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PREY-PREDATOR MODELS IN MARINE LIFE

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Abstract:—The paper discusses the mathematical model based on the prey-predator system, which diffuses in a near linear bounded region and the whole region is divided into two near annular patches with different physical conditions. The model has been employed to investigate the population densities of fishes depending on time and position. Finite Element Method has been used for the study and computation. The domain is discretized into a finite number of sub domains (elements) and variational functional is derived. Graphs are plotted between the radial distance and the population density of species for different value of time.
Key words:—Discretization, diffusion, intrinsic growth, Laplace transform, variational form.

I. 1 INTRODUCTION

The prey-predator relationship is the most well known of all the nutrient relationships in the ocean. [2] Fishes as a group provide numerous examples of predators. Thus the herbivorous fishes such as the Sardines and the Anchovy are sought after by predators of the pelagic region. These plankton feeding fishes form an important link between the plankton and the higher carnivores which have no means of directly utilizing the rich plankton as a source of food. Many of the predatory fishes, such as tuna, the barracuda and the salmon, have sharp teeth and they move at high speed in pursuit of their prey.[7] Many marine mammals such as the killer whales, porpoises, dolphins, seals, sea lions and walruses are all predators with well developed sharp teeth as an adaptation for predatory life. The pelagic fishes of great depths resort to predatory life as they have other source of food there. But owing to the scarcity of food available, they are comparatively smaller than the predators of the surface region. These fishes have developed many interesting contuvances and food habits. Thus many of the deep sea fishes such as the chiasmodus have disproportionately large mouths and enormously distensible stomachs and body walls which permit swallowing and digestion of fishes up to three times their own size. The mouth is well armed with formidable teeth to prevent the escape of the prey. Some of these fishes are provided with luminescent organs and special devices to make the best of the prevailing conditions. Many benthic ferns also are predaceous, living upon each other and other bottom animals.[5] Bottom fishes, especially the ray, live on crustaceans, shell fish, worms and coelentrates of the sea floor.

II. MATHEMATICAL MODEL

In this paper, we discuss the mathematical model based on the prey-predator system, which diffuses in a near linear bounded region and the whole region is divided into two patches. The model has been employed to investigate the population densities of fishes depending on time and position.[4] Finite Element Method has been used for the study and computation. This approach can be extended to the study of population in more patches.

Variational finite element method is also employed.[1] This method is an extension of Ritz method and is used for more complex boundary value problems. In the variational finite element method the domain τ is discretized into a finite number of sub domains (elements) and variational functional is obtained.[3],[6] The approximate solution for each element is expressed in terms of undetermined nodal values of the field variable, as appropriate shape functions (trial functions) or interpolating functions.

Here we have taken a prey-predator model, in which prey-predator populations diffuse between two circular patches in a given area. The system of non-linear partial differential equations for the above case is:

$$\frac{\partial N_i}{\partial t} = r_{1i} N_i \left(1 - \frac{N_i}{K_{1i}} - \frac{b_{1i}}{K_{1i}} P_i \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r D_{1i} \frac{\partial N_{1i}}{\partial r} \right)$$

$$\frac{\partial P_i}{\partial t} = r_{2i} P_i \left(-1 + \frac{b_{2i}}{K_{2i}} N_i \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r D_{2i} \frac{\partial P_{1i}}{\partial r} \right),$$

i=1, 2 (1)

The total area is divided into two patches. The first patch is assumed to lie along the radius $r_0 \le r \le r_1$ and second patch lies along the radius $r_1 \le r \le r_2$.

Here, for i^{th} patch we take

N_{1i} =Density of prey population

P_{1i} =Density of predator population

r_{1i} =Intrinsic growth rate of prey population

r_{2i} =Intrinsic growth rate of predator population

b_{1i} =Interspecific interaction coefficient

b_{2i} = Interspecific interaction coefficient

D_{1i} , D_{2i} =Diffusion coefficient of prey and predator populations (i=1,2)

Equilibrium points of the equations (1) are

$E_{1i}(0,0)$, $E_{2i}\left(N_i^*, P_i^*\right)$

where,

$$N_i^* = \frac{K_{2i}}{b_{2i}},\ P_i^* = \frac{K_{1i}}{b_{1i}} - \frac{K_2}{b_{2i}b_{1i}},\ i=1,2$$

We take small perturbations u_{1i} and u_{2i} from the non zero equilibrium point i.e.

$$N_i = N_i^* + u_{1i},\ P_i = P_i^* + u_{2i} \tag{2}$$

where $|u_{1i}| << 1$, $|u_{2i}| << 1$ and u_{1i} , u_{2i} are prey and predator populations.

