International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 3, Issue 6 (January 2014) PP: 41-45

Time Truncated Chain Sampling Plans for Inverse Rayleigh Distribution

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Abstract: *A Chain sampling plan is developed based on truncated lifetimes when the lifetime of an item follows a inverse Rayleigh distribution. The design parameters such as the minimum sample size and the acceptance number are obtained by satisfying the producer's and consumer's risks at the specified quality levels, under the assumption that the termination time and the number of items are pre-fixed. The results are illustrated by an example.*
Keywords: *Chain sampling plan, Consumer's risk, Inverse Rayleigh distribution, Operating characteristics, Producer's risk, truncated life test.*

I. Introduction

Acceptance sampling plans are used for quality control purposes in firms and industries. Keeping in view two basic factors of time and cost, it is not possible to check the life time of each and every electronic device. Whenever a sampling inspection is considered, the lot is either accepted or rejected along with associated producer and consumer's risk. In a time truncated sampling plan suppose n units are placed in a life test and the experiment is stopped at a predetermined time T, where the number of failures is recorded until the pre specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Sampling inspection in which the criteria for acceptance and nonacceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in Chain Sampling Plan and so the use of chain sampling plan is often recommended when an extremely high quality is essential. Using life tests one may find the probability of acceptance, minimum sample size put on test and the minimum ratio of true average life to the specified average life or quality level subject to the consumer's risk. These life tests are studied by many authors using the different statistical distributions, More recently, Aslam and Jun (2009) proposed the group acceptance sampling plan based on the truncated life test when the lifetime of an item follows the inverse Rayleigh and Log-logistic distribution, Kantam R.R. L., Rosaiah K. and Srinivasa Rao G. (2001) discussed acceptance sampling based on life tests with Log-logistic models. Rosaiah K. and Kantam R.R.L. (2005) discussed acceptance sampling based on the inverse Rayleigh distribution. Gupta and Groll(1961), Baklizi and EI Masri(2004), and Tsai, Tzong and Shou(2006), Balakrishnan, Victor Leiva & Lopez (2007). All these authors developed the sampling plans for life tests using single acceptance sampling. The purpose of this study is to find the probability of acceptance for chain sampling plan assuming the experiment is truncated at preassigned time and lifetime follows a Inverse Rayleigh distribution.

II. Glossary of Symbols

n	-	Sample size
λ	-	Shape parameter
T	-	Prefixed time
β	-	Consumer's risk
p*	-	Minimum probability
d	-	Number of defectives
i	-	Acceptance criteria
Pa(p)	-	Probability of Acceptance

III. Inverse Rayleigh Distribution

The probability density function g(z) and cumulative distribution function G(z) of the Rayleigh distribution are given by

$$g(z) = \frac{2}{z^3}\exp\left(\frac{-1}{z^2}\right) \quad \text{for } z > 0 \quad \text{------------------(1)}$$

$$G(Z) = \exp\left(\frac{-1}{z^2}\right) \quad \text{for } z > 0 \quad \text{-------------------(2)}$$

If a scale parameter $\lambda > 0$ is introduced, the distribution function and the density function of Inverse Rayleigh distribution, respectively are given by

$$f(t,\lambda) = \frac{2\lambda^2}{t^3}\exp\left(\frac{-\lambda^2}{t^2}\right) \quad \text{for } t > 0, \lambda > 0 \quad \text{---------------------(3)}$$

$$F(t,\lambda) = \exp\left(\frac{-\lambda^2}{t^2}\right) \quad \text{for } t > 0, \lambda > 0 \quad \text{----------------------(4)}$$

The failure rate of a single parameter inverse Rayleigh distribution is increasing for t < 1.0694543$\sqrt{\lambda}$ and decreasing for t > 1.0694543$\sqrt{\lambda}$ as shown by Mukherjee and Saran (1984). The mean of the inverse Rayleigh distribution is $2\lambda\sqrt{\pi}$.Voda (1972) studied the properties of the maximum likelihood estimator of λ, determined tolerance limits and confidence intervals of minimum length and discussed problems of hypothesis testing. Mukherjee and Maiti (1997) considered quantile estimation in Inverse Rayleigh distribution

