# Calibration of a Piezoresistive Pressure Sensor Using the 2D Progressive Polynomial Method

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**ABSTRACT:** The paper presents a 2D progressive polynomial model for sensor calibration. A piezoresistive pressure sensor was used for the implementation of this model, whose voltage output depends on both pressure and temperature. In the presented example, temperature and pressure values were measured at 10 points and compared with values obtained from the standard. The observed measurement error can be utilized to improve the transfer characteristics of the sensor. For the purpose of correcting the transfer characteristics of the sensor, calibration was performed in four steps using the proposed model. Using Matlab software support, a graphical representation of the corrected function was presented, where it can be observed that the linearization was completed after four steps. A problem may arise if the sensor's output function is nonlinear; in that case, it is necessary to perform calibration in many more steps than presented.

Keywords: sensor, progressive polynomial method, calibration, piezoresistive pressure sensor

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### I. INTRODUCTION

The sensor plays a crucial role in the modern technological world, the use of sensors has enabled the automation of devices or systems, which is ubiquitous in today's world.. The primary function of a sensor is to provide a mechanism for collecting various types of information in specific processes. Sensors are devices that change some of their properties (most commonly electrical or chemical) under the influence of a physical phenomenon, i.e., its numerous values or changes in these values. For example, a mercury thermometer converts the measured temperature into the expansion of mercury liquid, which can be read on a graduated scale on the tube. Sensors have found application in various measurement and control models and are key elements in research and development across various industries.

The practical application of sensors is present in almost every aspect of human activity. Sensors can be embedded in automobiles, airplanes, ships, mobile phones, industrial plants, and even implanted in human bodies. Measurement error is defined as the difference between the measured and true values. There are several reasons that lead to errors, such as calibration error, error due to input loading, error due to the sensor's sensitivity to the effects of other variables, etc. By performing appropriate sensor calibration, the accuracy of measurements can be significantly increased. Calibration is often defined as a set of procedures that, under specific conditions, establish a relationship between the values indicated by a measuring instrument and the corresponding values realized by a standard. According to the ISO definition, calibration is a set of operations to establish the relationship between the values indicated by a measuring instrument and the corresponding values realized by a standard. According to the ISO definition, calibration is a set of operations to establish the relationship between the values indicated by a measuring instrument and the corresponding values realized by standards. Each sensor has a 'characteristic curve' that defines the sensor's response to input. The calibration process maps the sensor response to the ideal linear response. The most successful way to accomplish calibration depends on the nature of the characteristic curve.

There are different approaches to sensor calibration, including single-point calibration, two-point calibration, or multi-point calibration [1], calibration using look-up tables [2], calibration based on piecewise-polynomial or spline interpolation [3], [4], calibration through error minimization [1], and calibration based on curve fitting. Most of these methods are used for calibrating sensors in one dimension (involving a single variable), while for a multidimensional approach to calibration, the progressive polynomial calibration (curve fitting) method has proven to be most effective [1], [6]. As the most suitable method for calibrating multidimensional sensors, a progressively polynomial approach to linearization was used in a study for the calibration of a two-dimensional sensor measuring both pressure and temperature. For the purposes of this research, a piezoresistive pressure sensor was used with the equation  $U(p, T) = -1.2\ln(1.7(1+0.01T)-P)$ , where U(p, T) is the output voltage dependent on temperature and pressure [7].

## **II. PROGRESSIVE POLYNOMIAL CALIBRATION METHOD - ONE DIMENSIONAL FUNCTION**

The progressive polynomial calibration method is performed in such a way that each calibration measurement can be individually used to directly calculate one programmable coefficient in the correction function. Correction is then automatically applied to modify the sensor output. The next calibration step allows the use of this corrected sensor signal. Each subsequent correction step is applied in a way that preserves the integrity of each previous calibration.

If the input variable is denoted as x and the (electrical) sensor output as y, the response of the uncalibrated sensor can be represented by the sensor transfer function y=f(x). The desired transfer function is denoted as y=g(x), assuming that it is a linear function of the input signal  $g(x) = K \cdot x$ .

The purpose of calibration is to obtain the transfer function of the calibrated sensor that is identical or close to the desired transfer function. This is achieved through calibration measurements with a set of known input signals xn, and by comparing the measured sensor output  $f(x_n)$  with the desired output value  $y_n = g(x_n)$ . The proposed approach is to calculate the corrected transfer curve of the sensor  $h_n(x)$  after each calibration measurement. Using the previously corrected transfer curves  $h_1(x)$  to  $h_{n-1}(x)$ , previously calibrated output values  $y_1$  to  $y_{n-1}$ , and the nth calibration measurement  $f(x_n)$ . In this way, calibration is expressed through a series of formulas:

 $h_n(a_n, h_1(x) \dots h_{n-1}(x), y_1 \dots y_{n-1})$  (1)

where  $a_n$  is the n-th calibration coefficient. At each calibration step, the calibration coefficient an must be calculated to achieve a representation where the corrected sensor output is equal to the desired output:  $h_n(x_n) = g(x_n) = y_n$ . The expression for the n-th calibration function in the progressive polynomial method is [1], [6]:

$$h_n(x) = h_{n-1}(x) + a_n \cdot y_{ref} \cdot \prod_{i=1}^{n-1} \left( \frac{h_i(x) - y_i}{y_{ref}} \right)$$
(2)

