International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 3, Issue 3 (October 2013) PP: 25-30

Research Analysis and Design of Geometric Transformations using Affine Geometry

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ABSTRACT: *In this research paper, we focus on affine transformations of the coordinate system. The affine transformation consists of linear transformations (rotation, scaling, and shear) and a translation (shift). It provides a good approximation for the changes in pose of objects that are relatively far from the camera. The affine transformation also serves as the basic block in the analysis and registration of general and non rigid geometric transformations. When the area considered is small enough, the affine transformation serves as a first-order Taylor series approximation of any differentiable geometric transformation*.

Keywords: *geometric transformation, global algorithms, geometric-radiometric estimation, structured images, textured images, affine space, collinearity.*

I. INTRODUCTION

The most popular methods for estimating the geometric transformations today are based on local features, such as intensity-based regions (IBR) and edge-based region (EBR) [4], and scale-invariant feature transform (SIFT) [5] and maximally stable extremal regions (MSER) [6]. These methods identify features of small regions in the image and extract the transformation from the correspondence of the features. The correspondence problem is solved by using features that are invariant to the geometric transformation. Affine invariant features include multiscale autoconvolution (MSA) [7], affine invariant moments [8], cross-weighted (CW) moments [9], and trace transform [10]. Affine and illumination invariant features for color images are presented in [11]; however, the illumination changes are global and not location dependent. Most methods employing local features handle illumination changes by normalization of the illumination in the considered area, or by using edge (corner) information, which is less sensitive to the variations in illumination. However, as shown in the following, the localization of the features and the success of the registration process are effected by the changes in lighting conditions.

II. EXPERIMENTATION & EVALUATION

Global registration methods estimate the parameters of the geometric transformation from the whole image without a prior stage of local feature extraction [12]–[14]. Since global algorithms treat the image as a whole, the background should be separated from the registered part of the image prior to the registration. Global algorithms tend to show robustness to noise. Sensitivity to radiometric changes, on the other hand, is a major disadvantage of most global approaches. Some affine registration methods avoid dealing with the illumination changes by discarding the radiometric information and treating the image as a binary image. A comparison of several binary registration methods is presented in [15]. It is shown in [15] that although the method complexity grows linearly with the number of pixels, the calculation of moments is only a marginal factor in the total calculation time, mainly due to the need to solve high-order polynomial equations.

The reported calculation time is about 1 s even for small images. The variation in illumination between images of the same object creates a major complication for recognition and registration algorithms. Most registration methods measure properties in the image that are either robust or invariant to the illumination changes [16]. However, such methods assume that the changes in the illumination are location independent; therefore, they are only applicable in small regions of the image.

The dominating approach for representing the effects of location dependent illumination changes, when no geometric transformation is involved (i.e., camera and object are fixed), is by a linear combination of basis images. The idea of representing illumination changes by a linear combination of images was proposed by Shashua [17]. Hallinan [18] represented illumination changes by a linear combination of basis images and used principal component analysis (PCA) to find an approximation of the basis images. In cases, where the shape of an object is convex and the object is Lambertian (the apparent brightness of the surface is the same regardless of the viewing angle), it was shown by Belheumer and Kriegman [16] that the set of all images under arbitrary illuminations forms a convex cone. The cone can be constructed from as few as three images. Further study was done by Basri and Jacobs [19], who used spherical harmonics to show that the set of images produced by a Lambertian convex surface lies close to a 9-D space.

As previously mentioned, the difficulties associated with the joint geometric–radiometric estimation problem have led to the current state, where only a few attempts have been made to solve it. The lack of pointwise correspondence (due to the geometric transformation) and the lack of intensitywise alignment (due to the radiometric mapping) do not allow for a simple direct usage of the intensity information of the images. Seemingly, the geometric and radiometric problems are strongly coupled and may not be answered separately. As such, straightforward approaches for solving this problem typically lead to a high-dimensional nonlinear nonconvex optimization problem. Only a few works have explicitly modeled joint geometric–radiometric deformations. Indeed, among these, most evade the inherent nonlinearity of this estimation problem through linear approximation and/or variational optimization-based approaches [20], [21]. An explicit solution to joint geometric–radiometric estimation problem, where the radiometric change is a nonlinear mapping of the image intensities is given [22]. The assumed radiometric changes, however, are not location dependent. The estimation of location-dependent radiometric changes in the presence of an affine transformation is described in [23]. It requires several images of different spectral bands. Therefore, it is not suitable for grayscale images.

The Hessian affine region detector is a feature detector used in the fields of computer vision and image analysis. Like other feature detectors, the Hessian affine detector is typically used as a preprocessing step to algorithms that rely on identifiable, characteristic interest points.

