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Some numerical methods for Schnackenberg model

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Abstract:- In this paper, Schnackenberg model has been solved numerically for finding an approximate solution by Finite difference method and Adomain decomposition method. Example showed that ADM more accurate than FDM and more efficient for this kind of problems as shown in tables (1,2) and figures (1-4).

Keywords:- Adomain Decomposition method, Partial Differential Equations, Schnackenberg equations.

I. INTRODUCTION

Many physical, chemical and engineering problems mathematically can be modeled in the form of system of partial differential equations or system of ordinary differential equations. Finding the exact solution for the above problems which involve partial differential equations is difficult in some cases. Here we have to find the numerical solution of these problems using computers which came into existence [11].
For nonlinear partial differential equations, however, the linear superposition principle cannot be applied to generate a new solution. So, because most solution methods for linear equations cannot be applied to nonlinear equations, there is no general method of finding analytical solutions of nonlinear partial differential equation, and numerical techniques are usually required for their solution[6].

1.1 MATHEMATICAL MODEL:

A general class of nonlinear-diffusion system is in the form

$$\left.\begin{aligned}\frac{\partial u}{\partial t} &= d_1\Delta u + a_1 u + b_1 v + f(u,v) + g_1(x)\\ \frac{\partial v}{\partial t} &= d_2\Delta v + a_2 u + b_2 v - f(u,v) + g_2(x)\end{aligned}\right\} \qquad (1)$$

With homogenous dirchlet or neumann boundary condition on a bounded domain Ω , $n\leq 3$, with locally lipschitz continuous boundary. It is well known that reaction and diffusion of chemical or biochemical species can produce a variety of spatial patterns. This class of reaction diffusion systems includes some significant pattern formation equations arising from the modeling of kinetics of chemical or biochemical reactions and from the biological pattern formation theory. In this group, the following four systems are typically important and serve as mathematical models in physical chemistry and in biology:

- **Brusselator model:**

$a_1 = -(b+1),\ b_1 = 0,\ a_2 = b,\ b_2 = 0,\ f = u^2 v,\ g_1 = a,\ g_2 = 0$

where a and b are positive constants.

- **Gray-Scott model:**

$a_1 = -(F+k),\ b_1 = 0,\ a_2 = 0,\ b_2 = -F,\ f = u^2 v,\ g_1 = 0,\ g_2 = F$

where F and k are positive constants.

- **Glycolysis model:**

$a_1 = -1,\ b_1 = k,\ a_2 = 0,\ b_2 = -k,\ f = u^2 v,\ g_1 = p,\ g_2 = \delta$

where k, p and δ are positive constants.

- **Schnackenberg model:**

$a_1 = -k,\ b_1 = a_2 = b_2 = 0,\ f = u^2 v,\ g_1 = a,\ g_2 = b$

Where k, a and b are positive constants [14].
Then one obtains the following system of two nonlinearly coupled reaction-diffusion equations,

$$\left.\begin{aligned}\frac{\partial u}{\partial t} &= d_1\Delta u - ku + u^2 v + a\\ \frac{\partial v}{\partial t} &= d_2\Delta v - u^2 v + b\end{aligned}\right\} \quad (2)$$

With initial and boundary conditions:

$$\left.\begin{aligned}u(t,x) = v(t,x) = 0, \quad t > 0, \quad x \in \partial\Omega\\ u(0,x) = u_0(x), \ v(0,x) = v_0(x) \quad x \in \partial\Omega\end{aligned}\right\} \quad (3)$$

And with Neumann boundary conditions:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0 \qquad at \quad x = 0 \quad and \quad x = L \quad (4)$$

Where d_1, d_2, k, a, b *and* L are positive constants [14].

Reaction-diffusion (RD) systems arise frequently in the study of chemical and biological phenomena and are naturally modeled by parabolic partial differential equations (PDEs). The dynamics of RD systems has been the subject of intense research activity over the past decades. The reason is that RD system exhibit very rich dynamic behavior including periodic and quasi-periodic solutions [4, 13].

