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# Magneto hydrodynamic Couple stress Squeeze Film Lubrication of Triangular Plates

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**Abstract:** *In this paper the analysis of the effect of transverse magnetic field on the couple stress squeeze film lubrication between triangular plates is presented. The modified Reynolds equation is derived. The closed form expressions are obtained for the MHD squeeze film pressure, load carrying capacity and squeeze film time. The results are presented for different values of operating parameters. It is observed, that the effect of applied magnetic field on the squeeze film lubrication between triangular plates with couple stress fluids is to increase the load carrying capacity significantly and to delay the time of approach as compared to the corresponding non-magnetic case.*

**Key words:** *Squeeze film, Triangular plates, Couple-stress fluid, Magnetic field.*

## I. INTRODUCTION

Magneto hydrodynamics (MHD) is the physical mathematical framework that concern the dynamics of electrically conducting fluids. Examples of such fluids include plasma, liquid metals, and salt water or electrolytes. The word magnetohydrodynamics (MHD) is derived from magneto-meaning magnetic field, hydro-meaning liquid and dynamics-meaning movements of an object by forces. Hartmann studied, theoretically and experimentally the flow of an incompressible fluid between parallel planes with magnetic field $B_0$ normal to them. Hence this type of flow is called Hartmann flow. Hartmann number is the constant which gives the ratio between the magnetic viscosity and the ordinary viscosity in the flow of a fluid.

In recent years considerable attention has been devoted to various problems on the motion of an electrically conducting liquid lubricant in MHD theory of lubrication. Magnetohydrodynamic lubrication flow between parallel rotating disks was analyzed by Huges and Elco [1]. They concluded that the load capacity of the bearing is dependent on the MHD interactions in the fluid and the frictional torque on the rotor can be made zero for both field configurations by supplying electrical energy through the electrodes to the fluid. Elco and Huges [1] studied MHD pressurization of liquid metal bearings. They found that the load capacity can be increased and with no flow or external pressurization a load can be obtained with reasonable values of magnetic field strength and current. Kuzma [2] presented the MHD journal bearing. It is shown that the load carrying capacity is increased by the application of a magnetic field. The MHD squeeze film was studied by Kuzma et.al [3]. Magnetohydrodynamics squeeze films are investigated theoretically and experimentally. The theory of MHD lubrication as applied to squeeze films is extended to include film inertia effects and buoyant forces. Excellent agreement is obtained between theory and experiment. Dodge et.al [4] presented MHD squeeze film bearings. They observed that the squeeze action is altered significantly when the electric field is symmetrical about the centre of the bearing and results are presented for various values of the Hartmann number. Malik and Singh [5] presented analysis of finite MHD journal bearings. Hamza [6] analyzed the MHD squeeze film. The results show that the electromagnetic forces increase the load carrying capacity considerably. The Magnetohydrodynamic effects on a fluid film squeezed between two rotating surfaces were presented by Hamza [7]. He concluded that, the combined effect of the magnetic forces and the centrifugal inertial forces on the velocity profiles, the load capacity and the torque that the fluid exerts on the surfaces are studied. In general, the results show that these two forces have opposite effects. Das [8] presented a study of optimum load bearing capacity for slider bearing lubricated with couplestress fluids in magnetic field. It is observed that the value of maximum load capacity depend on couplestress and magnetic parameters. Lin [9] analyzed MHD squeeze film characteristics between annular disks. He concluded that the influence of magnetic fields signifies an enhancement in the squeeze film pressure. On the whole, the magnetic field affect characterized by the Hartmann number provides an increase in values of load carrying capacity and the response time as compared to the classical non-conducting lubricant case. Finite difference solution for MHD flow of couplestress fluid between two concentric rotating cylinders with porous lining was studied by Murthy et.al [10]. They observed that as porous lining thickness increases, the slip velocity decreases. As magnetic field strength increases, velocity increases, slip velocity increases and there by skin friction increases. Dynamic characteristics for MHD wide slider bearings with an exponential film profile were studied by Lin and Lu [11]. They observed that the presence of externally applied magnetic fields signifies on enhancement in the film pressure. On the whole, the applied magnetic-field effects by the characterized by the Hartmann number provide a significant increase in values of load carrying capacity, the stiffness co-efficient and damping co-efficient as compared to the non-conducting lubricant case. Derivation of two dimensional couplestress hydromagnetic squeeze film Reynolds equation and application to wide parallel rectangular plates was analyzed by Lin et.al [12]. They found that the effects of couplestress and external magnetic fields provide an increase in the load capacity and response time as compared to the classical Newtonian hydrodynamic rectangular squeeze film plates. The problem of MHD couplestress squeeze film lubrication between triangular plates has not be studied so far in the literature known in this field. Hence, in this paper an attempt has been made to analyze the effect of applied magnetic field on the squeeze film lubrication between triangular plates with Stokes [13] couplestress fluid.

