Vibration Analysis of a Portal Frame Subjected To a Moving Concentrated Load

Ms. S. J. MODAK\textsuperscript{1}, H. V. HAZARE\textsuperscript{2},
\textsuperscript{1}Assistant Professor in Civil Engg., Ramdeoabba College of Engineering & Management, Nagpur – 440 009
\textsuperscript{2}Professor of Civil Engg., Priyadarshini College of Engineering, Nagpur – 440 019

Abstract:—The objective of the investigation is to mathematically simulate dynamics and vibrations of a portal frame subjected to a concentrated load moving on its horizontal member with a certain constant velocity. This portal frame is a basic structure of a low length single span bridge. The emphasis is on an approach to model forced vibrations of the vertical members of the portal frame.

Keywords:—Bridges, Columns, Portal Frame, Influence Line, Vibrations.

I. CONSTRUCTION OF A SHORT LENGTH BRIDGE

Fig. 1 is a schematic presentation of a short length bridge. The length is so short that the basic structure of a bridge is a simple one span portal frame \(0_1\ AB \ 0_2\). The width of the bridge is also fairly small so that it could be considered as a particular case of a girder bridge \([1]\). The material of the frame is Mild Steel (M.S.). The philosophy of the analysis is explained through a representative small scale structure with dimensions length of \(AB = 1005\) mm, width = 50 mm and thickness is = 5mm. The vertical members \(0_1A\) and \(0_2B\) are geometry wise identical. The material of \(0_1A\) & that of \(0_2B\) is also M.S. A vehicle with total weight \(W\) is moving on \(AB\) with a constant velocity.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{schematics of a portal frame for a short bridge}
\end{figure}

The objective of the investigation is to estimate vibration response of \(0_1A\ & \ 0_2B\).

II. FREE BODY DIAGRAMS OF A PORTAL FRAME

The free body diagrams of the members of the portal frame described in Fig. 1 are detailed in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Figure 2 (a)}
\end{figure}

* Numbers in the square brackets denote references listed at the end of the paper.
Fig. 2: free body diagrams of a portal frame

Fig. 2(a) describes Free Body Diagram of Horizontal beam portion AB of the portal frame. W is a concentrated load (in fact the weight of the vehicle) acting on AB at an arbitrary distance x’ from left end A. This x’ is all the while changing as the vehicle moves from left to right with a constant velocity V. W_A is a reaction of left support 0_A on AB at A whereas W_B is a reaction of right support 0_B on AB at B.

At a section of AB at a distance x = 5mm (in this specific case) anticlock wise couple is exerted by support 0_A on AB. x = 5mm because linear dimension of a cross section of 0_A at the top end = 10.0 mm. This meant top section of member 0_A is exerting on section X in the C.C.W. direction a moment because member 0_A is imposing a moment restraint on AB to see that complete portion worth 10mm from end A of the member AB of course on it’s bottom side has to remain straight. Hence, the slope of section x of AB from A at 5mm on account of probable free displacement of neutral axis of AB as a simply supported beam due to concentrated load W is prevented.

In view of above detailed analysis for free body diagram of member AB of the portal frame the free body diagrams of the other members 0_A and 0_B which are vertical supports of AB get deduced logically as described in Figures 2(b) and 2(c) respectively for 0_A & 0_B.

As per the free body diagrams of Figures 2(b) and 2(c) the total axial load on left support 0_A is summation of W_A & W_A’ whereas the same for the support 0_B is W_B & W_B’. The exact quantitative relationships for W_A & W_A’ and that of W_B & W_B’ is derived in the following section.

III. QUANTITATIVE FORCE ANALYSIS OF THE PORTAL FRAME

3.1 Analysis of Member AB

Redrawing the Free Body Diagram of Member AB (Fig. 2(a) for the sake of ready reference) as shown.

