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MHD Flow of a Rivlin - Ericksen Fluid through a Porous Medium in a Parallel Plate Channel under an Externally Applied Boundary Acceleration

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Abstract: *In this paper an initial value investigation has been made for a two dimensional MHD flow of a second order Rivlin-Ericksen type visco-elastic fluid through a porous medium in a parallel plate channel under externally applied boundary acceleration. Ihe exact solutions for the velocity, accelerations and shear stress have been obtained using separations of variables method. Their behaviour for different governing parameters has been discussed computationally based on the available physiological data.*
Keywords: *Rivlin-Ericksen fluid flow, boundary acceleration, porous medium and parallel plate channel.*

I. INTRODUCTION

External acceleration of large amplitudes imparted to the human body cause serious problems in the cardiovascular system, causing the impairment of certain physiological functions. Head ache, increase of pulse rate, loss of vision, venous pooling of blood in the lower extremities, congestion of liver and lungs hemorrhages in the face and neck, eyes, lungs and brain are some of the complications as a result of external accelerations [1, 3 & 4]. On the other hand, sudden accelerations are deliberately imparted to the body to obtain some beneficial effects. For example, body accelerations synchronous with heart beat, a proposed new method for assisting the failing heart, has been employed by Verdouw et.al [8] to reactive the failing heart. Experimental results obtained by Arntzentus et.al [2] on heart action in pigs indicate that blood pressure and cardiac output are raised when body accelerations synchronous with heart beat was applied in the forward direction in early systole. One of the channel walls in subjected to externally applied boundary acceleration. Against from this, blood is in constant circulation with in the blood vessels throughout the body. This circulatory system is the transport system of the body consisting of four type's viz., systematic circulation, the pulmonary circulation, the portal circulation and the coronary circulation. In these circulatory systems, the blood ultimately flows through a network of capillaries with porous wall having distinct functions. These capillaries act as loading or unloading stations in a transportation system with main function of nourishing the tissues in general. The flow through such net work of capillaries may be idealized as a two dimensional flow through a porous medium. The interaction of body accelerations with blood flow is desirable for controlling the ill effects of such body acceleration over blood circulation Lachlan [6]. Krishna and Rao [5] studied an initial value investigation has been made for a two dimensional flow of a second order Rivlin-Ericksen type visco-elastic fluid and the exact solutions for the velocity, acceleration, shear stress and volume flow rate have been obtained using transform method. Numerical computations have been carried out to discuss their behaviour for different parameters based on the available physiological data. Sud. V.K., et al [7] analyzed the blood flow in large and small arteries under the influence of externally applied periodic oscillations. The solutions for the blood velocity, fluid acceleration and shear stress are obtained. It has been show that high blood velocity and large shear stress are produced in large arteries. However, in the case of small arteries the flow is not disturbed. Recently Veera Krishna et.al [9] studied an initial value investigation and has been made for a two dimensional flow of a second order Rivlin-Ericksen type visco-elastic fluid in a parallel plate channel under externally applied boundary acceleration. Suneetha et.al [10] discussed the steady hydro magnetic flow of a couple stress fluid in a parallel plate channel bounded on one side by a porous bed under the influence of a transverse magnetic field and periodic body acceleration. In this paper an initial value investigation has been made for a two dimensional MHD flow of a second order Rivlin-Ericksen type visco-elastic fluid through a porous medium in a parallel plate channel under externally applied boundary acceleration.