## II. MATHEMATICAL FORMULATION

The geometry and the configuration of the bearing system is as shown fig. 1. The lower plate is assumed to be fixed while the upper plate moves normally towards the lower plate with a uniform velocity $\dot{h} = \frac{dh}{dt}$. The lubricant in the film region is assumed to be the couple stress fluid.

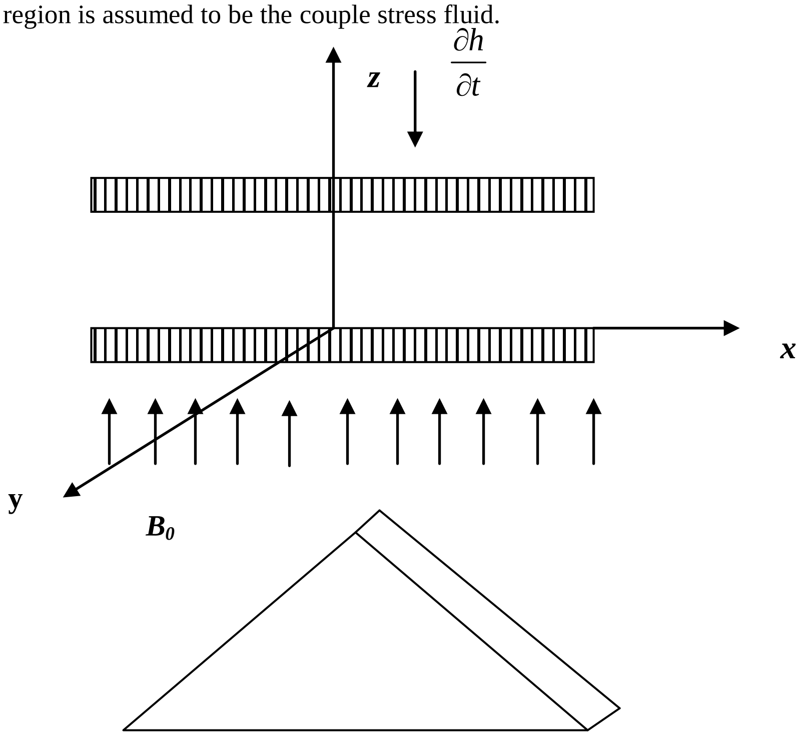


**Fig. 1 Schematic diagram of the Rectangular Bearing**

The basic equations governing the hydromagnetic flow of the couplestress lubricant in the fluid film region are given by

$$\frac{\partial^2 u}{\partial z^2} - \frac{\eta}{\mu}\frac{\partial^4 u}{\partial z^4} - \frac{\sigma B_0^2}{\mu} u = \frac{1}{\mu}\frac{\partial p}{\partial x}, \quad (1)$$

$$\frac{\partial^2 v}{\partial z^2} - \frac{\eta}{\mu}\frac{\partial^4 v}{\partial z^4} - \frac{\sigma B_0^2}{\mu} v = \frac{1}{\mu}\frac{\partial p}{\partial y}, \quad (2)$$

$$\frac{\partial p}{\partial z} = 0, \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (4)$$

The relevant boundary conditions are

i) At the upper surface ( z = h)

$$u = v = 0, \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0, \quad (5a)$$