Various orders are self-organized far from the chemical equilibrium. The theoretical procedures and notions to describe the dynamics of patterns formation have been developed for the last three decades [10]. Attempts have also been made to understand morphological orders in biology [5]. Clarification of the mechanisms of the formation of orders and the relationship among them has been one of the fundamental problems in non-equilibrium statistical physics [9].

Various finite difference algorithms or schemes have been presented for the solution of hyperbolic-parabolic problem or its simpler derivatives, such as the classical diffusion equation. It is well-known that many of these schemes are partially unsatisfactory due to the formation of oscillations and numerical diffusion within the solution [12].

Solution by the finite difference method, although more general, will involve stability and convergence problems, may require special handling of boundary conditions, and may require large computer storage and execution time. The problem of numerical dispersion for finite difference solutions is also difficult to overcome [7].

Adomian decomposition has been applied to solve many functional equations so far. In this article, we have used this method to solve the heat equation, which governs on numerous scientific and engineering experimentations. Some special cases of the equation are solved as examples to illustrate ability and reliability of the method. Restrictions on applying Adomian decomposition method for these equations are discussed [3]. The decomposition method can be an inactive procedure for analytical solution of a wide class of dynamical systems without linearization or weak nonlinearity assumptions, closure approximations, perturbation theory, or restrictive assumptions on stochasticitiy [1].

II. MATERIALS AND METHODS

2.1 FINITE DIFFERENCE APPROXIMATIONS

The finite difference Scheme, generally reduces a linear, nonlinear partial differential equations into system of linear, nonlinear equations and various methods were developed to find the numerical solution and acceleration the convergence [8]. Assume that the rectangle $R = \{(x,t) : 0 \le x \le a, \quad 0 \le t \le b\}$ is subdivided into *n*-1 by *m*-1 rectangle with sides $\Delta x = h$ and $\Delta t = k$. Start at the bottom row, where $t = t_1 = 0$, and the solution is $u(x_i, t_1) = f(x_i)$. The grid spacing is uniform in every row: $x_{i+1} = x_i + h$ and $(x_{i-1} = x_i - h)$, and it is uniform in every column: $t_{j+1} = t_j + k$ and $(t_{j-1} = t_j - k)$. And use the approximation $u_{i,j}$ for $u(x_i, t_j)$ to obtain [8].

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \quad (5)$$

$$\frac{\partial v}{\partial t} = \frac{v_{i,j+1} - v_{i,j}}{\Delta t} \quad (6)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta x)^2} \quad (7)$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{v_{i-1,j} - 2v_{i,j} + v_{i+1,j}}{(\Delta x)^2} \quad (8)$$

Substitute (5) – (8) in the Schnackenberg model (1) to get

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}=d_1\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta x)^2}-ku_{i,j}+(u_{i,j})^2v_{i,j}+a$$

$$\frac{v_{i,j+1}-v_{i,j}}{\Delta t}=d_2\frac{v_{i-1,j}-2v_{i,j}+v_{i+1,j}}{(\Delta x)^2}-(u_{i,j})^2v_{i,j}+b$$

$$u_{i,j+1}-u_{i,j}=\frac{d_1\Delta t}{(\Delta x)^2}[u_{i-1,j}-2u_{i,j}+u_{i+1,j}]-\Delta tku_{i,j}+\Delta t(u_{i,j})^2v_{i,j}+\Delta ta$$

$$v_{i,j+1}-v_{i,j}=\frac{d_2\Delta t}{(\Delta x)^2}[v_{i-1,j}-2v_{i,j}+v_{i+1,j}]-\Delta t(u_{i,j})^2v_{i,j}+\Delta tb$$

Let $r_1=\frac{d_1\Delta t}{(\Delta x)^2}$ and $r_2=\frac{d_2\Delta t}{(\Delta x)^2}$ then

$$u_{i,j+1}=u_{i,j}+r_1u_{i-1,j}-2r_1u_{i,j}+r_1u_{i+1,j}-\Delta tku_{i,j}+\Delta t(u_{i,j})^2v_{i,j}+\Delta ta$$

$$v_{i,j+1}=v_{i,j}+r_2v_{i-1,j}-2r_2v_{i,j}+r_2v_{i+1,j}-\Delta t(u_{i,j})^2v_{i,j}+\Delta tb$$

$$\left.\begin{aligned}u_{i,j+1}&=[1-2r_1-\Delta tk]u_{i,j}+r_1[u_{i-1,j}+u_{i+1,j}]+\Delta t(u_{i,j})^2v_{i,j}+\Delta ta\\v_{i,j+1}&=[1-2r_2]v_{i,j}+r_2[v_{i-1,j}+v_{i+1,j}]-\Delta t(u_{i,j})^2v_{i,j}+\Delta tb\end{aligned}\right\} \quad (9)$$

