ \Delta\beta]^T \quad (1)$$

The Maximum Likelihood method was used to estimate the compatibility factors [equation (1)] from flight data pertaining to lateral-directional control inputs. Table 1 presents the values of scale factor ( $K_\beta$ ) and biases ( $\Delta a_x \ \Delta a_y \ \Delta a_z \ \Delta p \ \Delta q \ \Delta r \ \Delta\beta$ ) estimated from lateral-directional flight data. The values mentioned in parentheses are the Cramer-Rao bounds suggesting the level of accuracy. It can be observed from Table 1 that the biases are negligible and scale factor is close to unity. Also, the values of Cramer-Rao bounds estimated along with compatibility factors are very low. The scale factor close to unity, negligible biases and very low values of Cramer-Rao bounds establish the high accuracy level of the data gathered during the flight testing.

Table 1: Data compatibility check: Lateral-directional flight data

Factors → Input ↓	$\Delta a_x$ (m/s <sup>2</sup> )	$\Delta a_y$ (m/s <sup>2</sup> )	$\Delta a_z$ (m/s <sup>2</sup> )	$\Delta p$ (rad/s)	$\Delta q$ (rad/s)	$\Delta r$ (rad/s)	$K_\beta$	$\Delta\beta$ (rad)
HLD1	0.2478 (0.0023)	0.1827 (0.0009)	0.083 (0.0005)	-0.0006 (0.0000)	-0.0011 (0.0000)	0.0023 (0.0000)	1.0167 (0.0005)	-0.0122 (0.0002)
HLD2	0.4557 (0.0027)	0.2303 (0.0015)	0.1257 (0.0006)	-0.0007 (0.0000)	-0.0008 (0.0000)	0.0018 (0.0000)	1.0804 (0.0102)	-0.0141 (0.0003)
HLD3	0.1730 (0.0030)	0.1649 (0.0034)	0.0201 (0.0011)	-0.0002 (0.0000)	-0.0008 (0.0000)	0.0021 (0.0000)	0.9816 (0.0094)	-0.0041 (0.0004)

( ) Cramer-Rao Bounds

#### IV. AERODYNAMIC MODEL

The following lateral-directional state equations (simplified case) were used to postulate the aerodynamic model for the estimation of lateral-directional parameters.

$$\dot{\beta} = -r + \frac{g}{V} \sin \phi - \frac{\rho V S_w}{2m} C_Y \quad (2a)$$

$$\dot{p} = \rho V^2 S_w \bar{c} \frac{(I_x C_l + I_{xz} C_n)}{2(I_x I_z - I_{xz}^2)} \quad (2b)$$

$$\dot{r} = \rho V^2 S_w \bar{c} \frac{(I_x C_n + I_{xz} C_l)}{2(I_x I_z - I_{xz}^2)} \quad (2c)$$

$$\dot{\phi} = p \quad (2d)$$

The side-force, rolling moment and yawing moment coefficient appearing in equation (2) are modeled as per equation (3).

$$C_Y = C_{Y_0} + C_{Y_\beta} \beta + C_{Y_p} \left(\frac{pb}{2V}\right) + C_{Y_r} \left(\frac{rb}{2V}\right) + C_{Y_{\delta_r}} \delta_r \quad (3a)$$

$$C_l = C_{l_0} + C_{l_\beta} \beta + C_{l_p} \left(\frac{pb}{2V}\right) + C_{l_r} \left(\frac{rb}{2V}\right) + C_{l_{\delta_a}} \delta_a + C_{l_{\delta_r}} \delta_r \quad (3b)$$

$$C_n = C_{n_0} + C_{n_\beta} \beta + C_{n_p} \left(\frac{pb}{2V}\right) + C_{n_r} \left(\frac{rb}{2V}\right) + C_{n_{\delta_r}} \delta_r \quad (3c)$$