$w = dh/dt$ , (5b)

ii) At the lower surface (z = 0)

$u = v = 0$ , $\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0$ (6a)

$w = 0$, (6b)

The solution of the equations (1) and (2) subject to the boundary conditions (5a) and (6a) is obtained in the form

$$u = -\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial x}\left\{\frac{1}{(A^2-B^2)}\left(\frac{B^2 Cosh\frac{A(2z-h)}{2l}}{Cosh\frac{Ah}{2l}} - \frac{A^2 Cosh\frac{B(2z-h)}{2l}}{Cosh\frac{Bh}{2l}}\right)+1\right\} \quad (7)$$

Similarly,

$$v = -\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial y}\left\{\frac{1}{(A^2-B^2)}\left(\frac{B^2 Cosh\frac{A(2z-h)}{2l}}{Cosh\frac{Ah}{2l}} - \frac{A^2 Cosh\frac{B(2z-h)}{2l}}{Cosh\frac{Bh}{2l}}\right)+1\right\} \quad (8)$$

Where, $M_0 = B_0 h_0 (\sigma/\mu)^{1/2}$ is the Hartmann number.

$$A = \left\{\frac{1+\left(1-4l^2\sigma B_0^2/\mu\right)^{1/2}}{2}\right\}^{1/2} , \quad B = \left\{\frac{1-\left(1-4l^2\sigma B_0^2/\mu\right)^{1/2}}{2}\right\}^{1/2}$$

Substituting (7) & (8) in the continuity equation (4) and integrating across the film thickness *h* and use of the boundary conditions (5b) and (6b) gives the modified Reynolds equation in the form

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{\mu M_0^2 dh/dt}{h_0^2 f(h,l,M_0)} \quad (9)$$

where

$$f(h,l,M_0) = \frac{2l}{A^2-B^2}\left(\frac{B^2}{A}\tanh\frac{Ah}{2l} - \frac{A^2}{B}\tanh\frac{Bh}{2l}\right) + h$$

The boundary conditions for the pressure field are

$p(x_1, y_1) = 0$ (10)

where

$$(x_1 - a)(x_1 - \sqrt{3}y_1 + 2a)(x_1 + \sqrt{3}y_1 + 2a) = 0$$

and $a$ is the length of the side of the equilateral triangle whose equation is

$$(x-a)(x-\sqrt{3}y+2a)(x+\sqrt{3}y+2a) = 0$$

The point of intersection of the medians of the triangle is selected as the origin Solution of (9) with condition (10) gives the pressure equation in the form

$$p = -\frac{\mu M_0^2 dh/dt}{h_0^2 f(h,l,M_0)}\frac{a^2}{3}\left(1 - \frac{3}{4a^2}x_1^2 - \frac{3}{4a^2}y_1^2 - \frac{1}{4a^3}x_1^3 + \frac{3}{4a^3}x_1 y_1^2\right) \quad (11)$$

The expression for the pressure distribution in dimensionless form is

$$P = -\frac{p h_0^3}{\mu (dh/dt) 3\sqrt{3}a^2}$$

$$= \frac{M_0^2}{9\sqrt{3}} \left\{ \frac{(1-x^*)\left(1+\frac{\sqrt{3}y^*}{2}+\frac{x^*}{2}\right)\left(1-\frac{\sqrt{3}y^*}{2}+\frac{x^*}{2}\right)}{F(h^*,l^*,M_0)} \right\} \quad (12)$$

Where

$$l^* = \frac{2l}{h_0},\ x^* = \frac{x}{a},\ y^* = \frac{y}{a},\ h^* = \frac{h}{h_0}$$

$$F(h^*,l^*,M_0) = \frac{l^*}{A^{*2}-B^{*2}}\left(\frac{B^{*2}}{A^*}\tanh\frac{A^*h^*}{l^*} - \frac{A^{*2}}{B^*}\tanh\frac{B^*h^*}{l^*}\right) + h^*$$

$$A^* = \left\{\frac{1+\left(1-l^{*2}M_0^2\right)^{1/2}}{2}\right\}^{1/2}, \qquad B^* = \left\{\frac{1-\left(1-l^{*2}M_0^2\right)^{1/2}}{2}\right\}^{1/2}$$