IV. Chain Sampling Plan

Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) making use of cumulative results of several samples help to overcome the shortcomings of the Single Sampling Plan. The distinguish feature is that the current lot under inspection can also be accepted if one defective unit is observed in the sample provided that no other defective units were found in the samples from the immediately preceeding i lots, i.e. the chain It avoids rejection of a lot on the basis of a single nonconfirming unit and improves the poor discrimination between good and bad quality. The conditions for application and operating procedure of chsp-1 are as follows

4.1 Conditions for application of ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.

1) The product to be inspected comprises a series of successive lots produced by a continuing process.
2) Normally lots are expected to be of essentially the same quality.
3) The consumer has faith in the integrity of the producer.

4.2 Operating Procedure

The plan is implemented in the following way:

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1.
3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceeding i samples of size n.

The Chain sampling Plan is characterized by the parameters n and i. We are interested in determining the sample size required for in the case of Inverse Rayleigh distribution and various values of acceptance number i.

The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability(β) of accepting a bad lot is known as the consumer's risk. Often the consumer risk is expressed by the consumer's confidence level. If the confidence level is p* then the consumer's risk will be β = 1- p*.We will determine the sample size so that the consumer's risk does not exceed a given value β. The probability of acceptance in the case of chain sampling plan is given by

$P_a(p) = (1-p)^{n} + np(1-p)^{n-1}(1-p)^{ni}$

Where $p = F(t,\lambda) = \exp\left(\frac{-\lambda^2}{t^2}\right)$

In Table 1 we present the minimum values of n, satisfying equation for p* = 0.75, 0.90, 0.95, 0.99 and for T/λ_0 = 0.628,0.942,1.257,1.571,2.356,3.141,4.712. These choices are consistent with Gupta and Groll (1961), Gupta (1962),Kantam et al (2001), Baklizi and EI Masri (2004), Balakrishnan et Al (2007).

V. Operating Characteristic Function

If the true but unknown life product deviates from the specified life of the product it should result in a considerable change in the probability of acceptance of the lot based on the sampling plan. Hence the probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function of the sampling plan. Once the minimum sample size is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. For a fixed T, p is a decreasing function of $\lambda \geq \lambda_0$.For a fixed i the operating chartacteristic function values as a function of λ/λ_0 are presented in Table 2 for different values of p* and for the chsp-1 and the corresponding OC curves are also drawn.

VI. Description Of Tables And An Example

Assume that the life time distribution is an Inverse Rayleigh distribution and that the experimenter is interested in knowing that the true mean life is atleast 1000 hours with confidence 0.99. It is assumed that the maximum affordable time is 767 hours and $T/\lambda_0 = 0.942$, from the table 1, we obtain n = 11. Therefore, for this sampling plan we accept the product if not more than 1 item fail out of 11 items and if no defectives are found in the immediately preceeding i samples before T = 767 units of time, with the assurance that the true median life is atleast 1000 with probability 0.99.

For the sampling plan(n = 11, i= 2, $T/\lambda_0 = 0.942$) and confidence level p* = 0.99 under inverse Rayleigh distribution the values of the operating characteristic function from Table 2 as follows

λ/λ_0	2	4	6	8	10	12
L(p)	0.965196	1	1	1	1	1

Table 1:
Minimum sample size for the proposed plan in case of Inverse Rayleigh distribution with probability p* and the corresponding acceptance number i .