From the boundary conditions (4) we have

$\frac{\partial u}{\partial x}=\frac{u_{i,j}-u_{i-1,j}}{2\Delta x}=0$, $so\ u_{i-1,j}=u_{i,j}\quad and\quad u_{1,j}=u_{2,j}$

And

$\frac{\partial u}{\partial x}=\frac{u_{i+1,j}-u_{i,j}}{2\Delta x}=0$, $so\ u_{i+1,j}=u_{i,j}\quad and\quad u_{11,j}=u_{10,j}$

And also for v

$\frac{\partial v}{\partial x}=\frac{v_{i,j}-v_{i-1,j}}{2\Delta x}=0$, $so\ v_{i-1,j}=v_{i,j}\quad and\quad v_{1,j}=v_{2,j}$

And

$\frac{\partial v}{\partial x}=\frac{v_{i+1,j}-v_{i,j}}{2\Delta x}=0$, $so\ v_{i+1,j}=v_{i,j}\quad and\quad v_{11,j}=v_{10,j}$

And from the initial condition:

$u_{1,1}=u_{2,1}=u_{3,1}=u_{4,1}=u_{5,1}=u_{6,1}=u_{7,1}=u_{8,1}=u_{9,1}=u_{10,1}=u_{11,1}=u_0(x)$

$v_{1,1}=v_{2,1}=v_{3,1}=v_{4,1}=v_{5,1}=v_{6,1}=v_{7,1}=v_{8,1}=v_{9,1}=v_{10,1}=v_{11,1}=v_0(x)$

The result equation (8) is the finite difference method for the Schnackenber model.

2.2 ADOMIAN DECOMPOSITION METHOD

Nonlinear differential equations are usually arising from mathematical modeling of many frontier physical systems. In most cases, analytic solutions of these differential equations are very difficult to achieve. Common analytic procedures linearize the system or assume the nonlinearities are relatively insignificant. Such procedures change the actual problem to make it tractable by the conventional methods. This changes, some times seriously, the solution. The above drawbacks of linearization and numerical methods arise the need to search for alternative techniques to solve the nonlinear differential equations, namely, the analytic solution methods, such as the perturbation method, the iteration variational method [11–14] and the Adomian decomposition method.

The decomposition method was first introduced by Adomian since the beginning of the 1980s.The Adomian decomposition method (ADM) is used to solve a wide range of physical problems. This method provides a direct scheme for solving linear and nonlinear deterministic and stochastic equations without the need for linearization this yields convergent series solutions rapidly. An advantage of this method is that, it can provide analytical approximation or an approximated solution to a rather wide class of nonlinear (and stochastic) equations without linearization, perturbation, closure approximation, or discretization methods. Unlike the common methods, i.e., weak nonlinearity and small perturbation which change the physics of the problem due to simplification, ADM gives the approximated solution of the problem without any simplification. Thus, its results are more realistic [1,2].

We define the operator $L_t=\frac{\partial}{\partial t}\quad\Rightarrow\quad L_t^{-1}=\int_0^t(.)dt$ and $L_{xx}=\frac{\partial^2}{\partial x^2}$ then

system (1) can be written as:

$$L_t u = d_1 L_{xx} u - ku + u^2 v + a \quad (10)$$

$$L_t v = d_2 L_{xx} v - u^2 v + b \quad (11)$$

By applying the inverse of operator L_t on (10,11) we get:

$$u(t,x) = u(0,x) + d_1 L_t^{-1}(L_{xx} u) - kL_t^{-1} u + L_t^{-1}(u^2 v) + L_t^{-1} a$$