The Hessian affine detector is part of the subclass of feature detectors known as affine-invariant detectors: Harris affine region detector, Hessian affine regions, maximally stable extremal regions, Kadir–Brady saliency detector, edge-based regions (EBR) and intensity-extrema-based (IBR) regions.

The Hessian affine detector algorithm is almost identical to the Harris affine region detector. In fact, both algorithms were derived by Krystian Mikolajczyk and Cordelia Schmid in 2002, ,[1] based on earlier work in,[2][3] see also [4] for a more general overview. See the Harris affine region detector entry for a more complete description of the detection algorithm.

The Harris affine detector relies on interest points detected at multiple scales using the Harris corner measure on the second-moment matrix. The Hessian affine also uses a multiple scale iterative algorithm to spatially localize and select scale & affine invariant points. However, at each individual scale, the Hessian affine detector chooses interest points based on the Hessian matrix at that point:

$$H(\mathbf{x}) = \begin{bmatrix} L_{xx}(\mathbf{x}) & L_{xy}(\mathbf{x}) \\ L_{xy}(\mathbf{x}) & L_{yy}(\mathbf{x}) \end{bmatrix}$$

Where $L_{aa}(\mathbf{x})$is second partial derivative in the $a$direction and $L_{ab}(\mathbf{x})$is the mixed partial second derivative in the $a$and $b$directions. It's important to note that the derivatives are computed in the current iteration scale and thus are derivatives of an image smoothed by a Gaussian kernel: $L(\mathbf{x}) = g(\sigma_I) \otimes I(\mathbf{x})$. As discussed in the Harris affine region detector article, the derivatives must be scaled appropriately by a factor related to the Gaussian kernel: σ_I^2.

At each scale, interest points are those points that simultaneously are local extrema of both the determinant and trace of the Hessian matrix. The trace of Hessian matrix is identical to the Laplacian of Gaussians (LoG)[5]:

$$DET = \sigma_I^2(L_{xx}L_{yy}(\mathbf{x}) - L_{xy}^2(\mathbf{x}))$$
$$TR = \sigma_I(L_{xx} + L_{yy})$$

As discussed in Mikolajczyk et al.(2005), by choosing points that maximize the determinant of the Hessian, this measure penalizes longer structures that have small second derivatives (signal changes) in a single direction.[6] This type of measure is very similar to the measures used in the blob detection schemes proposed by Lindeberg (1998), where either the Laplacian or the determinant of the Hessian were used in blob detection methods with automatic scale selection.

Like the Harris affine algorithm, these interest points based on the Hessian matrix are also spatially localized using an iterative search based on the Laplacian of Gaussians. Predictably, these interest points are called Hessian–Laplace interest points. Furthermore, using these initially detected points, the Hessian affine detector uses an iterative shape adaptation algorithm to compute the local affine transformation for each interest point. The implementation of this algorithm is almost identical to that of the Harris affine detector; however, the above mentioned Hessian measure replaces all instances of the Harris corner measure.

Mikolajczyk et al. (2005) have done a thorough analysis of several state of the art affine region detectors: Harris affine, Hessian affine, MSER,[7] IBR & EBR [8] and salient[9] detectors.[6] Mikolajczyk et al. analyzed both structured images and textured images in their evaluation. Linux binaries of the detectors and their test images are freely available at their webpage. The results of Mikolajczyk et al. (2005) show a comparison of affine region detectors for a more quantitative analysis.

Overall, the Hessian affine detector performs second best to MSER. Like the Harris affine detector, Hessian affine interest regions tend to be more numerous and smaller than other detectors. For a single image, the Hessian affine detector typically identifies more reliable regions than the Harris-Affine detector. The performance changes depending on the type of scene being analyzed. The Hessian affine detector responds well to textured scenes in which there are a lot of corner-like parts. However, for some structured scenes, like buildings, the Hessian affine detector performs very well. This is complementary to MSER that tends to do better with well structured (segmentable) scenes.

Affine geometry

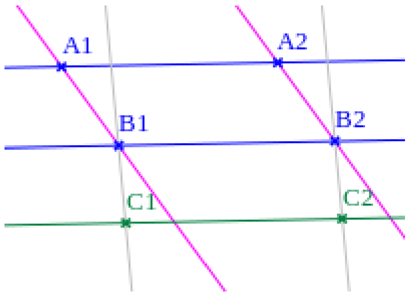

Figure 1: Playfair's Axiom

In affine geometry one uses Playfair's axiom to find the line through C1 and parallel to B1B2, and to find the line through B2 and parallel to B1C1. The intersection C2 is the result of the indicated translation.