II. FORMULATION AND SOLUTION OF THE PROBLEM

We consider an incompressible viscous and electrically conducting two dimensional flow of a second order Rivlin-Ericksen type visco-elastic fluid in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plates and the entire flow field is subjected to a uniform inclined magnetic field. Choose a Cartesian system (t, y) with boundary walls

$y = 0$ and h. The flow is uni-directional in view of the fact the pressure gradient and body acceleration are in the same direction. The governing equation of the motion in the non-dimensional flow with respect to the frame of reference is,

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma \mu_e^2 H_0^2}{\rho}u - \frac{\nu}{k}u \qquad (2.1)$$

With boundary conditions

$$\frac{du}{dt} = g.e^{int} \quad \text{at} \quad y = h \qquad (2.2)$$

$$u = 0 \quad \text{at} \quad y = 0 \qquad (2.3)$$

Equation (2.1) is to be solved subjected to the conditions (2.2) and (2.3), In view of Equation (2.2), we take

$$u = v(y).e^{int} \qquad (2.4)$$

And the applied pressure gradient

$$\frac{\partial p}{\partial x} = A.e^{int} \qquad (2.5)$$

Making use of (2.4) and (2.5) we solve (2.1) using the boundary conditions,

$$v = \frac{g}{n} Sinnt \quad \text{on} \quad y = h \qquad (2.6)$$

$$v = 0 \quad \text{on} \quad y = 0 \qquad (2.7)$$

Where, u is the velocity of the fluid, ρ is the density of fluid, μ is the coefficient of viscosity of the fluid, p is the pressure, A is an amplitude of pressure gradient, α_1 the coefficient of visco-elasticity g is the body acceleration (or) amplitude of the applied acceleration and 'n' its angular frequency in rad. sec^{-1}

From (2.1) and (2.4) we obtain

$$\frac{d^2 v(y)}{dy^2} - \frac{M_1}{\mu + \alpha_1 in} v(y) = \frac{A}{\mu + \alpha_1 in} \qquad (2.8)$$

Where, $M_1 = \rho in + \sigma \mu_e^2 H_0^2 + \frac{\mu}{k}$

The general solution of Equation (2.8) is

$$v(y) = C_1 \cosh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} y + C_2 \sinh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} y - \frac{A}{M_1} \qquad (2.9)$$

Now we find the arbitrary constants C_1 and C_2 by using equations (2.6) and (2.7) we get

$$v(y) = \frac{A}{M_1}\cosh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} y + \frac{\frac{A}{M_1}\left(1 - \cosh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} h\right) + \frac{g}{n}\sin nt}{\sinh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} h} .\sinh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} y - \frac{A}{M_1} \qquad (2.10)$$

Equation (2.10) is substituted in equation (2.4) we get

$$u = \left[\frac{A}{M_1}\cosh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} y + \frac{\frac{A}{M_1}\left(1 - \cosh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} h\right) + ing}{\sinh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} h} .\sinh\frac{\sqrt{M_1}}{\sqrt{\mu + \alpha_1 in}} y - \frac{A}{M_1}\right] e^{int} \qquad (2.11)$$

Taking the real part of the equation (2.11) obtained as

$$u = A_1 + u_1 + u_2 + \frac{u_3 + u_4}{A_2} \qquad (2.12)$$

where

$A_1 = - M_2 \sin nt$

$A_2 = \sinh^2 ah.\cos^2 bh + \cosh^2 ah.\sin^2 bh$

$u_1 = M_2$ sinha$y$. sinb$y$. cosnt

$u_2 = M_2$ cosha$y$.cosb$y$.sinnt

$u_3 = M_2$ [sinha$h$.cosb$h$. cosha$y$.sinb$y$-cosha$h$.sinb$y$.sinha$y$.cosb$y$ - cosha$h$.cos^{2}bh.sinha$h$.cosha$y$.sinb$y$+ +cosh^{2}ah.cosb$h$.sinb$h$.sinha$y$.cosb$y$ - sinh^{2}ah.sinb$y$.cosb$h$.sinha$y$.cosb$y$- -cosha$h$.sinha$h$.sin^{2}bh.cosha$y$.sinb$y$.]cosnt - ng.sinha$h$.cosb$h$. cosha$y$.sinb$y$.cosnt + ng.cosha$h$.sinb$h$.sinha$y$.cosb$y$.cosnt