$$v(t,x) = v(0,x) + d_2 L_t^{-1}(L_{xx} v) - L_t^{-1}(u^2 v) + L_t^{-1} b$$

By initial conditions in system (3) then (10,11) can be written as:

$$u(t,x) = u_0(x) + d_1 L_t^{-1}(L_{xx} u) - kL_t^{-1} u + L_t^{-1}(u^2 v) + L_t^{-1} a$$

$$v(t,x) = v_0(x) + d_2 L_t^{-1}(L_{xx} v) - L_t^{-1}(u^2 v) + L_t^{-1} b$$

By using Adomian decomposition method :

$$u(t,x) = \sum_{n=0}^{\infty} u_n(t,x) \quad \text{and} \quad v(t,x) = \sum_{n=0}^{\infty} v_n(t,x)$$

$$\sum_{n=0}^{\infty} u_n(t,x) = u_0(x) + d_1 L_t^{-1}(L_{xx} \sum_{n=0}^{\infty} u_n) - kL_t^{-1} \sum_{n=0}^{\infty} u_n + L_t^{-1} \sum_{n=0}^{\infty} A_n + L_t^{-1} a \quad (12)$$

$$\sum_{n=0}^{\infty} v_n(t,x) = v_0(x) + d_2 L_t^{-1}(L_{xx} \sum_{n=0}^{\infty} v_n) - L_t^{-1} \sum_{n=0}^{\infty} B_n + L_t^{-1} b \quad (13)$$

Where A_n and A_n are Adomian polynomials. But $A_n = B_n$ because both non-linear terms are $u^2 v$

Where $A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[F(\sum_{i=0}^{n} \lambda^i u_i) \right]_{\lambda=0}$

But here $A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[F(\sum_{i=0}^{n} \lambda^i u_i, \sum_{i=0}^{n} \lambda^i v_i) \right]_{\lambda=0}$ because non-linear term have two functions $u(t,x)$ and $v(t,x)$ then by equation (12):

$$u_0 = u_0(x)$$

$$u_{k+1} = d_1 L_t^{-1}(L_{xx} u_k) - kL_t^{-1} u_k + L_t^{-1} A_k + L_t^{-1} a \qquad k \geq 0$$

By equation (13):

$$v_0 = v_0(x)$$

$$v_{k+1} = d_2 L_t^{-1}(L_{xx} v_k) - L_t^{-1} B_k + L_t^{-1} b \qquad k \geq 0$$

$$A_0 = \frac{1}{0!} \frac{d^0}{d\lambda^0} \left[F(\sum_{i=0}^{0} \lambda^i u_i, \sum_{i=0}^{0} \lambda^i v_i) \right]_{\lambda=0}$$

$$= F(u_0, v_0) = u_0^2 v_0 = B \quad (14)$$

$$k = 0$$

$$u_1 = d_1 L_t^{-1}(L_{xx} u_0) - kL_t^{-1} u_0 + L_t^{-1} A_0 + L_t^{-1} a = d_1 L_t^{-1}(L_{xx} u_0) - kL_t^{-1} u_0 + L_t^{-1}(u_0^2 v_0) + L_t^{-1} a$$

$$= d_1 L_{xx} u_0 t - ku_0 t + u_0^2 v_0 t + at = [d_1 L_{xx} u_0 - ku_0 + u_0^2 v_0 + a]t$$

$$Let: U1 = d_1 L_{xx} u_0 - ku_0 + u_0^2 v_0 + a$$

$$u_1 = U1t \quad (15)$$

$$v_1 = d_2 L_t^{-1}(L_{xx} v_0) - L_t^{-1} B_0 + L_t^{-1} b = d_2 L_t^{-1}(L_{xx} v_0) - L_t^{-1}(u_0^2 v_0) + L_t^{-1} b$$

$$= d_2 L_{xx} v_0 t - u_0^2 v_0 t + bt = [d_2 L_{xx} v_0 - u_0^2 v_0 + b]t$$

$$Let: V1 = d_2 L_{xx} v_0 - u_0^2 v_0 + b$$

$$v_1 = V1t \quad (16)$$

$$A_1 = \frac{1}{1!} \frac{d^1}{d\lambda^1} \left[F(\sum_{i=0}^{1} \lambda^i u_i, \sum_{i=0}^{1} \lambda^i v_i) \right]_{\lambda=0} = \frac{d}{d\lambda} [F(u_0 + \lambda u_1, v_0 + \lambda v_1)]_{\lambda=0}$$

