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Matrix Representation of Graph Theory with Real Life Applications

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Abstract. *Graph theory is one of the most important and basic topics of discrete mathematics in Mathematics. In all sectors of science graph theory has a great impact. Most common use of graphs occurs in Physics and Chemistry except Mathematics. It is also used in the modeling of Biology, Finance and Computer science. Basic concepts of graphs are discussed here with classification with figures.*
In this paper matrix representation of graphs, applications of graphs in different sectors of science including real life applications are discussed.
Keywords: *Graphs, Matrix Representation, Incidence matrix, Adjacency Matrix, Cut-Set Matrix, Circuit Matrix, Real life Applications.*

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I. Introduction

Graphs are mathematical structures used to model pair-wise relations between objects from a certain collection. A graph $G = (V, E)$ consists of V, a nonempty set of vertices and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints. Vertices can be any abstract data types and can be presented with the points in the plane. These abstract data types are also called nodes. A line or line segment connecting these nodes is called an edge. Again, more abstractly saying, edge can be an abstract data type that shows relation between the nodes.

II. Matrix Representation of Graph theory

Graph can be represented in the form of matrix. The matrices that can be formed by a graph are given below.

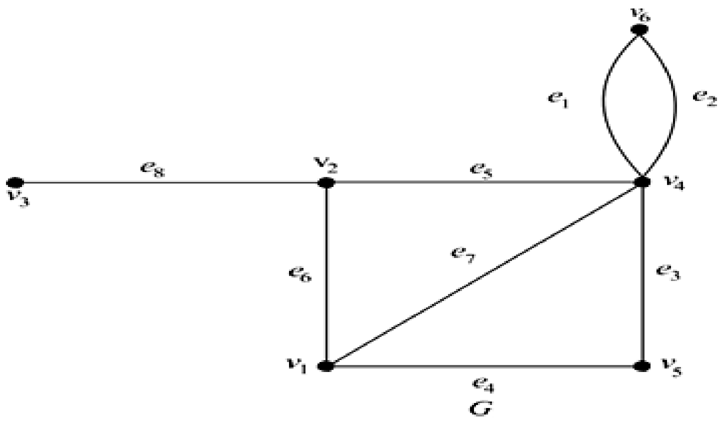
1. Incidence Matrix
2. Adjacency Matrix
3. Cut-Set Matrix
4. Circuit Matrix
5. Path Matrix

2.1 Incidence Matrix: An Edge connected to a vertex is known as incidence edge to that vertex. Let G be a graph with n vertices, m edges and without selfloops. The incidence matrix A of G is an $n \times m$ matrix $A = [a_{ij}]$ whose n rows correspond to the n vertices and the m columns correspond to m edges such that

$$a_{ij} = \begin{cases} 1, \text{if jth edge } m_j \text{ is incident on the ith vertex} \\ 0, \text{otherwise} \end{cases}$$

It is also called vertex-edge incidence matrix and is denoted by $A\ (G)$.

Example: Consider the Graph G below



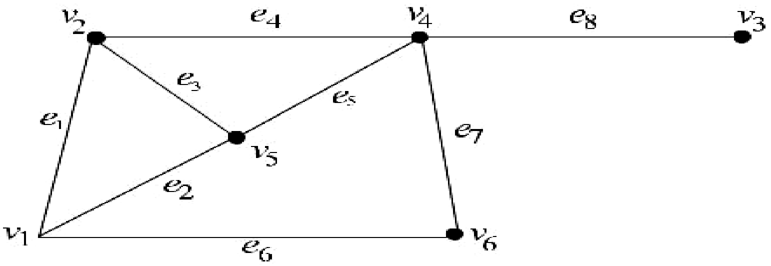
The incidence matrix of G is

$$A\,(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

2.2 Adjacency Matrix: When two vertices are connected by single path than they are known as adjacent vertices. If vertex is connected to itself then vertex is said to be adjacent to itself. Let G be a graph with n vertices, m edges. The adjacency matrix A of G is an $n \times m$ matrix $A = [a_{ij}]$ whose n rows correspond to the n vertices and the m columns correspond to m edges such that

$$a_{ij} = \begin{cases} 1, \text{if } \{v_i, v_j\} \text{is an edge of G} \\ 0, \text{otherwise} \end{cases}$$