The aim was to estimate the unknown parameter vector,  $\Theta$  [equation (4)] using Regression methods from the lateral-directional flight data corresponding to the doublet aileron and/or rudder control inputs.

$$\Theta = [C_{Y_0} \ C_{Y_\beta} \ C_{Y_p} \ C_{Y_r} \ C_{Y_{\delta_r}} \ C_{l_0} \ C_{l_\beta} \ C_{l_p} \ C_{l_r} \ C_{l_{\delta_a}} \ C_{l_{\delta_r}} \ C_{n_0} \ C_{n_\beta} \ C_{n_p} \ C_{n_r} \ C_{n_{\delta_r}} \ C_{n_\beta}]^T \quad (4)$$

#### V. PARAMETER ESTIMATION USING REGRESSION

The Neural Gauss-Newton [8-10] method is an algorithm that utilizes the Feed Forward Neural Network (FFNN) and Gauss-Newton optimization to estimate the aerodynamic parameters. The neural model has been used to predict the time histories of motion variables at  $(k+1)^{\text{th}}$  instant given the measured motion variables corresponding to  $k^{\text{th}}$  instant (where  $k = 1$  to  $n$ ;  $n$  is the total number of discrete data points). For all the practical purposes of parameter estimation, this approach helps in building flight dynamic model (in restricted sense) using measured input-output data and does not require an a priori postulation of the mathematical model or solution of equations of motion. The algorithm of NGN method used to estimate the parameters with the help of block diagram given in Fig. 8 has been summarized below.

a) As a first step, the measured flight data undergoes the data compatibly check. The measured motion variables are then transferred to the center of gravity for the further use during estimation process (Blocks 1-3 of Fig. 8).

b) The procedure followed for the neural network training using the FFNNs is explained in the blocks 3-8.

- c) The block 9 checks the convergence criteria for the FFNNs training. Once the training is accomplished the trained neural model is used for the parameter estimation.
- d) Using the chosen aerodynamic model, the already trained neural model is used to calculate the system output  $Y(k)$ . The input  $U(k)$  is constructed in the block 10 using aerodynamic model fed through block 15. The input  $U(k)$  is fed to block 5 to estimate the system output  $Y(k)$ .
- e) The computed response  $Y(k)$  and  $\partial Y(k)/\partial \Theta$  (from block 11) are fed to the blocks 12 and 13 to update the aerodynamic parameter. The aerodynamic model is updated using new set of aerodynamic parameters in block 15. The computation through steps (d) to (e) is continued till the convergence criterion (block 14) is achieved. Once the convergence is achieved, aerodynamic parameters are estimated along with associated standard deviation.