$$= \frac{d}{d\lambda}[(u_0 + \lambda u_1)^2 (v_0 + \lambda v_1)]_{\lambda=0} = \frac{d}{d\lambda}[(u_0^2 + 2\lambda u_0 u_1 + \lambda^2 u_1^2)(v_0 + \lambda v_1)]_{\lambda=0}$$

$$= \frac{d}{d\lambda}[u_0^2 v_0 + 2\lambda u_0 u_1 v_0 + \lambda^2 u_1^2 v_0 + \lambda u_0^2 v_1 + 2\lambda^2 u_0 u_1 v_1 + \lambda^3 u_1^2 v_1]_{\lambda=0}$$

$$= 2u_0 u_1 v_0 + u_0^2 v_1 = 2u_0 (U1t) v_0 + u_0^2 V1t$$

$$A_1 = [2u_0 v_0 U1 + u_0^2 V1]t = B_1 \qquad (17)$$

K=1

$$u_2 = d_1 L_t^{-1}(L_{xx} u_1) - k L_t^{-1} u_1 + L_t^{-1} A_1 + L_t^{-1} a = d_1 L_t^{-1}[L_{xx}(U1t)] - k L_t^{-1}(U1t) + L_t^{-1}([2u_0 v_0 U1 + u_0^2 V1]t)$$

$$= d_1 L_{xx}(U1)\frac{t^2}{2} - kU1\frac{t^2}{2} + [2u_0 v_0 U1 + u_0^2 V1]\frac{t^2}{2} = [d_1 L_{xx}(U1) - kU1 + 2u_0 v_0 U1 + u_0^2 V1]\frac{t^2}{2} + at$$

$$Let: U2 = d_1 L_{xx}(U1) - kU1 + 2u_0 v_0 U1 + u_0^2 V1$$

$$u_2 = U2\frac{t^2}{2} + at \qquad (18)$$

$$v_2 = d_2 L_t^{-1}(L_{xx} v_1) + F L_t^{-1}(1 - v_1) - L_t^{-1} B_1$$

$$= d_2 L_t^{-1}[L_{xx}(V1t)] + F L_t^{-1}(1 - V1t) - L_t^{-1}([2u_0 v_0 U1 + u_0^2 V1]t)$$

$$= d_2 L_{xx}(V1)\frac{t^2}{2} + F(t - V1\frac{t^2}{2}) - [2u_0 v_0 U1 + u_0^2 V1]\frac{t^2}{2}$$

$$v_2 = Ft + [d_2 L_{xx}(V1) - FV1 - 2u_0 v_0 U1 - u_0^2 V1]\frac{t^2}{2} + bt$$

$$Let: V2 = d_2 L_{xx}(V1) - FV1 - 2u_0 v_0 U1 - u_0^2 V1$$

$$v_2 = Ft + V2\frac{t^2}{2} + bt \qquad (19)$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[F(\sum_{i=0}^{2}\lambda^i u_i, \sum_{i=0}^{2}\lambda^i v_i)\right]_{\lambda=0} = \frac{1}{2!}\frac{d^2}{d\lambda^2}[F(u_0 + \lambda u_1 + \lambda^2 u_2, v_0 + \lambda v_1 + \lambda^2 v_2)]_{\lambda=0}$$

$$= \frac{1}{2!}\frac{d^2}{d\lambda^2}[(u_0 + \lambda u_1 + \lambda^2 u_2)^2 (v_0 + \lambda v_1 + \lambda^2 v_2)]_{\lambda=0}$$

$$= \frac{1}{2!}\frac{d^2}{d\lambda^2}[[u_0^2 + 2u_0(\lambda u_1 + \lambda^2 u_2) + (\lambda u_1 + \lambda^2 u_2)^2](v_0 + \lambda v_1 + \lambda^2 v_2)]_{\lambda=0}$$