Then system of equations (1) becomes

$$\frac{\partial u_{1i}}{\partial t} = r_{1i}N_i^*\left(\frac{u_{1i}}{K_{1i}} + \frac{b_{1i}u_{2i}}{K_{1i}}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{1i}\frac{\partial u_{1i}}{\partial r}\right)$$

$$\frac{\partial u_{2i}}{\partial t} = r_{2i}P_i^*\left(\frac{b_{2i}u_{1i}}{K_{2i}}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{2i}\frac{\partial u_{2i}}{\partial r}\right) \tag{3}$$

Initial conditions are taken as

$$u_{1i}(r,0) = G_{1i}(r),$$
$$u_{2i}(r,0) = G_{2i}(r) \tag{4}$$

Where $G_{ji}(r)$ are known functions.

Interface conditions at $r = r_1$ are assumed to be

$$D_{11}\frac{\partial u_{11}}{\partial r}\bigg|_{R_1} = -D_{12}\frac{\partial u_{12}}{\partial r}\bigg|_{R_2}$$
$$D_{21}\frac{\partial u_{21}}{\partial r}\bigg|_{R_1} = -D_{22}\frac{\partial u_{22}}{\partial r}\bigg|_{R_2} \tag{5}$$

where R_1 and R_2 denote region –I and region-II respectively.

Boundary conditions associated with the system of equations are

$$\left.\frac{\partial u_{11}}{\partial r}\right|_{r=r_0} = \left.\frac{\partial u_{12}}{\partial r}\right|_{r=r_2}$$

$$\left.\frac{\partial u_{21}}{\partial r}\right|_{r=r_0} = \left.\frac{\partial u_{22}}{\partial r}\right|_{r=r_2} \qquad (6)$$

III. SOLUTION

To solve this model, we apply the Finite Element Method. Comparing the system of equations (3) with Euler Lagrange's equations, we get

$$F_i = \left[A_i u_{1i}^2 + B_i u_{2i}^2 + C_i u_{1i} u_{2i} + D_i \left(u_{1i}^{'}\right)^2 + E_i \left(u_{2i}^{'}\right)^2\right] r, \quad \text{i=1,2} \qquad (7)$$

where $u_{1i}^{'} = \dfrac{\partial u_{1i}}{\partial r}$, $u_{2i}^{'} = \dfrac{\partial u_{2i}}{\partial r}$, i=1,2

Here, $A_i = \dfrac{1}{2}\left(\dfrac{\partial}{\partial t} - r_{1i}\dfrac{N_1^*}{K_{1i}}\right)$

$$B_i = \frac{1}{2}\frac{\partial}{\partial t}$$

$$C_i = \frac{r_{1i} N_i^* b_{1i}}{K_{1i}} - \frac{r_{2i} P_i^* b_{2i}}{K_{2i}} \qquad D_i = -\frac{D_{1i}}{2}, \; E_i = -\frac{D_{2i}}{2}$$

The corresponding variational form is

$$I_i = \int_{L_{i-1}}^{L_i} F_i \, dr \qquad (8)$$

We take

$$u_{1i} = A^i + B^i \log r$$

$$u_{2i} = C^i + D^i \log r \qquad (9)$$

$$u_{11} = u_{110} \text{ at } r = L_0$$

$$u_{11} = u_{111} = \mathrm{u}_{12} \text{ at } r = L_1 \qquad (10)$$

$$u_{12} = u_{112} \text{ at } r = L_2$$

$$u_{21} = u_{220} \text{ at } r = L_0$$

$$u_{21} = u_{221} = \mathrm{u}_{22} \text{ at } r = L_1 \qquad (11)$$

$$u_{22} = u_{222} \text{ at } r = L_2$$

Using these values we get

$$A^i = \frac{u_{11(i-1)}\log(L_i) - u_{11i}\log(L_{i-1})}{\log(L_i) - \log(L_{i-1})}$$