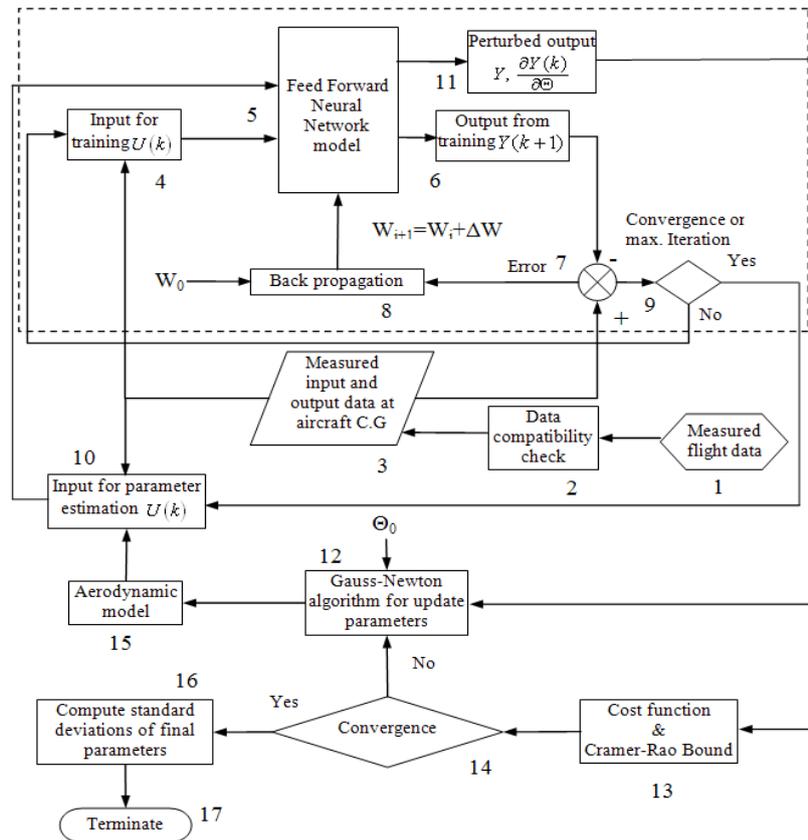


Fig. 8 The schematic of NGN method

The NGN algorithm was applied to the real lateral-directional flight data sets pertaining to doublet aileron-rudder control inputs to estimate the unknown parameter vector  $\Theta$  [Eqn 4]. The residual error between the measured flight data  $Z(k + 1)$  and the estimated neural output  $Y(k + 1)$  was minimized to estimate the unknown parameter vector ( $\Theta$ ). The network parameters varied were the number of hidden layers (1-3), the number of neuron in the hidden layers (2-10), the learning rate (0.1-0.8), the momentum rate (0.1-0.8) and the number of iterations (100-4000). The network parameters finally chosen gave a good match between the true and the predicted values of the time histories of the variables. The final FFNN structure consisted of one hidden layer having the five neurons with a learning rate of 0.3 and the number of iterations equal to 2000.

Figs. (9-11) present the measured and the trained response of the motion variables along y-body axis obtained during the process of training the neural model. It can be observed that the measured response of  $\beta$ ,  $\phi$ ,  $\psi$ ,  $p$ ,  $r$  and  $a_y$  matches well with the trained response.

Figs. (12-14) present the measured and the estimated response of the motion variables such as angle of sideslip ( $\beta$ ), bank angle ( $\phi$ ), yaw angle ( $\psi$ ), roll rate ( $p$ ), yaw rate ( $r$ ) and acceleration ( $a_y$ ) along y-body axis obtained during the process of parameter estimation from the real lateral-directional flight data sets using the NGN method. It can be observed that the measured response of  $\beta$ ,  $\phi$ ,  $\psi$ ,  $p$ ,  $r$  and  $a_y$  matches well with the estimated response for most of the motion variables for all the ten sets of flight data.

