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Cubic spline function application in some algebraic functions

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Abstract: *In this study, cubic spline functions used to investigate the relationships between dependent and independent variables and their use in algebraic functions were examined. For this purpose, cubic spline functions were obtained by giving various values in different definition intervals to linear, quadratic and cubic functions, and the values of the function were estimated by giving various values within the definition range.*
The observed and predicted values of the predicted algebraic functions were found to be very close to each other. This result shows that cubic spline functions are a very suitable method for calculating algebraic functions.
Keywords: *Cubic spline, function, definition range*

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I. Introduction

Cubic spline functions considers investigation of the shape of the time dependence without that requiring that the exact functional form first be specified. Cubic splines provide great flexibility in fitting data with relatively few parameters [1, 2].

Cubic spline functions allow one to graph time-by-covariate interactions, to test formally for the proportional hazards assumption, and also to test for non-linearity of the time-by-covariate interaction. The functions may be fitted with existing software using relatively few parameters; the regression coefficients are predicted using standard maximum likelihood methods [3].

The fitting of a polynomial curve to a set of data points has applications in computer assisted design, computer assisted manufacturing and computer graphic systems and robot path planning. The interpolating methods lead to higher order polynomials, usually of order equal to one less than the number of data points [4].

With respect to the literature review, studies with cubic spline functions were found [5] improved a new spline method for solving two-point second order boundary value problems. [6], presented a numerical scheme for solving a finite difference approximation for discretizing spatial derivatives, and used the cubic spline collocation technique for the time integration of the resulting nonlinear system of ordinary differential equations. Analytical shaping method for low-thrust rendezvous trajectory using cubic spline functions was carried out by [7].

[8] has discussed the applied of cubic spline smoothing for oil and gas data interpretation. The Seabed Loging Data was used for their purposed. From the results, they finalized that cubic splines smoothing gives quite impressive results.

In this study, it is aimed to express some algebraic functions such as polynomial functions and rational functions as cubic spline functions by giving examples.

II. Material and Method

The spline function is to replace a single function, f, defined over the entire range of t with several low-order polynomials (splines) defined over subintervals of the range of t [1]. The points that divide the subintervals are called knots. These splines are continuous piecewise polynomials of degree m with continuous derivatives through order m-1. The values and first m-1 derivatives of the splines agree at either side of each knot, apart from the end knots. Thus, spline functions are smoothly joined piecewise polynomials.

In general, if there are k knots at times t_i (i = 1, . . . , k) it can be wrote a spline function of time as follow [3].

$$S(t)=\sum_{j=0}^{m}\beta_j t^j+\sum_{i=1}^{k}\theta_i(t-t_i)^m$$

There are k+m+1 regression coefficients, β and θ, for this mth-degree spline function.

Other commonly encountered special cases of spline functions include piecewise constants, or step functions, and piecewise linear functions. Cubic splines are often used to increase the flexibility of the spline function. Reviews of the theory and application of spline functions can be found in numerous references [1, 9, 10, 11].

III. Results

Linear, quadratic, cubic and quaternary polynomial functions and algebraic functions are fitted with cubic spline functions in the defined tabulation range. Each lower value of the definition intervals is the nodal point of the function. More than one function is obtained by expressing them as piecewise functions.

For the values between -1 and 4 of a linear function in the form of y=4x-3, the cubic spline function is fitted according to the actual values of the function. Cubic spline functions in the given range for each node within the specified definition range are obtained as follows.

$$f(x_1) = -2.9569\ (x+1)^3 + 6.9569\ (x+1)^2 - 7 \qquad -1 \le x < 0$$
$$f(x_2) = 0.8708\ x^3 - 1.9139\ x^2 + 5.0431x - 3 \qquad 0 \le x < 1$$
$$f(x_3) = -0.5263\ (x-1)^3 + 0.6986\ (x-1)^2 + 3.8278\ (x-1) + 1 \qquad 1 \le x < 2$$
$$f(x_4) = 1.2344\ (x-2)^3 - 0.8804\ (x-2)^2 + 3.6459\ (x-2) + 5 \qquad 2 \le x < 3$$
$$f(x_5) = -4.4115\ (x-3)^3 + 2.8230\ (x-3)^2 + 5.5885\ (x-3) + 9 \qquad 3 \le x < 4$$

The y values obtained according to the cubic spline function and the actual values of the function are given in Table 1. Figure 1 shows the graphs of y=4x-3 and cubic spline functions.