$u_4 = M_2$ [sinha$h$.cosb$h$.sinha$y$.cosb$y$+ +cosha$h$.sinb$h$.sinha$y$.sinb$y$-cosha$h$.cos^{2}bh.sinha$h$.sinha$y$.cosb$y$- - cosh^{2}ah.sinb$h$.cosb$h$.cosha$y$.sinb$y$+sinh^{2}ah.sinb$h$.cosb$h$.cosha$y$.sinb$y$- -cosha$h$.sinha$h$.sin^{2}bh.sinha$y$.cosb$y$]sinnt - ng.sinha$h$.cosb$h$.sinha$y$ cosb$y$.sinnt-ng.cosha$h$.sinb$h$.cosha$y$.sinb$y$.sinnt

$$M_2 = \frac{A\left(\sigma\mu_e^2 H_0^2 + \frac{\mu}{k}\right)}{\rho^2 n^2 + \left(\sigma\mu_e^2 H_0^2 + \frac{\mu}{k}\right)^2}$$

$$a = \frac{1}{\sqrt{2}}\left\{\frac{\left[R(S-(M^2+D^{-1})) + \sqrt{R^2(S-(M^2+D^{-1}))^2+1}\right]^{\frac{1}{2}}}{\left[\sqrt{R^2(S-(M^2+D^{-1}))^2+1}\right]^{\frac{1}{4}}}\right\},$$

$$b = \frac{1}{\sqrt{2}}\left\{\frac{\left[\sqrt{R^2(S-(M^2+D^{-1}))^2+1} - R(S-(M^2+D^{-1}))\right]^{\frac{1}{2}}}{\left[\sqrt{R^2(S-(M^2+D^{-1}))^2+1}\right]^{\frac{1}{4}}}\right\}$$

Here, the non-dimensional parameters R, S, β, M, D^{-1} and G are given by

$R = \frac{h^2 n\rho}{\mu}$ is the Reynolds number,

$S = \frac{\alpha_1}{h^2\rho}$ is the visco-elastic parameter,

$\beta = \frac{A}{\rho h n^2}$ is the applied pressure gradient parameter,

$D^{-1} = \frac{h^2}{k}$ is the inverse Darcy parameter

$M^2 = \frac{\sigma\,\mu_e^2 H_0^2 h^2}{\rho\nu}$ is the Hartmann number (Magnetic field Parameter)

and $G = \frac{g}{hn^2}$ is the body acceleration parameter

The shear stresses on the upper and lower walls at y = 0 and h in the non-dimensional form are calculated by

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right) + \alpha_1\left(\frac{\partial^2 u}{\partial y\partial t}\right) \tag{2.13}$$

III. RESULTS AND DISCUSSION

The profiles (1-6) exhibit the nature of the velocity for variations in the governing parameters S, β, R, G, M and D^{-1}. S the Visco-elastic parameter, R the Reynolds number, β the Applied pressure gradient parameter, M the Hartmann number, D^{-1} the inverse Darcy parameter and G the body acceleration parameter. In all these profiles, we observe that the velocity rapidly increases from zero value on the lower boundary y = 0 to its maximum value on the upper boundary $y = h$. For fixed R and β the velocity enhances with increase in the visco-elastic parameter and this enhancement in the velocity is plotted in the figure (1) for various values of β and R. For sufficiently large R the magnitude of the velocity is high comparable to smaller R and once again exhibits increase in trend with increase in S. The figure (2) correspond to decrease in β for fixed R and S, we notice that an enhancement in the velocity for reduction in β. Also higher values of R the magnitude of the velocity relatively increases for all β and S. The figure (3) correspond to behaviour of the velocity for variation in R fixing β and S, we once again notice that an increase in R enhances the velocity for fixed β and S while its magnitude exhibits a slight growth for increase in the visco-elastic parameter S at all corresponding values of β and R. Also the figure (4) correspond to behaviour of the velocity for variation in G fixing R, β and S, we once again notice that the velocity enhances increase in for fixed values of R, β and S. From figures (5& 6), the behavior of the velocity always reduces to increase the intensity of the magnetic field M, while the magnitude of the velocity continuously decreases with increase in the inverse Darcy parameter D^{-1}.