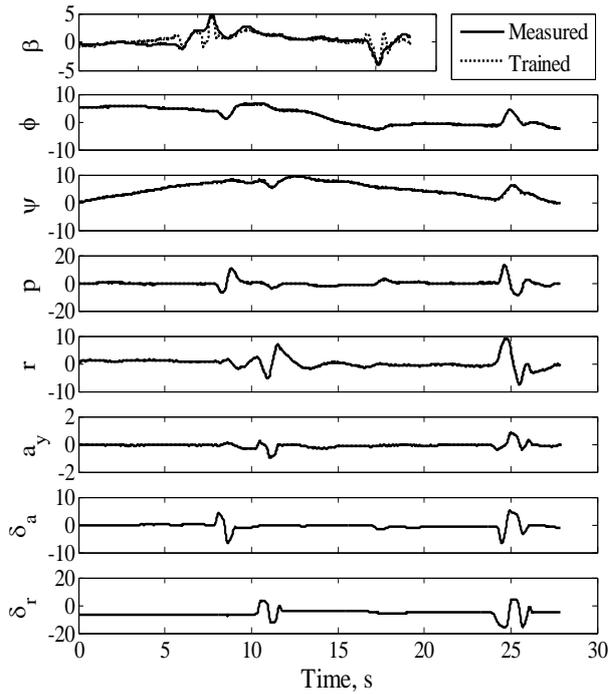


Fig. 9 Measured and Trained response: HLD1

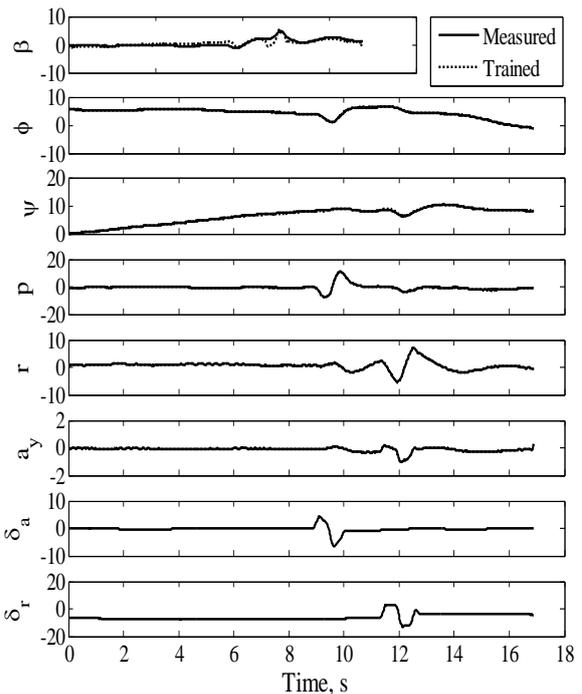


Fig. 10 Measured and Trained response: HLD2

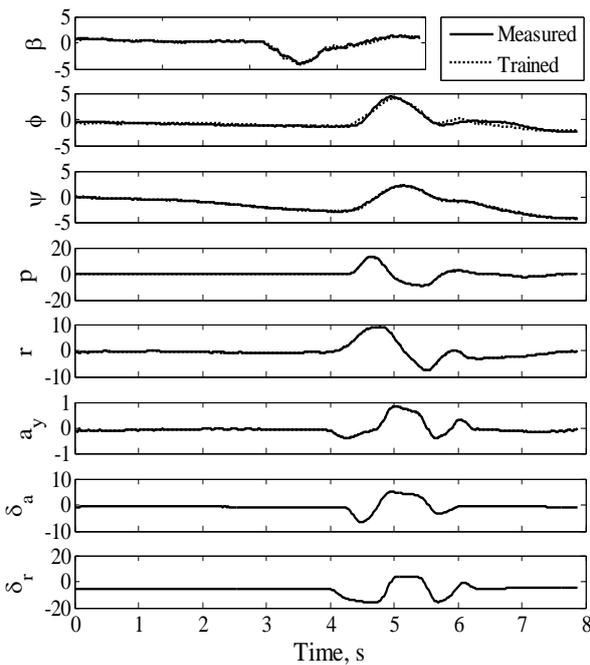


Fig. 11 Measured and Trained response: HLD3

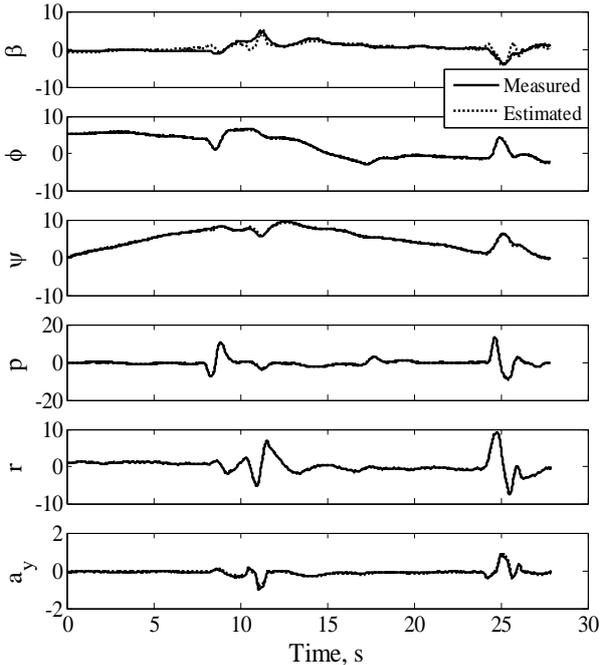


Fig. 12 Measured and Estimated response: HLD1

The parameter vector ( $\Theta$ ) given in Eqn 4 was estimated from the compatible real flight data by minimizing the cost function using NGN method. Table 2 presents the estimated lateral-directional aerodynamic parameters along with their Cramer-Rao bounds. The estimated parameters are compared to the wind tunnel estimates (col. 2). It can be observed that the estimated aerodynamic parameters such as  $C_{Y\beta}$ ,  $C_{Y\delta_r}$ ,  $C_{l\beta}$ ,  $C_{l\delta_a}$ ,  $C_{n\beta}$  and  $C_{n\delta_r}$  are consistent and in close agreement with the wind tunnel estimates. The flight data sets gave consistent values of the estimated damping ( $C_{l_p}$  and  $C_{n_r}$ ) and the cross ( $C_{l_r}$  and  $C_{n_p}$ ) derivatives (parameters). The obtained values of aerodynamic parameters such as  $C_{Y_p}$  and  $C_{Y_r}$  were also consistent. However, the values of the estimated parameters such as  $C_{l_0}$  and  $C_{n_0}$  are having opposite sign in contrast to the wind tunnel estimates but their value is quite small or negligible as desired for most of flight data sets. The aerodynamic parameter  $C_{Y_0}$  could not be estimated correctly. The estimated value of the parameter  $C_{Y\delta_r}$  was consistent but on a higher side for most of the flight data sets. The value of the parameter  $C_{l\delta_r}$  also could not be estimated correctly.