$$= \frac{1}{2!}\frac{d^2}{d\lambda^2}[(u_0^2 + 2\lambda u_0 u_1 + 2\lambda^2 u_0 u_2 + \lambda^2 u_1^2 + 2\lambda^3 u_1 u_2 + \lambda^4 u_2^2)(v_0 + \lambda v_1 + \lambda^2 v_2)]_{\lambda=0}$$

$$= \frac{1}{2!}\frac{d^2}{d\lambda^2}[u_0^2 v_0 + 2\lambda u_0 u_1 v_0 + 2\lambda^2 u_0 u_2 v_0 + \lambda^2 u_1^2 v_0 + 2\lambda^3 u_1 u_2 v_0 + \lambda^4 u_2^2 v_0$$
$$+ \lambda u_0^2 v_1 + 2\lambda^2 u_0 u_1 v_1 + 2\lambda^3 u_0 u_2 v_1 + \lambda^3 u_1^2 v_1 + 2\lambda^4 u_1 u_2 v_1 + \lambda^5 u_2^2 v_1$$
$$+ \lambda^2 u_0^2 v_2 + 2\lambda^3 u_0 u_1 v_2 + 2\lambda^4 u_0 u_2 v_2 + \lambda^4 u_1^2 v_2 + 2\lambda^5 u_1 u_2 v_2 + \lambda^6 u_2^2 v_2]_{\lambda=0}$$

$$= \frac{1}{2}(4u_0 u_2 v_0 + 2u_1^2 v_0 + 4u_0 u_1 v_1 + 2u_0^2 v_2) = 2u_0 u_2 v_0 + 2u_0 u_1 v_1 + u_0^2 v_2 + u_1^2 v_0 \qquad (20)$$

$$A_2 = 2u_0(U2\frac{t^2}{2} + at)v_0 + 2u_0(U1t + at)(V1t + bt) + u_0^2(Ft + V2\frac{t^2}{2}) + (U1t + at)^2 v_0$$

By eqs (16) , (17) , (19) and (20)

$$A_2 = u_0 v_0 U2t^2 + 2u_0 U1V1t^2 + u_0^2 V2\frac{t^2}{2} + (U1)^2 v_0 t^2 + (u_0^2 bt 2u_0 a)t$$

$$A_2 = [u_0 v_0 U2 + 2u_0 U1V1 + \frac{1}{2}u_0^2 V2 + (U1)^2 v_0]t^2 + (u_0^2 bt 2u_0 a)t = B_2 \qquad (21)$$

$k = 2$

$u_3 = d_1 L_t^{-1}(L_{xx} u_2) - k L_t^{-1} u_2 + L_t^{-1} A_2 + L_t^{-1} a$

By equations (18) and (21)

$u_3 = d_1 L_t^{-1}[L_{xx}(U2\frac{t^2}{2})] - k L_t^{-1}(U2\frac{t^2}{2}) + L_t^{-1}([u_0 v_0 U2 + 2u_0 U1V1 + \frac{1}{2}u_0^2 V2$

$+ (U1)^2 v_0]t^2)$

$= \frac{1}{2} d_1 L_{xx}(U2)\frac{t^3}{3} - \frac{1}{2} kU2\frac{t^3}{3} + [u_0 v_0 U2 + 2u_0 U1V1 + \frac{1}{2}u_0^2 V2 + (U1)^2 v_0]\frac{t^3}{3}$

$u_3 = [\frac{1}{2} d_1 L_{xx}(U2) - \frac{1}{2} kU2 + u_0 v_0 U2 + 2u_0 U1V1 + \frac{1}{2}u_0^2 V2 + (U1)^2 v_0]\frac{t^3}{3}$

$+ (a + u_0^2 b + 2u_0 a)\frac{t^2}{2} + at)$ (22)

$v_3 = d_2 L_t^{-1}(L_{xx} v_2) - L_t^{-1} B_2 + L_t^{-1} b$

By eqs (19) and (21)

$v_3 = d_2 L_t^{-1}[L_{xx}(V2\frac{t^2}{2})] - L_t^{-1}([u_0 v_0 U2 + 2u_0 U1V1 + \frac{1}{2}u_0^2 V2 + (U1)^2 v_0]t^2) + u_0^2 b + 2u_0 a)t) + bt$