The expression for the load-supporting capacity is obtained by

$$W = \int_{-2a}^{a} \int_{-\frac{2a+x_1}{\sqrt{3}}}^{\frac{2a+x_1}{\sqrt{3}}} p\,dy_1\,dx_1 \quad (13)$$

$$W = -\frac{\mu M_0^2\,dh/dt}{h_0^2\left\{f(h,l,M_0)+h\right\}}\left(\frac{27\sqrt{3}a^4}{60}\right) \quad (14)$$

The expression for the load-supporting capacity in dimensionless form is

$$W^* = -\frac{Wh_0^3}{27\mu\left(dh/dt\right)a^4} = \frac{\sqrt{3}M_0^2}{60}\left\{\frac{1}{F(h^*,l^*,M_0)+h^*}\right\} \quad (15)$$

The expression for the time-height relation is

$$t = -\frac{27\sqrt{3}a^4}{60W}\int_{h_0}^{h_1}\frac{\mu M_0^2\,dh}{h_0^2\left\{f(h,l,M_0)+h\right\}} \quad (16)$$

The expression for the time-height relation in dimensionless form is

$$T = \int_1^{t_1/t_0}\frac{Wh_0^2\,dt}{27\mu a^4} = -\frac{\sqrt{3}M_0^2}{60}\int_1^{h_1^*}\frac{dh^*}{\left\{F(h^*,l^*,M_0)+h^*\right\}} \quad (17)$$

where $h_1^* = \frac{h_1}{h_0}$

## III. RESULTS AND DISCUSSION

The squeeze film characteristics between the triangular parallel plates are analyzed on the basis of the Stokes couplestress fluid theory in the presence transverse magnetic fluid. The results are presented for various values of the non dimensional parameters like the couplestress parameter$l^*$, the magnetic Hartmann number $M_o$.

The variation of non-dimensional squeeze film pressure P, with$x^*$, is shown in the fig. 2 for different values of $M_o$, $l^*$ with the parametric values of $h^* = 0.4$ and $y^*$= 0. It is observed that P increases for the increaseing values of $l^*$. The effect of $M_o$ on the variation of P with $x^*$ is depicted in fig. 3. The dotted curves in the graph correspond to the Newtonian case ($l^*$= 0). It is found that P increases for the increasing values of $M_o$. Further, it is to be noted that the applied tranverse magnetic field increases the squeeze film pressure as compared to the non magnetic case ($M_o = 0$).

The variation of non dimensional load carrying capacity W with $h^*$ for different values $M_o$and $l^*$ is shown in the fig. 4 for various values of the couplestress parameter$l^*$. It is observed that the couplestress lubricant $(l^* > 0)$ provides the increased load carrying capacity as compared to the corresponding Newtonian case $(l^* = 0)$. The effect of Hartmann number $M_o$ on the variations of W with $h^*$ is depicted in the fig. 5 for different two values of $l^*$. It is observed that the couplestress lubricant in the presence of magnetic field $(M_o > 0)$ provides the larger load carrying capacity as compare to the corresponding non magnetic case.

The squeeze film time is an important characteristic of the bearing characteristics in the design of squeeze film bearings. The effect of couplestress parameter $l^*$ on the variation of non dimensional squeeze film time T with $h_1^*$ is depicted in the fig. 6. It is observed that the couplestress lubricant provides the longer squeeze film time as compare to the corresponding Newtonian lubricants. From fig. 7 it is clear that the presence transverse magnetic field increasing the squeeze film time significantly as compared to the corresponding non-magnetic case. Hence the improved squeeze film characteristics are observed for the couple stress lubricant in the presence of transverse magnetic field.

## IV. CONCLUSIONS

The following conclusions can be drawn on the basis of the result presented above

1. The significant increase in the load carrying capacity and delayed time of approach is observed for the couple stress lubricant.
2. The effect of transverse magnetic field on the squeeze film characteristics is to increase the load carrying capacity and squeeze film time as compare the non-magnetic case.