In mathematics **affine geometry** is the study of parallel lines. The parallel postulate is an example of an axiom using the concept of parallel lines, but there are a variety of approaches to affine geometry. In affine geometry the relation of parallelism may be adapted so as to be an equivalence relation. Comparisons of figures in affine geometry are made with dilations which are mappings comprising the affine group A. Since A lies between the Euclidean group E and the group of projectivities P, affine geometry is sometimes mentioned[1] in connection with the Erlangen program, which is concerned with group inclusions such as $E \subset A \subset P$.

Affine geometry can be developed on the basis of linear algebra. One can define an affine space as a set of points equipped with a set of transformations, the translations, which forms (the additive group of) a vector space (over a given field), and such that for any given ordered pair of points there is a unique translation sending the first point to the second. In more concrete terms, this amounts to having an operation that associates to any two points a vector and another operation that allows translation of a point by a vector to give another point; these operations are required to satisfy a number of axioms (notably that two successive translations have the effect of translation by the sum vector). By choosing any point as "origin", the points are in one-to-one correspondence with the vectors, but there is no preferred choice for the origin; thus this approach can be characterized as obtaining the affine space from its associated vector space by "forgetting" the origin (zero vector).

History

In 1748, Euler introduced the term *affine*[2][3] (Latin *affinis*, "related") in his book Introductio in analysin infinitorum. In 1827, August Möbius wrote on affine geometry in his book *Der barycentrische Calcul*.

Only after Felix Klein's Erlangen program was affine geometry recognized for being a generalization of Euclidean geometry.[4]

In 1918, Hermann Weyl referred to affine geometry for his text *Space, Time, Matterr*. He uses affine geometry to introduce vector addition and subtraction[5] at the earliest stages of his development of mathematical physics.

Systems of axioms

Several axiomatic approaches to affine geometry have been put forward:

Pappus' law

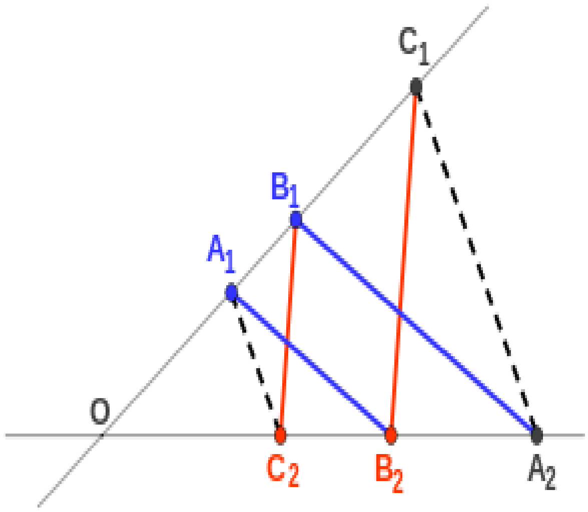

Figure 2: Pappu's Law

Pappus Law: If the red lines are parallel and the blue lines are parallel, then the dotted black lines must be parallel.

As affine geometry deals with parallel lines, one of the properties of parallels noted by Pappus of Alexandria has been taken as a premise:[6][7]

- If A, B, C are on one line and A', B', C' on another, then
$$(AB' \parallel A'B \ \wedge \ BC' \parallel B'C) \Rightarrow CA' \parallel C'A.$$

The full axiom system proposed has *point*, *line*, and *line containing point* as primitive notions:

- Two points are contained in just one line.
- For any line *l* and any point *P*, not on *l*, there is just one line containing *P* and not containing any point of *l*. This line is said to be *parallel* to *l*.
- Every line contains at least two points.
- There are at least three points not belonging to one line.

According to H. S. M. Coxeter, "The interest of these five axioms is enhanced by the fact that they can be developed into a vast body of propositions, holding not only in Euclidean geometry but also in Minkowski's geometry of time and space (in the simple case of 1 + 1 dimensions, whereas the special theory of relativity needs 1 + 3). The extension to either Euclidean or Minkowskian geometry is achieved by adding various further axioms of orthogonality, etc." [8]

The various types of affine geometry correspond to what interpretation is taken for *rotation*. Euclidean geometry corresponds to the ordinary idea of rotation, while Minkowski's geometry corresponds to hyperbolic rotation. With respect to perpendicular lines, they remain perpendicular when the plane is subjected to ordinary rotation. In the Minkowski geometry, lines that are hyperbolic-orthogonal remain in that relation when the plane is subjected to hyperbolic rotation.

Ordered structure

An axiomatic treatment of plane affine geometry can be built from the axioms of ordered geometry by the addition of two additional axioms.[9]

1. (Affine axiom of parallelism) Given a point A and a line r, not through A, there is at most one line through A which does not meet r.
2. (Desargues) Given seven distinct points A, A', B, B', C, C', O, such that AA', BB', and CC' are distinct lines through O and AB is parallel to A'B' and BC is parallel to B'C', then AC is parallel to A'C'.