Table 1. Values of y=4x-3 function and cubic spline function in the range [-1,4]

x	$y = 4x - 3$	Cubic spline
-1	-7	-7
-0.75	-6	-6
-0.5	-5	-5
0	-3	-3
0.6	-0.6	-0.6
1	1	1
1.25	2	2
1.5	3	3
2	5	5
2.5	7	7
3	9	9
3.25	10	10
3.5	11	11
4	13	13
4.25	14	14
4.5	15	15

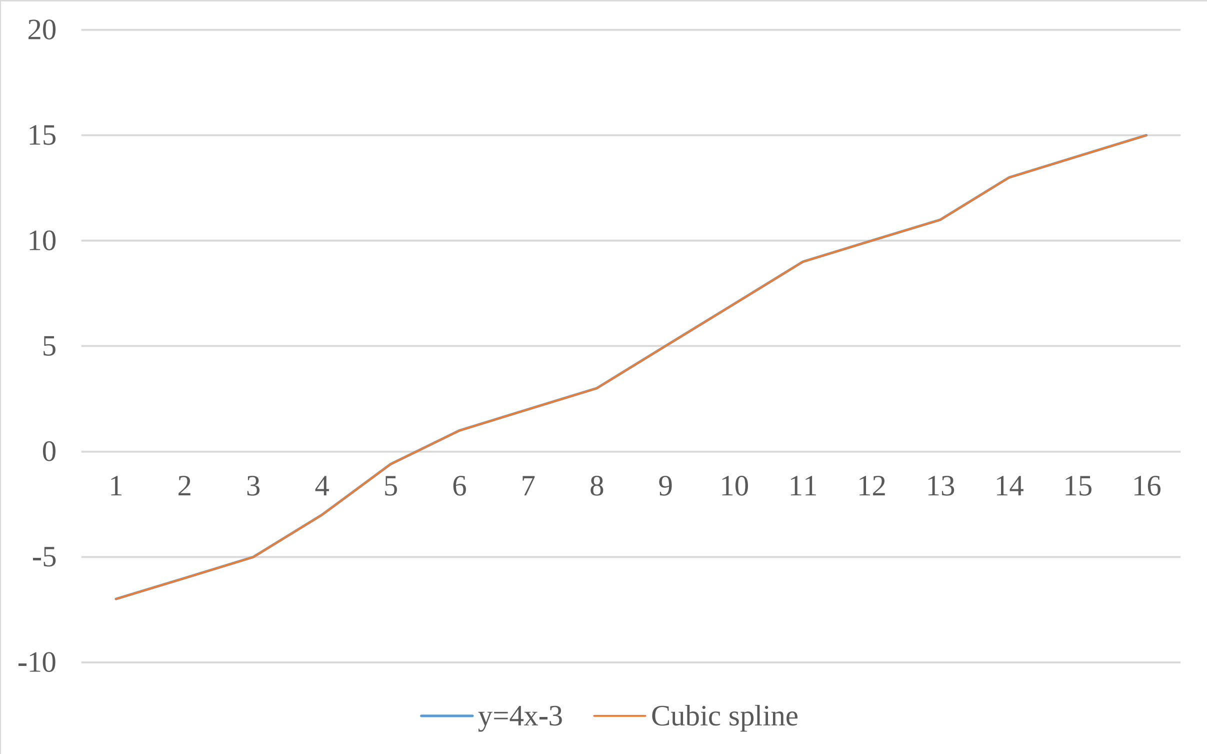

Figure 1. $y = 4x - 3$ and graphs of cubic spline functions

As seen in Figure 1, the values of y=4x-3 and cubic spline functions were found to be equal. This shows that the cubic spline function gives very good results.
The cubic spline functions created to express the $y=x^2+5$ function as a Cubic Spline function in the range [-1, 4.5] are as follows.