$$B^i = \frac{u_{11i} - u_{11(i-1)}}{\log(L_i) - \log(L_{i-1})}$$

$$C^i = \frac{u_{22(i-1)}\log(L_i) - u_{22i}\log(L_{i-1})}{\log(L_i) - \log(L_{i-1})}$$

$$D^i = \frac{u_{22i} - u_{22(i-1)}}{\log(L_i) - \log(L_{i-1})} \qquad (12)$$

$$i = 1,2$$

Accordingly

$$I_i = A_i\left[\left(A^i\right)^2 \alpha_{1i} + \left(B^i\right)^2 \alpha_{2i} + 2A^i B^i \alpha_{3i}\right] + B_i\left[\left(C^i\right)^2 \alpha_{1i} + \left(D^i\right)^2 \alpha_{2i} + 2C^i D^i \alpha_{3i}\right]$$
$$+ C_i\left[A^i C^i \alpha_{1i} + B^i D^i \alpha_{2i} + (A^i D^i + B^i C^i)\alpha_{3i}\right] + D_i\left(B^i\right)^2 \alpha_{4i} + E_i\left(D^i\right)^2 \alpha_{4i} \qquad (13)$$

where $I = I_1 + I_2$ (14) Now differentiating I with respect to nodal points u_{111} and u_{221}; and putting

$\dfrac{\partial I}{\partial u_{111}} = 0$, $\dfrac{\partial I}{\partial u_{221}} = 0$, we get

$$u_{110}\left(A_1\beta_{17} + D_1\beta_{18}\right) + u_{111}\left(A_1\beta_{11} + A_2\beta_{12} + D_1\beta_{13} + D_2\beta_{14}\right) + u_{112}\left(A_2\beta_{110} + D_2\beta_{111}\right)$$
$$+ u_{220}\left(C_1\beta_{19}\right) + u_{221}\left(C_1\beta_{15} + C_2\beta_{16}\right) + u_{222}\left(C_2\beta_{112}\right) = 0 \qquad (15)$$

$$u_{110}\left(C_1\beta_{27}\right) + u_{111}\left(C_1\beta_{21} + C_2\beta_{22}\right) + u_{112}\left(C_2\beta_{210}\right) + u_{220}\left(B_1\beta_{28} + E_1\beta_{29}\right)$$
$$+ u_{221}\left(B_1\beta_{23} + B_2\beta_{24} + E_1\beta_{25} + E_2\beta_{26}\right) + u_{222}\left(B_2\beta_{211} + E_2\beta_{212}\right) = 0 \qquad (16)$$

Substituting values of A_i, B_i, C_i, D_i, E_i ,(i=1,2) and taking Laplace transform of (15)and (16), we get

$$\left(\gamma_{11} p + \gamma_{12}\right)\bar{u}_{111} + \gamma_{13}\bar{u}_{221} = \frac{1}{p}\gamma_{14} + \gamma_{11} u_{111}(0) \qquad (17)$$

$$\gamma_{21}\bar{u}_{111} + \left(\gamma_{22} p + \gamma_{23}\right)\bar{u}_{221} = \frac{1}{p}\gamma_{24} + \gamma_{22} u_{221}(0) \qquad (18)$$