$= \frac{1}{2} d_2 L_{xx}(V2)\frac{t^3}{3} - [u_0 v_0 U2 + 2u_0 U1V1 + \frac{1}{2}u_0^2 V2 + (U1)^2 v_0]\frac{t^3}{3} + (u_0^2 b + 2u_0 a)\frac{t^2}{2} + bt$

$v_3 = [\frac{1}{2} d_2 L_{xx}(V2) - u_0 v_0 U2 - 2u_0 U1V1 - \frac{1}{2}u_0^2 V2 - (U1)^2 v_0]\frac{t^3}{3} + (u_0^2 b + 2u_0 a)\frac{t^2}{2} + bt$ (23)

III. APPLICATION (NUMERICAL EXAMPLE)

We solved the following example numerically to illustrate efficiency of the presented methods.

$\frac{\partial u}{\partial t} = d_1 \Delta u - ku + u^2 v + a \quad , t > 0, \qquad x \in \Omega$

$\frac{\partial v}{\partial t} = d_1 \Delta v - u^2 v + b \quad , t > 0, \qquad x \in \Omega$

We the initial conditions

$u(x, 0) = Us + 0.01 \sin(\pi x / L)$ for $0 \le x \le L$

$v(x, 0) = Vs - 0.12 \sin(\pi x / L)$ for $0 \le x \le L$

$u(0, t) = Us$, $u(L, t) = Us$ and $v(0, t) = Vs$, $v(L, t) = Vs$

We will take

$d_1 = d_2 = 0.01$, a=b= 0.09 , k=-0.004, Us=0, Vs=1

IV. FIGURES AND TABLES

Table 1 Comparison between the FDM and ADM for the values of concentration V.

	t = 1		t=2		t=3	
x	ADM	FDM	ADM	FDM	ADM	FDM
0	1.0001	1.0000	1.0001	1.0000	1.0035	1.0000
0.1	1.0014	1.0008	1.0022	0.9993	1.0048	0.9971
0.2	1.0020	1.0010	1.0022	0.9976	1.0043	0.9864
0.3	1.0021	1.0019	1.0012	0.9946	1.0021	0.9787
0.4	1.0018	1.0012	0.9994	0.9906	0.9984	0.9704
0.5	1.0011	1.0005	0.9971	0.9861	0.9936	0.9624
0.6	1.0003	0.9996	0.9945	0.9817	0.9881	0.9553
0.7	0.9995	0.9982	0.9921	0.9777	0.9826	0.9499
0.8	0.9988	0.9968	0.9900	0.9746	0.9781	0.9464
0.9	0.9984	0.9954	0.9887	0.9726	0.9750	0.9452
1.0	0.9982	0.9942	0.9882	0.9719	0.9739	0.9464
1.1	0.9984	0.9954	0.9887	0.9726	0.9750	0.9499

1.2	0.9988	0.9968	0.9900	0.9746	0.9781	0.9553
1.3	0.9995	0.9982	0.9921	0.9777	0.9827	0.9624
1.4	1.0003	0.9996	0.9945	0.9817	0.9881	0.9704
1.5	1.0011	1.0005	0.9971	0.9861	0.9936	0.9787
1.6	1.0018	1.0012	0.9994	0.9906	0.9984	0.9864
1.7	1.0021	1.0019	1.0012	0.9946	1.0021	0.9927
1.8	1.0020	1.0010	1.0022	0.9976	1.0043	0.9971
1.9	1.0014	1.0008	1.0022	0.9993	1.0048	0.9993
2.0	1.0001	1.0000	1.0010	1.0000	1.0035	1.0000

Table 2 Comparison between the FDM and ADM for the values of concentration U.