$$f(x_1) = 1.4163\,(x+1)^3 - 2.4163\,(x+1)^2 + 6 \qquad -1 \le x < 0$$
$$f(x_2) = -0.2488\,x^3 + 1.8325\,x^2 - 0.5837\,x + 5 \qquad 0 \le x < 1$$
$$f(x_3) = -0.4211\,(x-1)^3 + 1.0861\,(x-1)^2 + 2.3349\,(x-1) + 6 \qquad 1 \le x < 2$$
$$f(x_4) = 1.933\,(x-2)^3 - 0.177\,(x-2)^2 + 3.244\,(x-2) + 9 \qquad 2 \le x < 3$$
$$f(x_5) = -7.311\,(x-3)^3 + 5.622\,(x-3)^2 + 8.689\,(x-3) + 14 \qquad 3 \le x < 4$$

$y=x^2+5$ and cubic spline function values and graphs are shown in Table 2 and Figure 2.

Table 2. $y=x^2+5$ and cubic spline functions values

x	$y=x^2+5$	Cubic spline
-1	6	6
-0.75	5.5625	5.5625
-0.5	5.25	5.25
0	5	5
0.6	5.36	5.36
1	6	6
1.25	6.5625	5.5625
1.5	7.25	7.25
2	9	9
2.5	11.25	11.25
3	14	14
3.25	15.5625	15.5625
3.5	17.25	17.25
4	21	21