The calibration coefficient an is calculated based on the n-th calibration measurement, providing  $h_1(x_n)$  to  $h_{n-1}(x_n)$ , expressed as:

$$a_n = \frac{y_n - h_{n-1}(x_n)}{y_{ref}} \cdot \prod_{i=1}^{n-1} \left( \frac{y_{ref}}{h_i(x_n - y_1)} \right)$$
(3)

### III. PROGRESSIVE POLYNOMIAL CALIBRATION METHOD - TWO DIMENSIONAL FUNCTION

In this approach, the sensor output depends not only on one variable, but the output is also influenced from two independent variable.

As an example, consider a sensor in which the output voltage is not determined solely by the pressure applied to the sensor but is also to some extent influenced by the working temperature of the sensor, such that errors offset, gain, and nonlinearity are temperature-dependent to a certain extent. Such a sensor must be calibrated for both pressure and temperature, hence the term two-dimensional calibration. To implement this approach, a normalized input variable x for the desired sensitivity and z for the cross-sensitive variable were used. The uncalibrated sensor transfer function is given by y=f(x, z). For the calibration of the second variable z, we assume that an additional sensor can be used to measure z independently of other variables, z' = k(z). The sensor transfer function will be calibrated using:

- 1. NxM calibration measurements of the sensor output at different input vectors, denoted as  $f(x_n, z_m)$ , where n = 1 to N and m = 1 to M.
- 2. M simultaneous measurements of the output of the additional z-sensor, denoted as  $z'_m = k(z_m)$ , m = 1 to M.
- 3. N desired values of the sensor output signal, based on the ideal linear transfer curve  $y_n = g(x_n)$ , n = 1 to N. After each calibration measurement, a corrected transfer curve  $h_{nm}(x, z)$  is constructed based on the

previous calibrated transfer functions and the calibration coefficient  $a_{nm}(x, z)$  is constructed based on the previous calibrated transfer functions and the calibration coefficient  $a_{nm}$ . Each coefficient can be calculated based on additional values obtained in the aforementioned list. Table 1 presents the functions of the progressive polynomial calibration method [1], [6].

Table 1. Two-dimensional function of the progressive polynomial method								
m	1		M					
n	$T=T_I$		$T{=}T_M$					
1								
$p=p_1$	$h_{11} = f(p,T) + a_{11}$		$h_{1M}(p,T) = h_{1,M-1}(p,T) + a_{1m} \prod_{m=1}^{M} (T_M - T_m)$					
2			$h_{2M}(p,T) = h_{2,M-1}(p,T) + a_{2m}(h_{1m}(p,T))$					
<i>p</i> = <i>p</i> <sub>2</sub>	$h_{21} = h_{1M}(p,T) + a_{21}(h_{1M}(p,T)-y_1)$		$(-y_1) \prod_{m=1}^{M-1} (T_M - T_m)$					
n								
$p=p_N$	$h_{N1}(p,T) = h_{N-1,M}(p,T) + a_{N1}\{h_{nm}(p,T)\}$		$h_{NM}(p,T) = h_{N,M-1}(p,T) + a_{NM} \prod_{m=1}^{N} (T_M)$					
	$-y_n$ }		$(-T_m)\prod_{m=1}^{M-1} \{h_{NM}(P,T) - y_n\}$					

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Table 2. Calculation of coefficients for the two-dimensional function in the progressive polynomial calibration method

т	1	М
n	$T=T_1$	 T=T <sub>M</sub>
1 p=p1	a11=y1 – f(p1, T1)	 $a_{1M} = \frac{y_1 - h_{1,M-1}(p_1, T_M)}{\prod_{m=1}^{M-1} (T_M - T_m)}$
2 p=p2	$a_{21=\frac{y_2-h_{1,M-1}(p_2,T_1))}{\{h_{1,M-1}(p_2,T_1)-y_1\}}}$	 $a_{1M} = \frac{y_2 - h_{2,M-1}(p_2, T_M)}{\{h_{1,M-1}(p_2, T_1) - y_1\} \prod_{m=1}^{M-1} (T_M - T_m)}$
п р=р <sub>N</sub>	$a_{N1} = \frac{y_N - h_{N-1,M}(p_N, T_1))}{\prod_{n=1}^{N-1} (h_{nM}(n, T_1) - y_n)}$	 $a_{1M} = \frac{y_N - h_{M-1}(p_N, T_M)}{\{h_{m,M-1}(p_n, T_M) - y_n\} \prod_{m=1}^{M-1} (T_M - T_m)}$

# **IV.** Experimental result

The method has been successfully tested on various types of functions. In our case, the validity of the presented method was examined by correcting the transfer function of a piezoresistive pressure sensor U(p, T) = $-1.2\ln(1.7(1+0.01T)-P)$ , where U(p, T) is the output voltage dependent on temperature and pressure.