	t =1		t=2		t=3	
x	ADM	FDM	ADM	FDM	ADM	FDM
0	0.0015	0	0.0048	0	0.0081	0
0.1	-0.0150	-0.0232	-0.0102	-0.0276	-0.0108	-0.0319
0.2	-0.0306	-0.0456	-0.0241	-0.0539	-0.0221	-0.0623
0.3	-0.0450	-0.0664	-0.0367	-0.0783	-0.0316	-0.0902
0.4	-0.0579	-0.0851	-0.0476	-0.0999	-0.0390	-0.1149
0.5	-0.0692	-0.1014	-0.0568	0.1185	-0.0444	-0.1359
0.6	-0.0786	-0.1151	-0.0643	-0.1339	-0.0480	-0.1532
0.7	-0.0861	-0.1258	-0.0700	-0.1459	-0.0502	-0.1666
0.8	-0.0915	-0.1336	-0.0741	-0.1545	-0.0514	-0.1761
0.9	-0.0948	-0.1383	-0.0766	-0.1596	-0.0520	-0.1818
1.0	-0.0959	-0.1398	-0.0776	-0.1613	-0.0522	-0.1837
1.1	-0.0949	-0.1383	-0.0770	-0.1596	-0.0523	-0.1818
1.2	-0.0916	-0.1336	-0.0749	-0.1545	-0.0519	-0.1761
1.3	-0.0863	-0.1258	-0.0710	-0.1459	-0.0509	-0.1666
1.4	-0.0788	-0.1151	-0.0654	-0.1339	-0.0488	-0.1532
1.5	-0.0693	--0.1014	-0.0578	0.1185	-0.0451	-0.1359
1.6	-0.0580	-0.0851	-0.0484	0.1185	-0.0395	-0.1149
1.7	-0.0450	-0.0664	-0.0372	-0.0999	-0.0319	-0.0902
1.8	-0.0306	-0.0456	-0.0244	-0.0783	-0.0223	-0.0623
1.9	-0.0150	-0.0232	-0.0103	-0.0539	-0.0108	-0.0319
2.0	0.0015	-0.0000	0.0048	-0.0000	0.0023	-0.0000

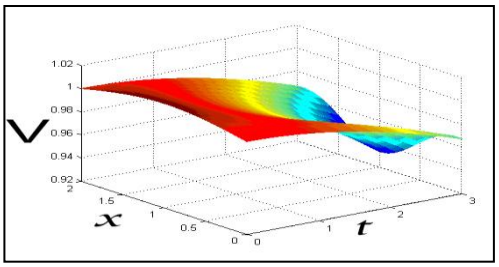

Fig. 1 ADM for the values of concentration V with 0<x<2 and 0<t<3

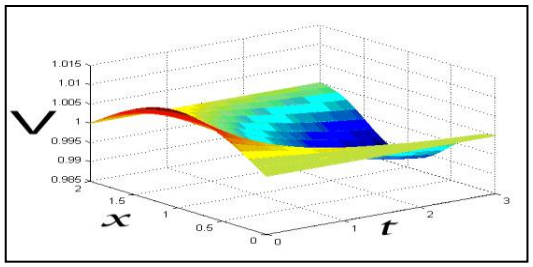

Fig. 2 FDM for the values of concentration V with 0<x<2 and 0<t<3

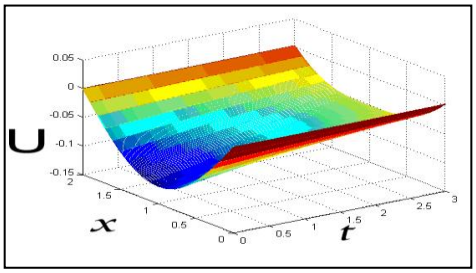

Fig. 3 ADM for the values of concentration U with 0<x<2 and 0<t<3

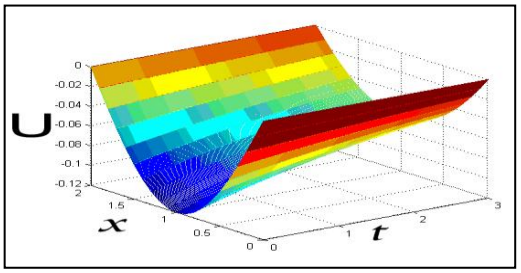

Fig. 4 FDM for the values of concentration U with 0<x<2 and 0<t<3

V. CONCLUSION

The Schnackenberg model solved Numerically using finite difference method and Adomain decomposition method, and we found that's finite difference method is earlier that ADM but ADM is more accurate than FDM and more efficient as show in tables (1-2) and figures (1-4).

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