## ACKNOWLEDGEMENT

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**Nomenclature:**

$a$ the length of the side of the equilateral triangle

$B_0$ applied magnetic field

$h$ film thickness

$h^*$ dimensionless film thickness $(= h/h_0)$

$h_0$ minimum film thickness (when time t = 0)

$h_1$ film thickness after time $\Delta t$

$h_1^*$ dimensionless film thickness after time $\Delta t$

$l$ couplestress parameter $(\eta/\mu)^{1/2}$

$l^*$ dimensionless couplestress parameter $(2l/h_0)$

$M_0$ Hartmann number $\left(= B_0 h_0 (\sigma^*/\mu)^{1/2}\right)$

$p$ pressure in the film region

$P$ dimensionless pressure $\left(= -ph_0^3 \big/ \mu(dh/dt)3\sqrt{3}a^2\right)$

$x, y, z$ Cartesian coordinates

$t$ time

$\Delta t$ time required for the film thickness to decrease to a value $h_1$

$T$ dimensionless time of approach $\left(= -Wh_0^3 / 27\mu\left(dh/dt\right)a^4\right)$ $u, v, w$ velocity components in $x, y, z$ directions

$V$ squeezing velocity $(= dh/dt)$

$w$ load carrying capacity

$w^*$ dimensionless load carrying capacity $\left(= -Wh_0^3 / 27\mu\left(dh/dt\right)a^4\right)$

η Material constant responsible for couplestress

$\mu$ viscosity of the fluid
$\sigma$ electrical conductivity

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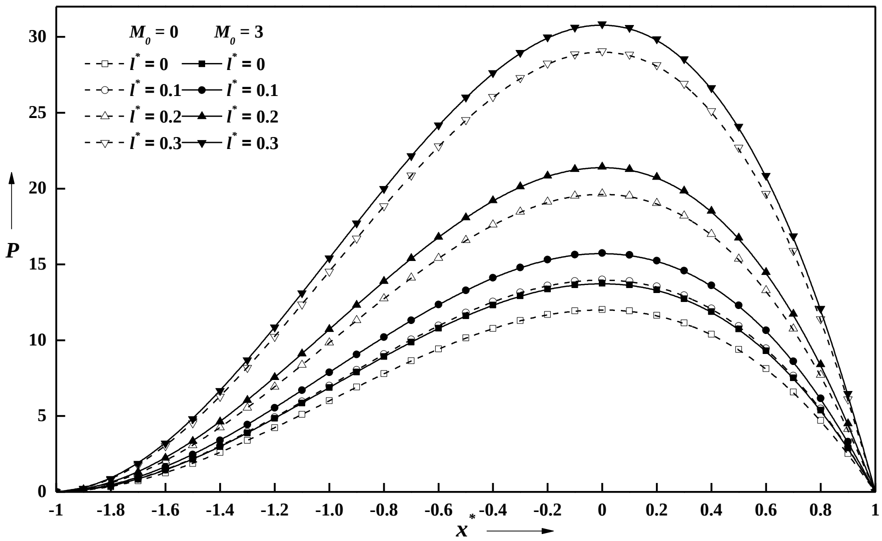


Figure 2. Variation of nondimensional Pressure $P$ with $x^*$ for different values of $M_0$ and $l^*$ with $h^* = 0.4, y^* = 0$.