Fig. 2(a) : Pepete FDB of AB redrawn for read reference as above

Refer Fig. 2(a) Pepete. The reactions R_A and R_B would be as under. Taking moments about point A and applying the condition of \( \sum \text{M}=0 \) of static equilibrium.

\[-W_{x'} + R_B \times L = 0\]

\[R_B = \frac{W_{x'}}{L} \]

........................ (3.1)
Similarly, it can be proved that

\[ R_A = W \left( 1 - \frac{x'}{L} \right) \]  
\[ Mx' = -W \frac{(L-x')}{L} x \]  
\[ Mx' (Near \ support \ B) = \frac{Wx'}{L} x \]

The bending moment at \( x = x' = 5\text{mm} \) in this specific case in which numerically \( x = x' = 5\text{mm} \) is denoted as \( M_x \).

\[ \therefore Mx' = R_A x \]

Substituting for \( R_A \) from Equation (3.2)
\[ \therefore Mx' = -W \frac{(L-x')}{L} x \]

Similarly, the bending moment near support B at \( x = x' = 5\text{mm} \), in this specific case in which numerically \( x = x' = 5\text{mm} \) is denoted as \( Mx' \) (near support B)

\[ \therefore Mx' (near \ support \ B) = R_B x \]

Substituting the value of \( R_B \) from Equation (3.1) in the above equation

These two bending moments are trying to induce slopes \( dy/dx \) at sections \( x = x' \) from both the supports A & B respectively in member AB treating AB as a simply supported beam. These moments and associated transverse deflection y at these sections are such that they are trying to turn the cross sections of member AB respectively C.W. and C.C.W. However, these elastic deformations are restrained by members 01A & 02B. Hence, top sides of 01A & 02B are trying to resist these deformations. This is only possible if top side of 01A is exerting C.C.W. moment at \( x = x' \) near A and if top side of 02B is exerting C.W. moment at \( x = x' \) near B respectively. To this action of 01A & 02B on member AB, the member AB will exert equal & opposite reactions on 01A & 02B.

With the result, member AB will exert moments \( Mx' \) given by Eq. (3.3) on top side of 01A C.W. and member AB will exert moment \( Mx' \) given by Eq. (3.4) C.C. W. on top side of member 02B.

These two bending moments are \( Mx' \) near A and \( Mx' \) (near support B) respectively will have senses C.W. and C.C. W. for supports 01A and 02B as described in Figures 2(b) and 2(c) respectively.

Similarly, the axial forces as exerted by AB on supports 01A and 02B will be both downwards and with magnitudes as given respectively by Equations (3.2) and (3.1) but denoted as \( W_{A}' \) and \( W_B \) in figures 2(b) and 2(c) respectively.

Thus, total vertically downwards action of AB on 01A is if denoted by \( W_{TD01A} \)

\[ W_{TD01A} = W_A' + W_{A} = W \left( 1 - \frac{x'}{L} \right) + W \left( 1 - \frac{x}{L} \right) \frac{x}{x+1} \]  

Similarly if \( W_{TD02B} \) is total downwards action as exerted by AB on 02B then

\[ W_{TD02B} = \frac{Wx'}{L} \left[ 1 + \frac{x}{x+1} \right] \]  

In equations (3.5) and (3.6) while deciding force actions of members AB these moments are assumed to be equivalent to two forces of equal magnitudes and opposite in senses acting along the axis of members 01A & 02B. These are shown in free body diagrams Fig. 2(b) and Fig. 2(c) respectively by forces \( W_A \) and \( W_B \). The forces \( W_A \) and \( W_B \) are assumed to have moment arm slightly more than \( x \) say more only by 1mm. Hence, they come out be equal to the second terms in both the equations (3.5) and (3.6) given above.

Further Equations (3.5) and (3.6) can be rearranged in the forms given below.

\[ W_{TD01A} = W \left( 1 - \frac{x'}{L} \right) \left[ 1 + \frac{x}{x+1} \right] \]  
\[ W_{TD02B} = \frac{Wx'}{L} \left[ 1 + \frac{x}{x+1} \right] \]  

In fact if it is assumed that the velocity of the vehicle moving over the bridge is \( V \) and it is constant and \( t \) is the time elapsed from the instant the vehicle has entered the bridge from left end till the time it has covered the distance \( x' \) where of-course \( 0 < x' < L \) then one can say that

\[ X' = Vt \]