4.25	23.0625	23.0625
4.5	25.25	25.25

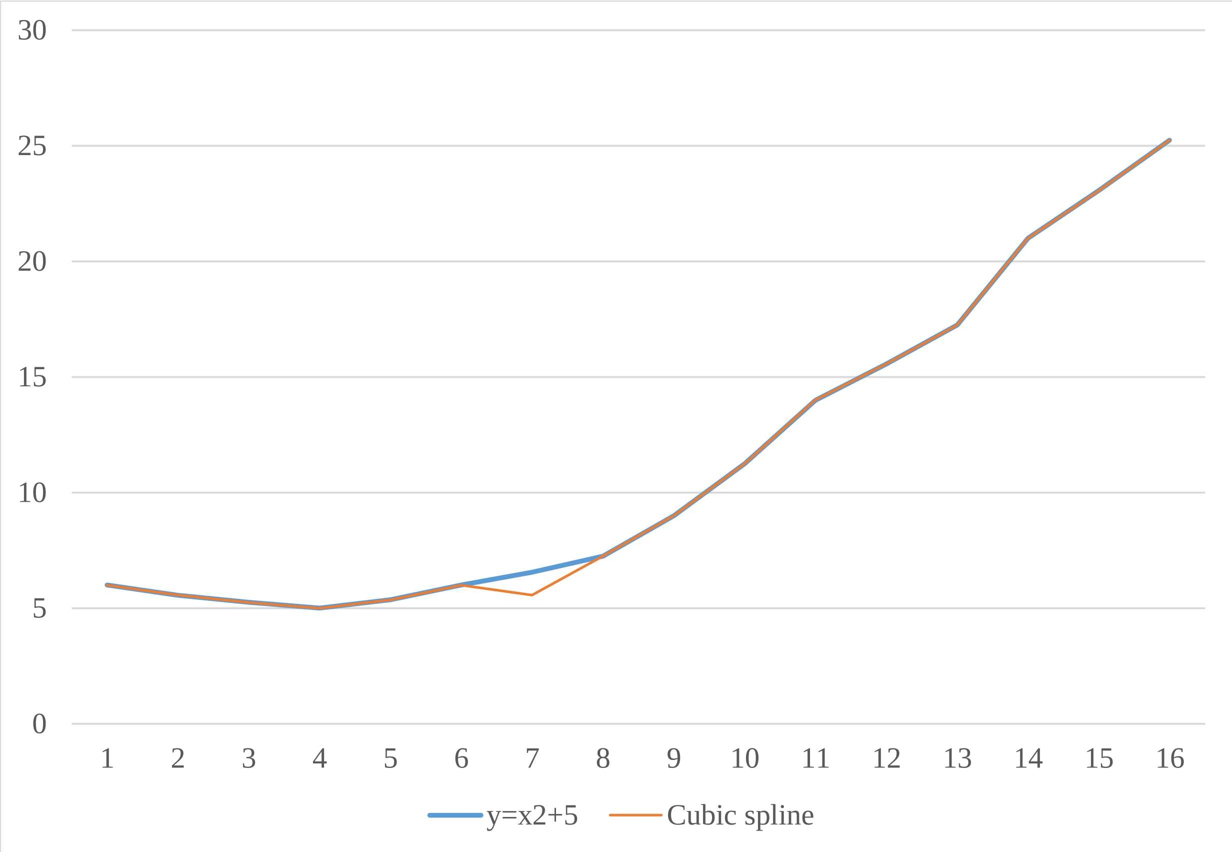

Figure 2. Graphs of $y=x^2+5$ function and cubic spline functions

When Table 2 and Figure 2 are examined carefully, it is seen that the function and cubic spline function values are very close to each other.

The expression of the $y=x^2+2x+3$ function in the form of a Cubic Spline function in the range [-1, 4.5] is as follows.

$$f(x_1) = -0.0574\,(x+1)^3 + 1.0574\,(x+1)^2 + 2 \qquad -1 \le x < 0$$
$$f(x_2) = 0.1722\,x^3 + 0.8852\,x^2 + 1.9426\,x + 3 \qquad 0 \le x < 1$$
$$f(x_3) = -0.6316\,(x-1)^3 + 1.4019\,(x-1)^2 + 4.2297\,(x-1) + 6 \qquad 1 \le x < 2$$
$$f(x_4) = 2.3541\,(x-2)^3 - 0.4928\,(x-2)^2 + 5.1388\,(x-2) + 11 \qquad 2 \le x < 3$$
$$f(x_5) = -8.7847\,(x-3)^3 + 6.5694\,(x-3)^2 + 11.2153\,(x-3) + 18 \qquad 3 \le x < 4$$

$y=x^2+2x+3$ and cubic spline function values and graphs are presented in Table 3 and Figure 3.

Table 3. Values of $y=x^2+2x+3$ and cubic spline functions

x	$y=x^2+2x+3$	Cubic spline
-1	2	2
-0.75	2.0625	2.0625
-0.5	2.25	2.25
0	3	3
0.6	4.56	4.56
1	6	6
1.25	7.0625	7.0625

1.5	8.25	8.25
2	11	11
2.5	14.25	14.25
3	18	18
3.25	20.0625	20.0625
3.5	22.25	22.25
4	27	27
4.25	29.5625	29.5625
4.5	32.25	32.25