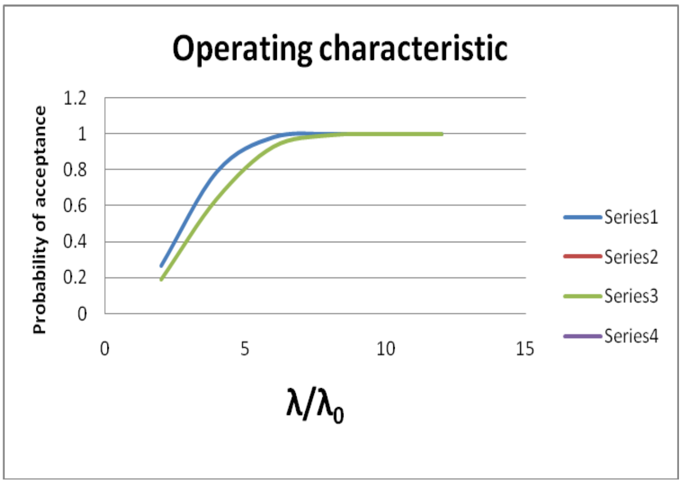
p*	i	T/λ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	1	12	5	3	2	2	1	1	1
	2	18	4	2	2	1	1	1	1
	3	18	4	2	2	1	1	1	1
	4	17	4	2	2	1	1	1	1
	5	17	4	2	2	1	1	1	1
	6	17	4	2	2	1	1	1	1
0.90	1	31	7	4	3	2	2	2	1
	2	29	6	4	3	2	2	1	1
	3	28	6	4	3	2	2	1	1
	4	28	6	4	3	2	1	1	1
	5	28	6	4	3	2	1	1	1
	6	28	6	4	3	2	1	1	1
0.95	1	38	9	5	3	2	2	2	2
	2	37	8	4	3	2	2	2	1
	3	37	8	4	3	2	2	2	1
	4	37	8	4	3	2	2	2	1
	5	37	8	4	3	2	2	2	1
	6	37	8	4	3	2	2	2	1
0.99	1	57	12	7	5	3	3	2	2
	2	56	12	7	5	3	2	2	2
	3	56	12	7	5	3	2	2	2
	4	56	12	7	5	3	2	2	2

	5	56	12	7	5	3	2	2	2
	6	56	12	7	5	3	2	2	2

Table 2: Operating Characteristic values for the time truncated chain sampling plan (n, i, T// λ_0) for a given p*, when i = 2 .

p*	n	$T/\lambda_m^{\ 0}$	λ / λ_0					
			2	4	6	8	10	12
0.75	8	0.628	0.99999876	1	1	1	1	1
	4	0.942	0.995662	1	1	1	1	1
	2	1.257	0.952362	1	1	1	1	1
	2	1.571	0.775024	0.999979	1	1	1	1
	1	2.356	0.641846	0.993905	0.998475	1	1	1
	1	3.141	0.407383	0.929656	0.973981	0.999995	1	1
	1	3.927	0.268744	0.793386	0.903134	0.999507	0.999995	1
	1	4.712	0.187559	0.641846	0.802379	0.993905	0.999756	0.999995
0.90	12	0.628	0.999996769	1	1	1	1	1
	6	0.942	0.990437	1	1	1	1	1
	3	1.257	0.845694	1	1	1	1	1
	2	1.571	0.618109	0.999951	1	1	1	1
	1	2.356	0.298488	0.975102	0.999979	1	1	1
	1	3.141	0.116585	0.775383	0.99425	0.999979	1	1
	1	3.927	0.268744	0.793386	0.982143	0.999507	0.999995	1
	1	4.712	0.187559	0.641846	0.92961	0.993905	0.999756	0.999995
0.95	16	0.628	0.999994736	1	1	1	1	1
	7	0.942	0.983483	1	1	1	1	1
	4	1.257	0.845694	1	1	1	1	1
	3	1.571	0.618109	0.999951	1	1	1	1
	2	2.356	0.298488	0.975102	0.999979	1	1	1
	1	3.141	0.116585	0.775383	0.99425	0.999979	1	1
	1	3.927	0.05316	0.496411	0.932054	0.997841	0.999995	1
	1	4.712	0.187559	0.641846	0.92961	0.993905	0.999756	0.999995
0.99	24	0.628	0.999987934	1	1	1	1	1
	11	0.942	0.965196	1	1	1	1	1
	6	1.257	0.665948	1	1	1	1	1
	4	1.571	0.377526	0.999861	1	1	1	1
	2	2.356	0.142502	0.947188	0.999951	1	1	1
	1	3.141	0.116585	0.775383	0.99425	0.999979	1	1
	1	3.927	0.05316	0.496411	0.932054	0.997841	0.999979	1
	1	4.712	0.027382	0.298488	0.775263	0.975102	0.998925	0.999979