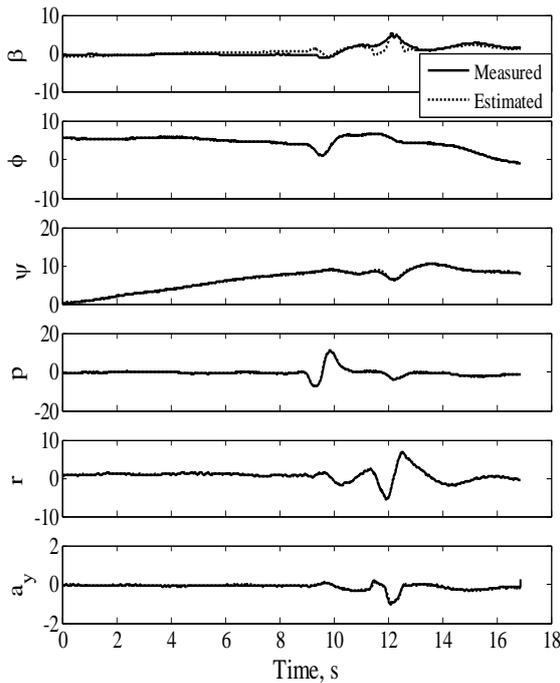


Fig. 13 Measured and Estimated response: HLD2

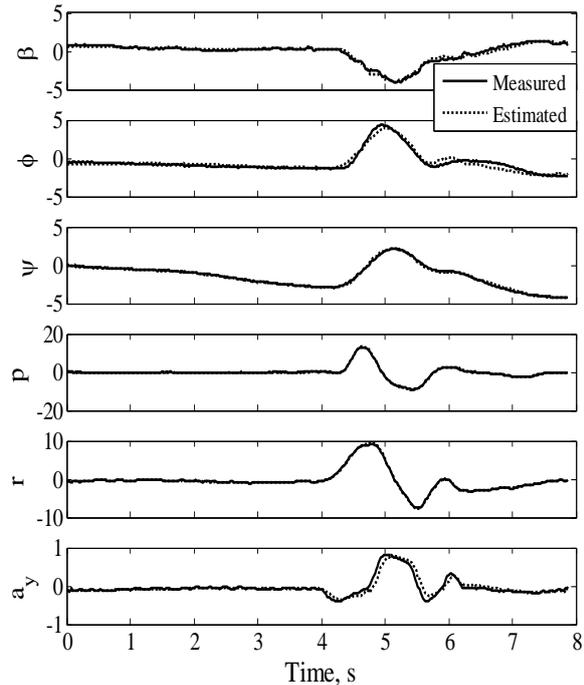


Fig. 14 Measured and Estimated response: HLD3

Table 2: Parameter Estimation using NGN Method

Parameters	WT Value	HLD1	HLD2	HLD2
$C_{Y_0}$	-0.013	0.0168 (0.0003)	0.0298 (0.0005)	0.0154 (0.0006)
$C_{Y_\beta}$	-0.531	-0.8096 (0.0063)	-0.8975 (0.0076)	-0.6154 (0.0156)
$C_{Y_p}$	-	0.1825 (0.0219)	0.0375 (0.0248)	-0.0004 (0.0001)
$C_{Y_r}$	-	0.6598 (0.0273)	1.3262 (0.0350)	0.0002 (0.0001)
$C_{Y_{\delta_r}}$	0.150	0.2254 (0.0031)	0.3733 (0.0044)	0.2021 (0.0049)
$C_{l_0}$	0.0015	0.0007 (0.0001)	0.0012 (0.0002)	0.0008 (0.0002)
$C_{l_\beta}$	-0.031	-0.0290 (0.0021)	-0.0253 (0.0032)	-0.0394 (0.0047)
$C_{l_p}$	-	-0.2865 (0.0088)	-0.3056 (0.0137)	0.0000 (0.0000)
$C_{l_r}$	-	0.1610 (0.0091)	0.1815 (0.0141)	0.0000 (0.0000)
$C_{l_{\delta_a}}$	-0.153	-0.1549 (0.0029)	-0.1760 (0.0046)	-0.1258 (0.0054)
$C_{l_{\delta_r}}$	0.005	0.0202 (0.0011)	0.0252 (0.0017)	0.0218 (0.0021)
$C_{n_0}$	0.001	-0.0076 (0.0001)	-0.0109 (0.0001)	-0.0042 (0.0002)
$C_{n_\beta}$	0.061	0.0323 (0.0021)	0.0462 (0.0029)	0.0495 (0.0056)
$C_{n_p}$	-	-0.1548 (0.0072)	-0.1427 (0.0096)	0.0001 (0.0000)
$C_{n_r}$	-	-0.1431 (0.0092)	-0.1654 (0.0131)	-0.0001 (0.0000)
$C_{n_{\delta_r}}$	-0.05	-0.0727 (0.0010)	-0.0997 (0.0016)	-0.0442 (0.0018)

## VI. CONCLUSION

The NGN method was used to model the lateral-directional aerodynamics using flight data of Hansa-3 aircraft. The following points were observed during the aerodynamic modeling in time domain.

1. The correct calibration is necessary before the flight test program to acquire flight data of good quality.
2. Any estimation method (output error methods) will give reasonably good estimates of parameters, if real flight data is of good quality.
3. The estimated compatibility factors established the high quality of flight data gathered during flight testing.
4. The NGN methods resulted in satisfactory estimation of lateral-directional aerodynamic parameters.
5. The reason for the deviation of some parameters being the insufficient information content in the data generated.
6. Weak parameters are difficult to estimate. Therefore, this could be the second reason for non-estimation few parameters (weak parameters).

The NGN method does not require any *a priori* postulation of mathematical model or solution of equations of motion and act as functional approximator which can collectively model any nonlinear relationship between the inputs and the outputs and thereby provide overall characterization of a system.

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