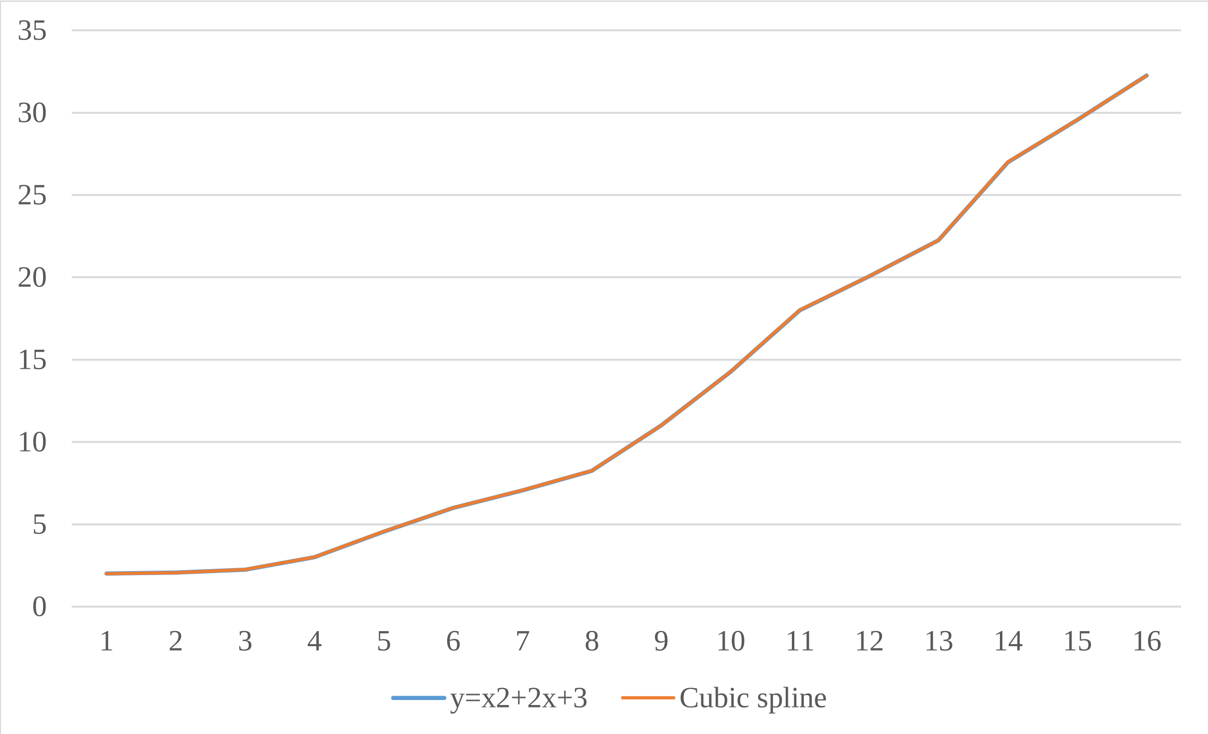

Figure 3. Graphs of $y=x^2+2x+3$ and cubic spline functions

The Cubic Spline function in the range [-1, 4] of the

$$y = \frac{3x-2}{x^2+1}$$

function can be expressed as follows.

$$f(x_1) = 0.8136\,(x+1)^3 - 0.3136\,(x+1)^2 - 2.5 \qquad -1 \le x < 0$$
$$f(x_2) = -1.4407\,x^3 + 2.1271\,x^2 + 1.8136\,x - 2 \qquad 0 \le x < 1$$
$$f(x_3) = 0.7493\,(x-1)^3 - 2.195\,(x-1)^2 + 1.7457\,(x-1) + 0.5 \qquad 1 \le x < 2$$
$$f(x_4) = 0.2437\,(x-2)^3 + 0.0528\,(x-2)^2 - 0.3965\,(x-2) + 0.8 \qquad 2 \le x < 3$$
$$f(x_5) = -1.3359\,(x-3)^3 + 0.7838\,(x-3)^2 + 0.4401\,(x-3) + 0.7 \qquad 3 \le x < 4$$

$y=(3x-2)/(x^2+1)$ and cubic spline function values and graphs are presented in Table 4 and Figure 4.

Table 4. $y=(3x-2)/(x^2+1)$ and cubic spline functions values

x	Observed: $y=(3x-2)/(x^2+1)$	Cubic Spline
-1	-2.5	-2.5
-0.75	-2.72	-2.8848
-0.5	-2.8	-2.8684

0	-2	-2
0.6	-0.147058824	-0.3628
1	0.5	0.5
1.25	0.682926829	0.7733
1.5	0.769230769	0.8699
2	0.8	0.8
2.5	0.75862069	0.7272
3	0.7	0.7
3.25	0.67027027	0.6863
3.5	0.641509434	0.6698
4	0.588235294	0.5882
4.25	0.563934426	0.5126
4.5	0.541176471	0.4061