The shear stresses (τ) are evaluated for variations in R, S and β at both the lower and upper boundaries. In general, the shear stress in the upper boundary is higher in magnitude compared to its value on the lower boundary at the corresponding sets of the parameters. From tables (1) and (2) we find on the lower and upper boundaries and increase in R slightly increases τ for all fixed S, M, D^{-1} and β Tables (3) and (4) represent the variation in τ for increase S fixing R and β. We notice that τ enhances with S for all sets of R, M, D^{-1} and β. Likewise, the stress τ reduces with increase in β on the both boundaries for different sets R, M, D^{-1} and S as shown in tables (5&6).

IV. FIGURES AND TABLES

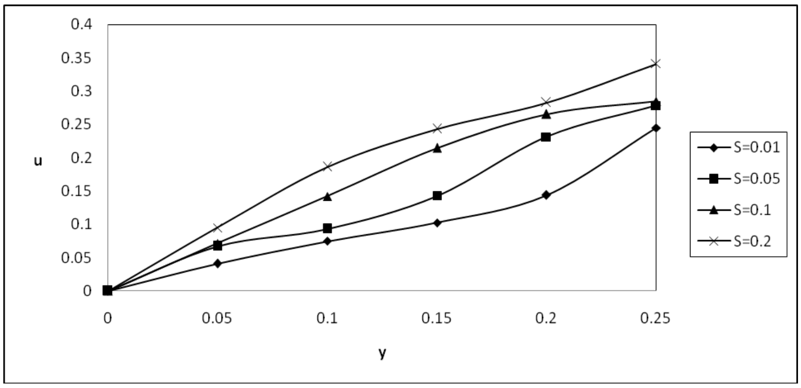

Fig 1: the velocity profile for u against S with β =0.77135, R=257.8125, M=2, D^{-1} =1000

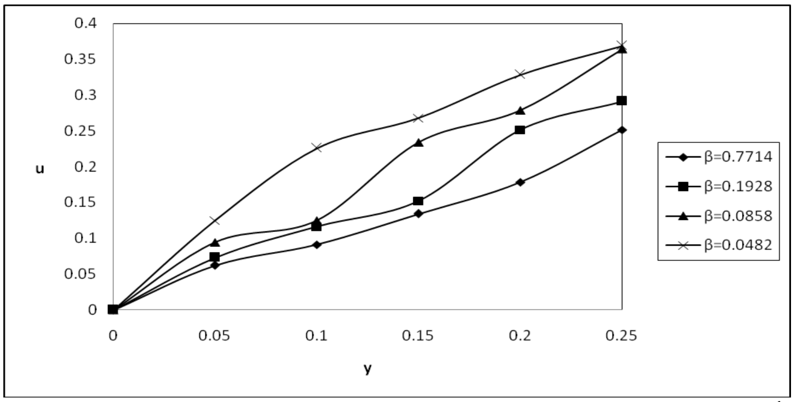

Fig 2: the velocity profile for u against β with S=0.01, R=257.8125, M=2, D^{-1} =1000

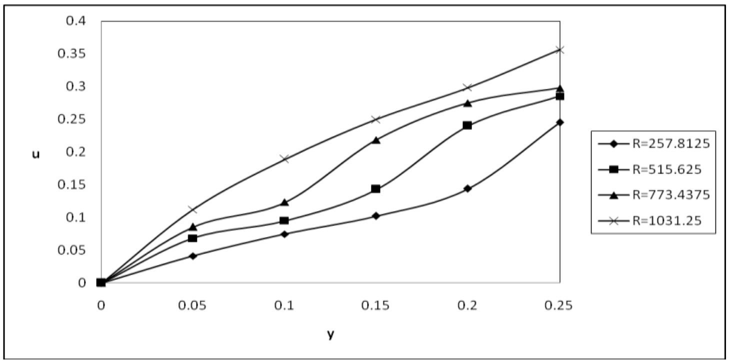

Fig 3: the velocity profile for u against R with β =0.77135, S=0.01, M=2, D^{-1} =1000