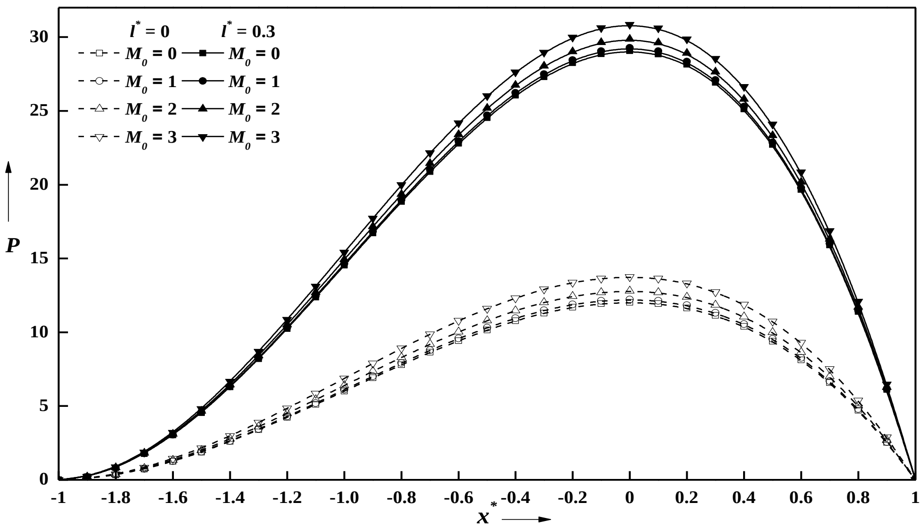


Figure3. Variation of nondimensional Pressure $P$ with $x^*$ for different values of $M_0$ and $l^*$ with $h^* = 0.4, y^* = 0$.

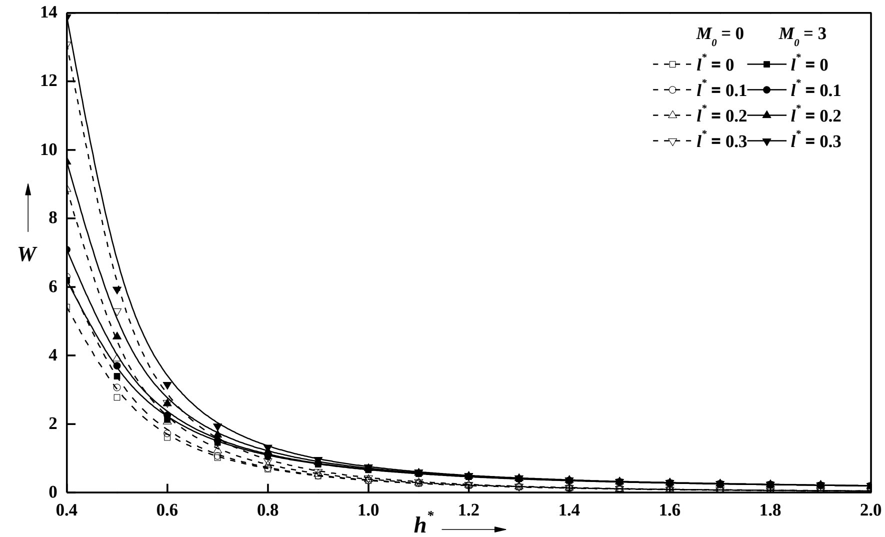


Figure 4. Variation of nondimensional Load carrying capacity $W$ with $h^*$ for different values of $M_0$ and $l^*$.

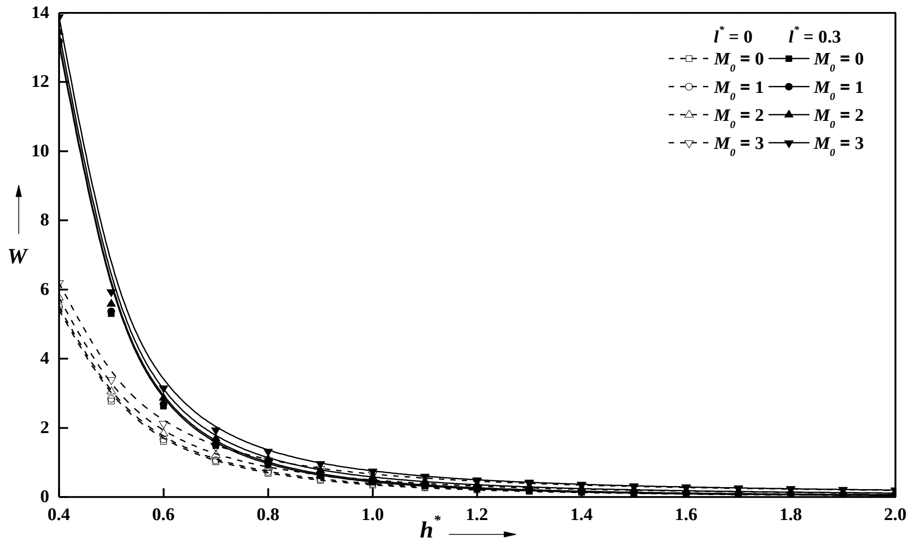


Figure 5. Variation of nondimensional Load carrying capacity $W$ with $h^*$ for different values of $l^*$ and $M_0$.