Here $\bar{u}_{111}$, $\bar{u}_{221}$ are Laplace transforms of u_{111} , u_{221} respectively and $u_{110}, u_{112}, u_{220}, u_{222}$ are independent of t and $u_{111}(0)$ and $u_{221}(0)$ are initial values. Solving (17 and (18), we get,

$$\bar{u}_{111} = \frac{(-\gamma_{11}\gamma_{22} p^2 + \gamma_{14}\gamma_{22} p - \gamma_{11}\gamma_{23} u_{111}(0) p + \gamma_{22}\gamma_{13} u_{221}(0) + \gamma_{14}\gamma_{23} - \gamma_{24}\gamma_{13})}{p(\gamma_{11}\gamma_{22} p^2 + \gamma_{11}\gamma_{23} p + \gamma_{12}\gamma_{22} p + \gamma_{12}\gamma_{23} - \gamma_{21}\gamma_{13})}$$

$$\bar{u}_{221} = \frac{(\gamma_{11}\gamma_{22} u_{221}(0) p^2 - \gamma_{24}\gamma_{11} p - \gamma_{11}\gamma_{21} u_{111}(0) p + \gamma_{22}\gamma_{12} u_{221}(0) p + \gamma_{14}\gamma_{21} - \gamma_{24}\gamma_{12})}{p(-\gamma_{11}\gamma_{22} p^2 - \gamma_{11}\gamma_{23} p - \gamma_{12}\gamma_{22} p - \gamma_{12}\gamma_{23} + \gamma_{21}\gamma_{13})}$$

(19)

Taking inverse Laplace transform of $\bar{u}_{111}$ and $\bar{u}_{221}$,we obtain the expressions for u_{111} and u_{221} .Substituting these values in (13), we get values of u_{1i} and u_{2i} (i=1,2).Using these values we can get the values of N_i and P_i (i=1,2).

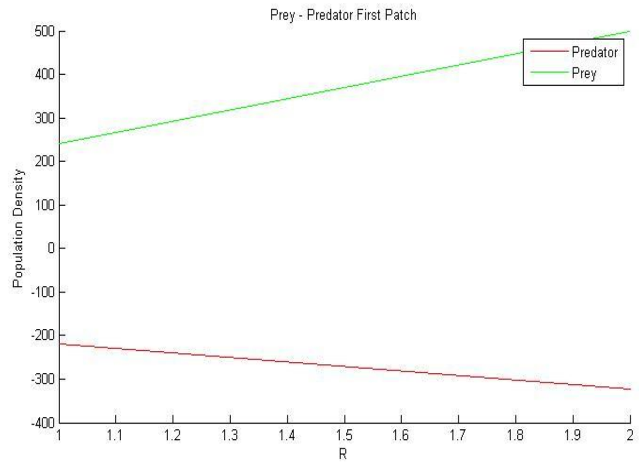
IV. NUMERICAL COMPUTATION

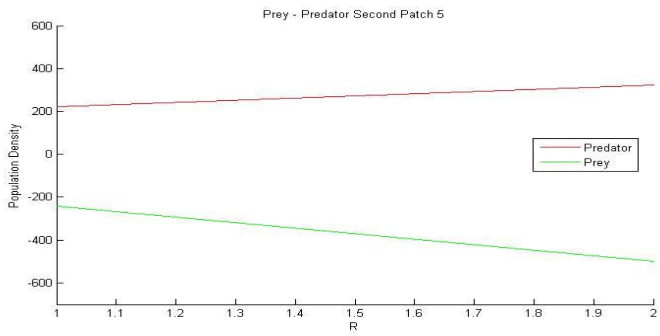
We make use of the following values of parameters and constants.

$r_{12} = .04$, $b_{11} = .5$, $b_{12} = .4$, $K_{11} = 100$, $K_{12} = 125$, $d_{11} = .9$, $d_{12} = .8$, $r_{21} = .025$, $r_{22} = .035$, $b_{21} = .5$, $b_{22} = .4$, $K_{21} = 110$, $K_{22} = 130$, $d_{21} = .8$, $d_{22} = .9$, $r_0 = 10$, $r_1 = 20$, $r_2 = 30$, $u_{1110} = .5$, $u_{2210} = .5$, $u_{1112} = .9$, $u_{222} = .9$, $u_{110} = .2$, $u_{220} = .2$.

V. CONCLUSION

Graphs are plotted between radius r and population density of species for different values of time. Graphs show that density of prey increases with radius while density of predator decreases with radius at any time in the first patch. In the second patch density of prey decreases with radius while density of predator increases. Graphs also show that density of prey increases with time and density of predator decreases with time in the first patch. In the second patch density of prey decreases with time while density of predator increases with time.





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