FIGURE 1:



VII. Conclusions

In this paper, chain sampling plan for the truncated life test was proposed in the case of Inverse Rayleigh distribution. The construction yields the minimum sample size required to test the items to decide upon whether a submitted lot is good having more median life or not.The operating characteristics values of the plan against a specified producer's risk are also presented. From the figure 1 we can see that the probability of acceptance increases when λ / λ_0 increases and it reaches the maximum value 1 when λ / λ_0 is greater than 2 .

REFERENCES

[1] Baklizi,a. and El Masri,A.E.K. (2004), "Acceptance sampling plans based on truncated life tests in the Birnbaum Saunders model", Risk Analysis, vol.24,1453-1457.

[2] Baklizi,a. (2003), "Acceptance sampling plans based on truncated life tests in the Pareto distribution of second kind ", Advances and Applications in Statistics, vol.3,33-48.

[3] Balakrishnan, N., Leiva,V. and Lopez, J. (2007), "Acceptance sampling plans from truncated life tests based on generalized Birnbaum Saunders distribution", communications in statistics – simulation and computation, vol.36,643-656.

[4] Dodge,H.F.(1955): Chain Sampling Plan. Industrial quality control.

[5] Epstein,B.(1954), "Truncated life test in the exponential case", Annals of mathematical statistics,vol.25,555-564.

[6] Gupta,S.S. and Groll,P.A. (1961), "Gamma distribution in acceptance sampling based on life tests", Journal of the American Statistical Association,vol.56,942-970.

[7] Gupta,R.D.and Kundu,D. (2007), "Generalized exponential distribution : existing methods and recent developments", Journal of Statistical planning and inference, vol.137,3537 - 3547.

[8] Muhammed Aslam, (2007)"Double acceptance sampling based on truncated life tests in Rayleigh distribution", European Journal of Scientific Research,Vol.17 No.4 ,pp.605-610.

[9] M. Aslam and C.H. Jun. A group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions. Pakistan Journal of Statistics 25, 1-13, 2009.

[10] Srinivasa Rao (2010), "Double acceptance sampling plans based on truncated life tests for Marshall – olkin extended exponential distribution",Austrian journal of Statistics , Vol.40, (2011),Number 3, 169-176.

[11] R.R.L. Kantam, K. Rosaiah and G. Srinivasa Rao.(2001`) Acceptance sampling based on life tests: Log-logistic models. Journal of Applied Statistics 28, 121-128.

[12] K. Rosaiah and R.R.L. Kantam,.(2001), "Acceptance sampling based on the inverse Rayleigh distributions", Economic Quality control, vol.20,151-160.

[13] Kundu, D. and Raqab,M.Z. (2005). " Generalised Rayleigh distribution; different methods of estimation",Computational Statistics and Data Analysis, vol 49,187-200.

[14] Mukherjee, S.P. and Saran, L.K. (1984). Bivariate inverse Rayleigh distributions in reliability studies. Journal of Indian Statistical Association,22,23-31.

[15] Voda. V.Gh.(1972). On the inverse Rayleigh distributed random variable.Rep statist.App.Res.,JUSE,19,13-21.

[16] Mukherjee, S.P. and Maiti, S.S. (1997) A percentile estimator of the Inverse Rayleigh parameter.IAPQR Transactions,21, 63-65.