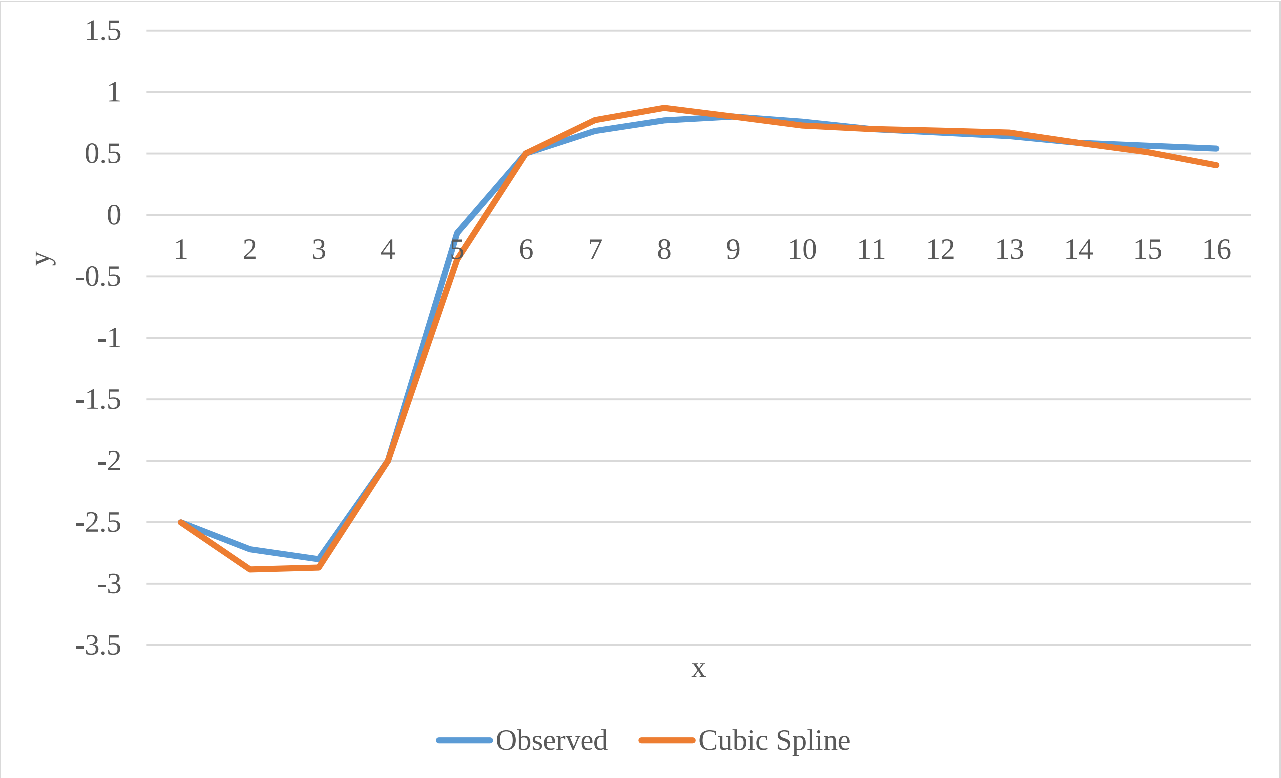

Figure 4. y=(3x-2)/(x²+1) and graphs of cubic spline functions

When both Table 4 and Figure 4 are examined, it is seen that the values of the functions give very close results when compared.

$$y = 2x^3 + 4x^2 - 5x + 10$$

the expression of the function in the form of a Cubic Spline function is as follows.

The expression of the

$$y = 2x^3 + 4x^2 - 5x + 10$$

function as a Cubic Spline function is as follows.

$$f(x_1) = 6.5263\ (x+1)^3 - 13.5263\ (x+1)^2 + 17 \qquad -1 \le x < 0$$
$$f(x_2) = 2.4211\ x^3 + 6.0526\ x^2 - 7.4737\ x + 10 \qquad 0 \le x < 1$$
$$f(x_3) = -4.2105\ (x-1)^3 + 13.3158\ (x-1)^2 + 11.8947\ (x-1) + 11 \qquad 1 \le x < 2$$
$$f(x_4) = 26.4211\ (x-2)^3 + 0.6842\ (x-2)^2 + 25.8947\ (x-2) + 32 \qquad 2 \le x < 3$$
$$f(x_5) = -89.4737\ (x-3)^3 + 79.9474\ (x-3)^2 + 106.5263\ (x-3) + 85 \qquad 3 \le x < 4$$

The values and graphs obtained with the $y = 2x^3 + 4x^2 - 5x + 10$ function and the cubic spline function are presented in Table 5 and Figure 5.

The values and graphs obtained with the $y = 2x^3 + 4x^2 - 5x + 10$ function and the cubic spline function are presented in Table 5 and Figure 5.