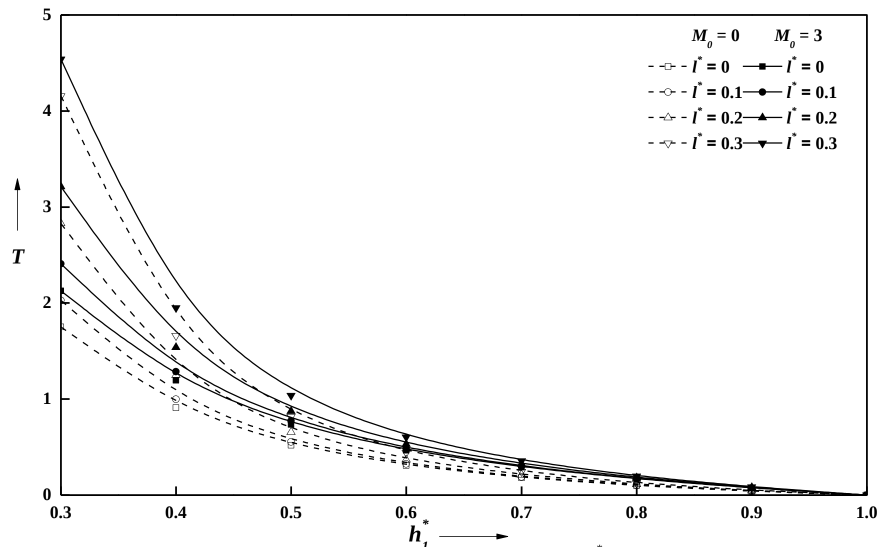


Figure 6. Variation of nondimensional squeeze film time $T$ with $h_1^*$ for different values of $M_0$ and $l^*$.

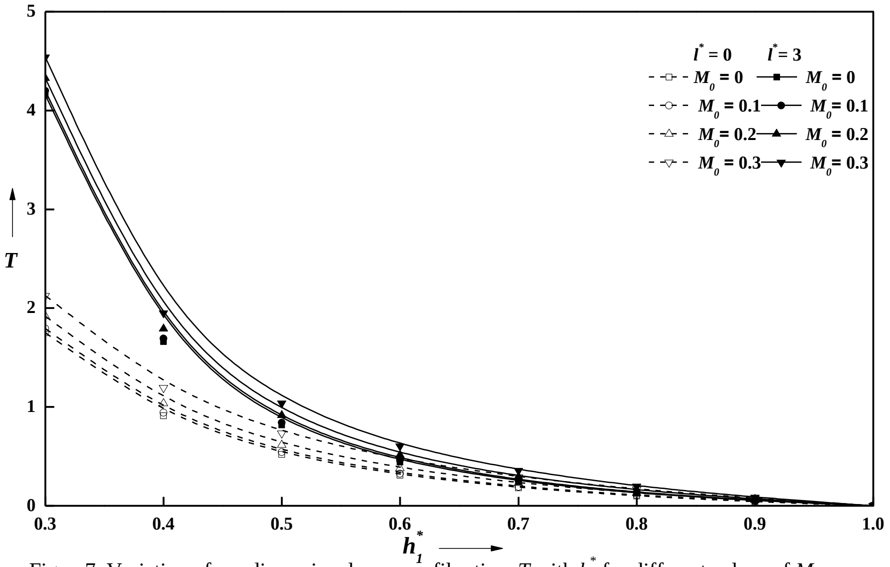


Figure7. Variation of nondimensional squeeze film time $T$ with $h_1^*$ for different values of $M_0$ and $l^*$.