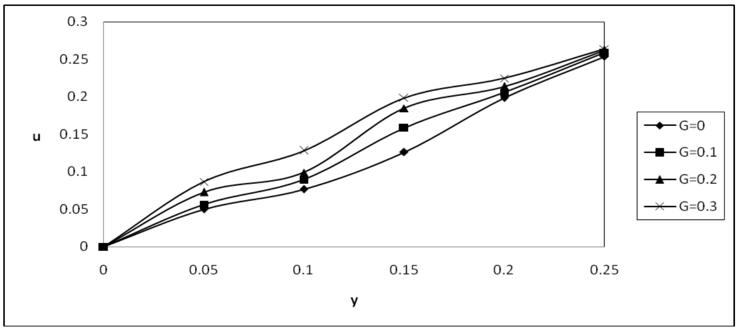

Fig 4: the velocity profile for u against G with β =0.77135, R=257.8125, S=0.01, M=2, D^{-1} =1000

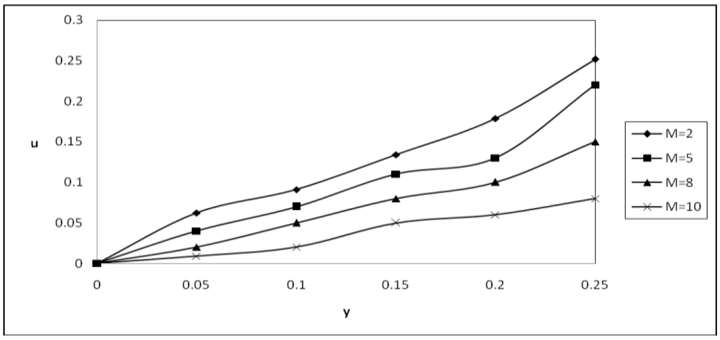

Fig 5: the velocity profile for u against M with β =0.77135, R=257.8125, S=0.01, D^{-1} =1000

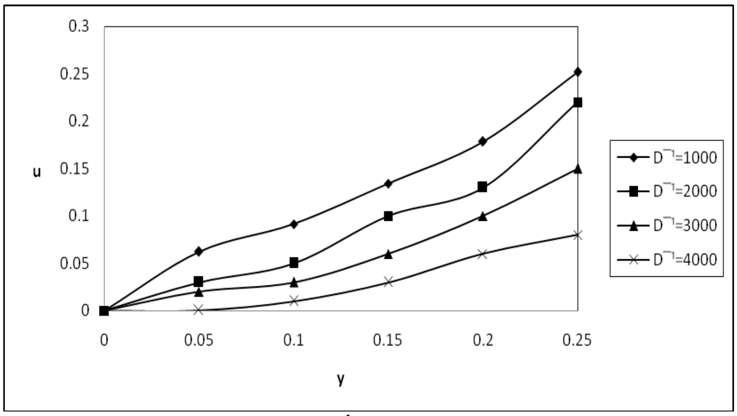

Fig 6: the velocity profile for u against D^{-1} with β =0.77135, R=257.8125, S=0.01, M=2

R	I	II	III	IV
257.8125	0.2755	0.4623	0.6728	0.7228
515.6250	0.4769	0.5145	0.7552	0.7899
773.4375	0.5586	0.7668	0.8014	0.8846
1031.2499	0.7742	0.8388	0.8871	0.9389

	I	**II**	**III**	**IV**
S	0.01	0.05	0.1	0.2
β	0.771350	0.192817	0.085706	0.048209
M	2	5	8	10
$\mathbf{D^{-1}}$	1000	2000	3000	4000

Table-1: The shear stress at y=0 level

R	**I**	**II**	**III**	**IV**
257.8125	0.3799	0.5225	0.6585	0.6795
515.6250	0.5528	0.6715	0.7288	0.7874
773.4375	0.6885	0.7481	0.8045	0.8455
1031.2499	0.7885	0.8828	0.9118	0.9785