Table 5. $y = 2x^3 + 4x^2 - 5x + 10$ and cubic spline functions values

x	$y = 2x^3 + 4x^2 - 5x + 10$	Cubic spline
-1	17	17
-0.75	15.15625	15.1563
-0.5	13.25	13.25
0	10	10
0.6	8.872	8.872
1	11	11
1.25	13.90625	13.9063
1.5	18.25	18.25
2	32	32
2.5	53.75	53.75
3	85	85
3.25	104.65625	104.6563
3.5	127.25	127.25
4	182	182
4.25	214.53125	214.5313
4.5	250.75	250.75

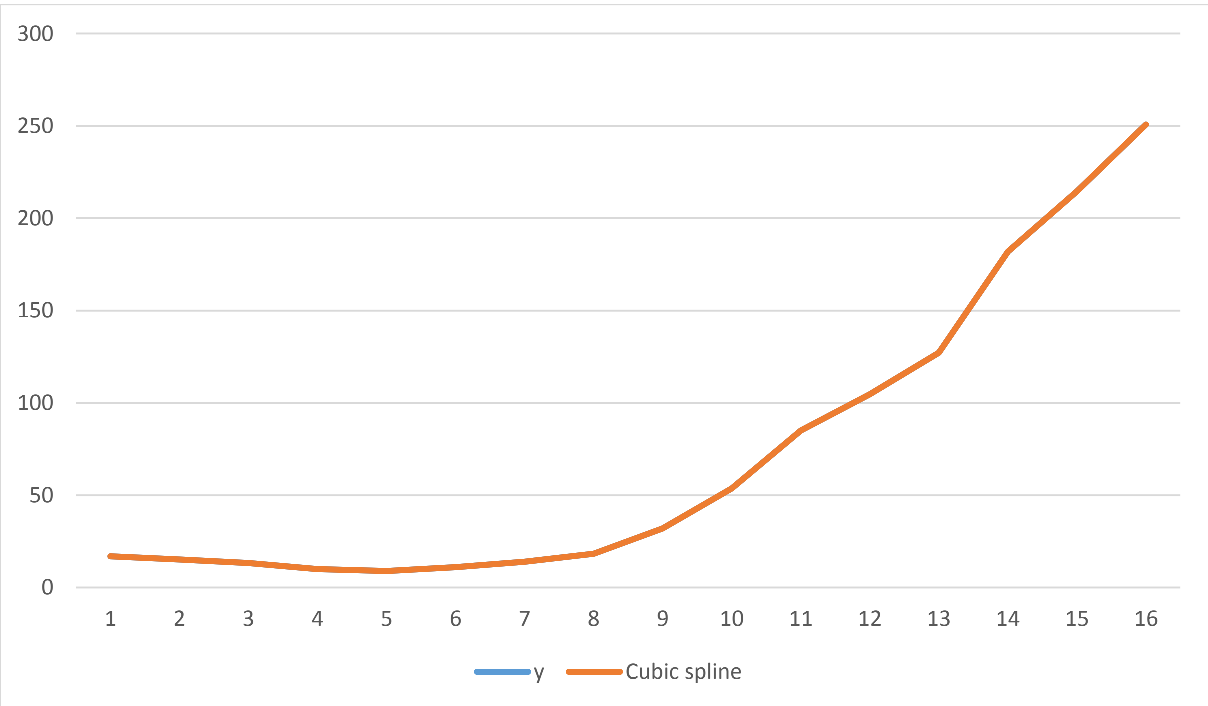

Figure 5. $y = 2x^3 + 4x^2 - 5x + 10$ and graphs of cubic spline functions

The Cubic Spline function of

$$y = \frac{x^4 + 2x - 1}{x^2 + 3}$$

can be written as follows.

$$f(x_1) = 0.0278\,(x+1)^3 + 0.1389\,(x+1)^2 - 0.5 \qquad -1 \le x < 0$$
$$f(x_2) = 0.25\,x^3 + 0.2222\,x^2 + 0.3611\,x - 0.3333 \qquad 0 \le x < 1$$
$$f(x_3) = -0.3134\,(x-1)^3 + 0.9722\,(x-1)^2 + 1.5555\,(x-1) + 0.5 \qquad 1 \le x < 2$$
$$f(x_4) = 1.8608\,(x-2)^3 + 0.0319\,(x-2)^2 + 2.5596\,(x-2) + 2.7143 \qquad 2 \le x < 3$$
$$f(x_5) = -7.1449\,(x-3)^3 + 5.6144\,(x-3)^2 + 8.206\,(x-3) + 7.1667 \qquad 3 \le x < 4$$

y=(x^4+2x-1)/(x^2+3) and cubic spline function values and graphs are given in Table 6 and Figure 6.