The affine concept of parallelism forms an equivalence relation on lines. Since the axioms of ordered geometry as presented here include properties that imply the structure of the real numbers, those properties carry over here so that this is an axiomatization of affine geometry over the field of real numbers.

Ternary fields

In 1984 Wanda Szmielew published a fundamental study of affine systems. As an algebraic preliminary, axioms are stated for several algebraic structures from loops to fields. **Ternary fields** are introduced as a ternary operation $T(u, x, v)$ that satisfies nine axioms that make it behave like $ux + v$, the archetype of an affine transformation of *x*. Ternary fields are also characterized as strong quasifields. Szmielew considers Desarguean as well as Pappian affine plane in the research work namely *From affine to Euclidean geometry*.

Affine transformations

Geometrically, affine transformations (affinities) preserve collinearity. So they transform parallel lines into parallel lines and preserve ratios of distances along parallel lines.

We identify as *affine theorems* any geometric result that is invariant under the affine group (in Felix Klein's Erlangen programme this is its underlying group of symmetry transformations for affine geometry). Consider in a vector space *V*, the general linear group GL(*V*). It is not the whole *affine group* because we must allow also translations by vectors *v* in *V*. (Such a translation maps any *w* in *V* to *w* + *v*.) The affine group is generated by the general linear group and the translations and is in fact their semidirect product $V \rtimes \mathrm{GL}(V)$. (Here we think of *V* as a group under its operation of addition, and use the defining representation of GL(*V*) on *V* to define the semidirect product.)

For example, the theorem from the plane geometry of triangles about the concurrence of the lines joining each vertex to the midpoint of the opposite side (at the *centroid* or *barycenter*) depends on the notions of *mid-point* and *centroid* as affine invariants. Other examples include the theorems of Ceva and Menelaus.

Affine invariants can also assist calculations. For example, the lines that divide the area of a triangle into two equal halves form an envelope inside the triangle. The ratio of the area of the envelope to the area of the triangle is affine invariant, and so only needs to be calculated from a simple case such as a unit isosceles right angled triangle to give $\frac{3}{4}\log_e(2) - \frac{1}{2}$, i.e. 0.019860... or less than 2%, for all triangles.

Familiar formulas such as half the base times the height for the area of a triangle, or a third the base times the height for the volume of a pyramid, are likewise affine invariants. While the latter is less obvious than the former for the general case, it is easily seen for the one-sixth of the unit cube formed by a face (area 1) and the midpoint of the cube (height 1/2). Hence it holds for all pyramids, even slanting ones whose apex is not directly above the center of the base, and those with base a parallelogram instead of a square. The formula further generalizes to pyramids whose base can be dissected into parallelograms, including cones by allowing infinitely many parallelograms (with due attention to convergence). The same approach shows that a four-dimensional pyramid has 4D volume one quarter the 3D volume of its parallelopiped base times the height, and so on for higher dimensions.

III. RESULTS & DISCUSSION

Affine geometry can be viewed as the geometry of **affine space**, of a given dimension *n*, coordinatized over a field *K*. There is also (in two dimensions) a combinatorial generalization of coordinatized affine space, as developed in synthetic finite geometry. In projective geometry, *affine space* means the complement of the points (the hyperplane) at infinity (see also projective space). *Affine space* can also be viewed as a vector space whose operations are limited to those linear combinations whose coefficients sum to one, for example $2x-y$, $x-y+z$, $(x+y+z)/3$, $\mathbf{i}x+(1-\mathbf{i})y$, etc.

Synthetically, affine planes are 2-dimensional affine geometries defined in terms of the relations between points and lines (or sometimes, in higher dimensions, hyperplanes). Defining affine (and projective) geometries as configurations of points and lines (or hyperplanes) instead of using coordinates, one gets examples with no coordinate fields. A major property is that all such examples have dimension 2. Finite examples in dimension 2 (**finite affine planes**) have been valuable in the study of configurations in infinite affine spaces, in group theory, and in combinatorics.

Despite being less general than the configurational approach, the other approaches discussed have been very successful in illuminating the parts of geometry that are related to symmetry.

Projective view

In traditional geometry, affine geometry is considered to be a study between Euclidean geometry and projective geometry. On the one hand, affine geometry is Euclidean geometry with congruence left out, and on the other hand affine geometry may be obtained from projective geometry by the designation of a particular line or plane to represent the points at infinity.[10] In affine geometry there is no metric structure but the parallel postulate does hold. Affine geometry provides the basis for Euclidean structure when perpendicular lines are defined, or the basis for Minkowski geometry through the notion of hyperbolic orthogonality.[11] In this viewpoint, an affine transformation geometry is a group of projective transformations that do not permute finite points with points at infinity.

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