The progressive polynomial model algorithm was tested using the Matlab computer program. For this purpose, a sample of 10 measured sensor values was presented, both for pressure and temperature values. The adopted static characteristics in the plane and space, in the temperature range of 0-20°C and pressure range of 0-1000mbar, are presented in Figures 1, 2, 3, 4, and 5, respectively.

Table 5. Values of the sensor/sensor standard										
Measured value temp. [°C]	0.2	2.4	4.6	8.8	9	11.2	13.4	15.6	17.8	20
Standard temp. [°C]	0	2	4	6	8	10	12	14	16	18
Measured pressure value P [bar]	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.1
Standard pressure value P [bar]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

 Table 3. Values of the sensor/sensor standard

For the transfer function  $U(p, T) = -1.2\ln(1.7(1+0.01T)-P)$ , the graph of temperature and pressure variation before correction is first presented (Figure 1).



# Figure 1. Transfer function before correction

What can be noticed from the graph is that the deviation of pressure and temperature from the real value is linear, which will significantly simplify the calibration process.

1. In the first step, offset calibration is performed:  

$$h_{1M}(p,T) = h_{1,M-1}(p,T) + a_{1M} \cdot \prod_{m=1}^{M-1} (T_M - T_m)$$
(4)
where the calibration coefficient:  

$$a_{1M} = \frac{y_1(p,T) - h_{1,M-1}(p,T)}{\prod_{m=1}^{M-1} (T_M - T_m)}$$
(5)

The calibration procedure is the same as in the one-dimensional approach, with the difference that in this process, there is also a variable T. Where  $T_m$  represents the measured temperature value of the sensor, and  $T_M$  represents the desired temperature value of the sensor. At the beginning of the calibration process, the pressure value is selected, starting from the initial value. This pressure value remains fixed while correcting the temperature by selecting values according to an established procedure.



Figure 2. Offset correction

2. After the offset correction, the next calibration step involves correcting the gain error (full-scale error) without affecting the previous step. This is achieved by rotating the function around the previous calibration point. The pressure values are selected, which, in the second calibration step, represent the end of the signal. Analogous to the previous step, the pressure value remains fixed, while further correction of the temperature is performed in the procedure. In this process, the transfer function rotates around the line (x=x1, z=z1) moving along the z-axis. The gain correction of the transfer function is performed without compromising the previous calibration step:

$$h_{2M}(p,T) = h_{2,M-1}(p,T) + a_{2M} \cdot \{h_{1M}(p,T) - y_1\} \prod_{m=1}^{M-1} (T_M - T_m)$$
(6)

where the calibration coefficient:

$$a_{2M} = \frac{y_2(p,T) - h_{2,M-1}(p,T)}{\{h_{1M}(p,T) - y_1\} \prod_{m=1}^{M-1} (T_M - T_m)}$$
(7)

Graphical representation, Figure 3



Figure 3. Gain correction of the transfer function

3. The third calibration step involves selecting a fixed pressure value at point three. These calibration measurements are used to correct the second-order linearity by 'bending' the function so that the previous calibration points remain fixed.

$$h_{3M}(p,T) = h_{3,M-1}(p,T) + a_{3M} \cdot \{h_{1M}(p,T) - y_1\} \{h_{2M}(p,T) - y_2\} \prod_{m=1}^{M-1} (T_M - T_m)$$
(8)

where the calibration coefficient:



Figure 4. Nonlinearity calibration

4. Through a continuous process of temperature correction, we obtain the final graph of the two-dimensional calibration method, where Figure 5 represents third-order linearization.



Figure 5. Corrected transfer function

Multiple calibration measurements can be performed in the same way for further linearization of the sensor transfer function. This explains the gradual correction of the transfer function until it fits the desired function. In fact, at each calibration step, the calibrated transfer function progresses towards the desired transfer,

hence the name progressive polynomial calibration. In this situation a simpler example was used, so the calibration expression is not 'cumbersome', more precisely, the calibration expression in four steps was applied. Very often, the situation is not so simple, and to achieve a high-quality solution, more than four calibration steps may be needed.

# V. Conclusion

The progressive polynomial calibration method represents the most powerful algorithm for calibrating smart sensors. Taking this into account, this paper focuses on a more detailed presentation of this algorithm for the calibration of two-dimensional sensors. Through the presented example of a two-dimensional function, it can be observed that the advantage lies in the simplicity of the sensor calibration process, at each calibration step, the calibrated transfer function progresses towards the desired transfer, while on the other hand, the disadvantage is the 'cumbersome' expression in the case of a nonlinear sensor output. U ovom slučaju korišćen je jednostavniji primer senzora, gde je kompletan proces okončan u četiri kalibraciona koraka. For more complex functions, the process is much more complicated and takes place in multiple steps, and the need itself depends on the level of measurement precision required. Additionally, this algorithm has proven to be a very efficient tool for multidimensional sensor calibration. To improve the curve fitting technique with the progressive polynomial method, two strategies can be employed. The first is further development and improvement of this method, and the second is the smart selection of calibration points during the implementation of each calibration step.

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