Table 6. y=(x^4+2x-1)/(x^2+3) and cubic spline function values

x	y=(x^4+2x-1)/(x^2+3)	Cubic Spline
-1	-0.5	-0.5
-0.75	-0.6129386	-0.487
-0.5	-0.59615385	-0.4614
0	-0.33333333	-0.33333
0.6	0.098095238	0.0471
1	0.5	0.5
1.25	0.863869863	0.8864
1.5	1.345238095	1.3718
2	2.714285714	2.714286
2.5	4.655405405	4.6465
3	7.166666667	7.166667
3.25	8.631624424	8.6372
3.5	10.23360656	10.2426
4	13.84210526	13.84211
4.25	15.84588279	15.8282
4.5	17.9811828	17.9331

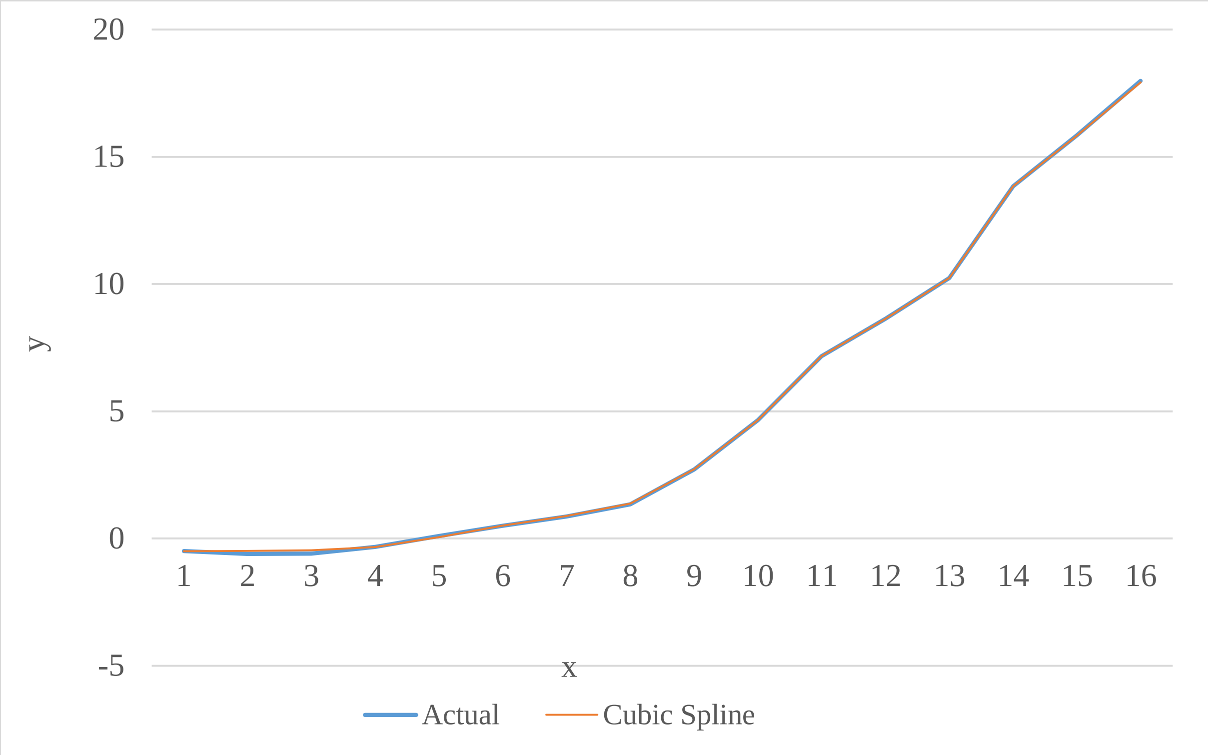

Figure 6. $y=(x^4+2x-1)/(x^2+3)$ and graphs of the cubic spline function

IV. Conclusion

In this study, some algebraic functions were expressed as cubic spline functions. The values of these functions according to the values in the interval [-1, 4.5] were calculated with cubic spline functions. The values of a large number of randomly selected algebraic functions calculated with cubic spline functions were found very close to their true values. It has been seen that the graphs of the selected algebraic functions and the cubic spline functions are in good agreement. It has been revealed that the cubic spline functions give very good results in calculating the values of algebraic functions in the definition intervals.

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