	I	**II**	**III**	**IV**
S	0.01	0.05	0.1	0.2
β	0.771350	0.192817	0.085706	0.048209
M	2	5	8	10
$\mathbf{D^{-1}}$	1000	2000	3000	4000

Table-2: The shear stress at y=1 level

S	**I**	**II**	**III**	**IV**
0.01	0.2755	0.4735	0.6529	0.7334
0.05	0.4883	0.5145	0.7652	0.8144
0.1	0.5333	0.7625	0.8014	0.8715
0.2	0.8028	0.8486	0.8898	0.9389

	I	**II**	**III**	**IV**
R	257.8125	515.6250	773.4375	1031.2499
β	0.771350	0.192817	0.085706	0.048209
M	2	5	8	10
$\mathbf{D^{-1}}$	1000	2000	3000	4000

Table-3: The shear stress at y=0 level

S	**I**	**II**	**III**	**IV**
0.01	0.3799	0.5327	0.6672	0.7015
0.05	0.5682	0.6715	0.7224	0.8124
0.1	0.7225	0.7562	0.7562	0.8525
0.2	0.7989	0.8878	0.9026	0.9785

	I	**II**	**III**	**IV**
R	257.8125	515.6250	773.4375	1031.2499
β	0.771350	0.192817	0.085706	0.048209
M	2	5	8	10
$\mathbf{D^{-1}}$	1000	2000	3000	4000

Table-4: The shear stress at y=1 level

β	**I**	**II**	**III**	**IV**
0.771350	0.2755	0.4866	0.6682	0.7535
0.192817	0.4835	0.5145	0.7648	0.7986
0.085706	0.6726	0.7825	0.8014	0.8852
0.048209	0.8002	0.8746	0.9028	0.9389
	I	**II**	**III**	**IV**
S	0.01	0.05	0.1	0.2
R	257.8125	515.6250	773.4375	1031.2499
M	2	5	8	10
$\mathbf{D^{-1}}$	1000	2000	3000	4000

Table-5: The shear stress at y=0 level

β	I	II	III	IV
0.771350	0.3799	0.5482	0.6882	0.7478
0.192817	0.5788	0.6715	0.7525	0.8128
0.085706	0.7583	0.7852	0.8045	0.8945
0.048209	0.8235	0.8925	0.9396	0.9785

	I	II	III	IV
S	0.01	0.05	0.1	0.2
R	257.8125	515.6250	773.4375	1031.2499
M	2	5	8	10
D^{-1}	1000	2000	3000	4000

Table-6: The shear stress at y=1 level

V. CONCLUSION

Based on the results of the numerical calculations, it can be concluded that:

[1]. For sufficiently large values of R, the magnitude of the velocity is high comparable to smaller R and once again exhibits increase in trend with increase in S.

[2]. To decrease in β for fixed R and S, an enhancement in the velocity for reduction in β. Also higher values of R the magnitude of the velocity relatively increases for all β and S.

[3]. The behaviour of the velocity for variation in R fixing β and S, an increase in R enhances the velocity for fixed β and S while its magnitude exhibits a slight growth for increase in the visco-elastic parameter S at all corresponding values of β and R.

[4]. The behaviour of the velocity for variation in G fixing R, β and S, the velocity enhances increase in for fixed values of R, β and S.

[5]. The behavior of the velocity always reduces to increase the intensity of the magnetic field M, while the magnitude of the velocity continuously decreases with increase in the inverse Darcy parameter D^{-1}.

[6]. On the lower and upper boundaries and increase in R slightly increases τ for all fixed S, M, D^{-1} and β **.** The variation in τ for increase S fixing R, M, D^{-1} and β . τ enhances with S for all sets of R, M, D^{-1} and β **.** The stress τ reduces with increase in β on the both boundaries for different sets of R, M, D^{-1} and S.

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