Substituting for \( x' \) from Equation (3.7) in to equation (3.5.a) and (3.6.a) one gets final action on members 01A & 02B simulated by the below stated equations

\[ W_{TD01A} = W \left( 1 - \frac{Vt}{L} \right) \left[ 1 + \frac{x}{x+1} \right] \]  
\[ W_{TD02B} = W \left( \frac{Vt}{L} \right) \left[ 1 + \frac{x}{x+1} \right] \]
IV. VIBRATION RESPONSE OF 0₁A AND 0₂B

The members 0₁A and 0₂B the supports of a bridge are subjected to time varying forces as described by Equations (3.8) and (3.9) respectively. Since, the material of supports 0₁A & 0₂B is elastic and that it will experience material hysteresis, the supports are subjected to longitudinal vibrations. The longitudinal vibrations can be ascertained by various approaches as regards various distributed mass, elasticity and damping of the material of column.

1) Considering entire column represented by single mass, single stiffness, single damper system, popularly abbreviated in the science of vibrations of structures as SDF system.

OR

2) Considering entire column represented by multi mass, multi stiffness, multi damper system say considering 5 Lumped mass system or 5 DOF system.

OR

3) Considering distributed mass, distributed elasticity, distributed damping system.

In the three cases stated above the column excitation i.e. the external force acting on the two columns is as described in Equations (3.8) and (3.9) respectively.

4) In the force analysis discussed so far it is considered as if only one vehicle is entering the bridge and it moves over the bridge with constant velocity.

It may so happen that the entry of vehicles on the bridge may be in a random manner, their weights may be different, their velocities may be different, same may be moving with some acceleration and retardation over the bridge. The accelerations or retardations may be constant or variable. For each one of the above cases the loading pattern on bridge supports 0₁A and 0₂B may be different. Further, for all these loading patterns in view of all the three styles of vibrations described earlier, it may become imperative to ascertain vibration response. This may be considered as situation of random excitation & consequently the random vibrations of the columns of a bridge.

5) Harmonic Excitation : One more interesting pattern of vehicle entry could be such that vehicles of the same make are getting admitted from left end with same velocities and with a definite spacing. In this case the loading pattern on 0₁A and 0₂B could be as shown below in Fig. 3 and Fig. 4.

![Fig. 3: Load pattern on 0₁A](image)

T = time to complete travel on bridge
L = Length of the bridge
V₁ = Vehicle Velocity
T’ = Constant time gap between two consecutive entries.

![Fig. 4: Load pattern on 0₂B](image)

T = time to complete travel on bridge
L = Length of the bridge
V₁ = Vehicle Velocity
T’ = Constant time gap between two consecutive entries

![Figure 5: Influence lines for the bridge](image)
As the load patterns described in Fig. 3 and Fig. 4 one can say that the load pattern is a periodically varying function. Such a function can be represented by Harmonic Series [2]. In view of every harmonic component one can decide what could probably be the resonant frequencies. It should then be seen that none of these resonant frequencies are close to the natural frequencies of either 0;A or 0;B in order to avoid induction of higher value of stress under vibrations [3] due to complete or partial resonance.

This aspect should be checked in view of all the 1 to 3 patterns of assumptions of distribution of mass, elasticity and damping.

The paper essentially addresses the issue of vibrations of bridge column which is much less addressed so far [4 to 69].

V. CONCLUSION

The first section of the paper details the scope of bridge column vibrations of a fairly small length bridge which may be considered as a portal frame. Sections 2 & 3 respectively detail qualitative and quantitative force analysis of a portal frame with a concentrated load acting only such that the load changes continuously its position from one end to the other. Section 4 details fairly detailed possibility of deciding vibration response of bridge columns. Detailing every possibility of vibration excitation and OR mass-stiffness-damping, distribution of two columns will precipitate individual paper. This is what is planned as a future extension of this work.

In addition the paper includes the influence line [70] of the bridge as depicted in Fig. 5.

REFERENCES

2. P.A. Pipes, “Applied Mathematics for Engineers”, Chapter -....
3. J.P. Denherdogt, “Mechanical Vibrations”


70. Keval Pujara, Vibrations for Engineers, Dhanpat Rai & Sons, Nai Sarak, Delhi

71. A reference on influence line