Example: Consider the Graph G below



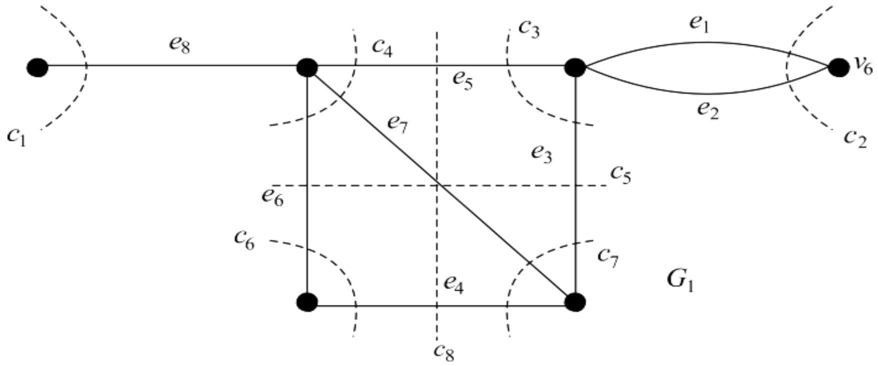
The adjacency matrix of G is

$$A\,(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

2.3 Cut-Set Matrix: Cut set is a Set of edges in a graph whose removal leaves the graph disconnected. Let G be a graph with m edges and q cutsets. The cut-set matrix $C = [c_{ij}]_{q \times m}$ of G is a matrix with

$$c_{ij} = \begin{cases} 1, \text{if ith cutset contains jth edge} \\ 0, \text{otherwise} \end{cases}$$

Example: Consider the graph shown in the figure below



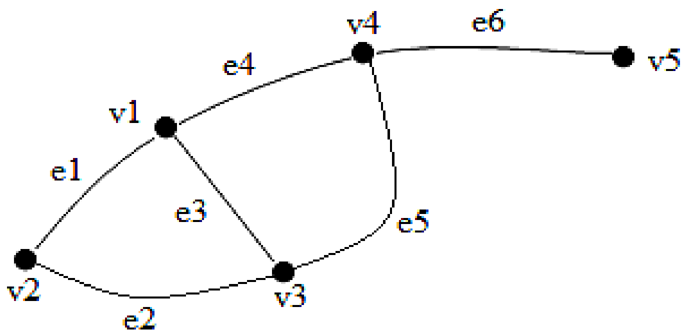
In the graph $G_1, E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$.
The cut-sets are $c_1 = \{e_8\}$, $c_2 = \{e_1, e_2\}$, $c_3 = \{e_3, e_5\}$, $c_4 = \{e_5, e_6, e_7\}$, $c_5 = \{e_3, e_6, e_7\}$, $c_6 = \{e_4, e_6\}$, $c_7 = \{e_3, e_4, e_7\}$ *and* $c_8 = \{e_4, e_5, e_7\}$.
Thus the cut-set matrices are given by

$$C\ (G_1) = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

2.4 Circuit Matrix: Circuit is a close walk in which no vertex/edge can appear twice. Consider a loopless graph $G = (V, E)$ which contains circuits. We enumerate the circuits of G: $C_1, C_2, \dots\dots\dots\dots\dots\dots, C_l$. The circuit matrix of G is an $\ell \times m$ matrix $C = [c_{ij}]$ where

$$c_{ij} = \begin{cases} 1, \text{if ith circuit includes jth edge} \\ 0, \text{otherwise} \end{cases}$$

Example: Consider the graph shown in the figure below



In the graph the circuits are $c_1 = \{e_1, e_2, e_3\}, c_2 = \{e_3, e_4, e_5\}, c_3 = \{e_1, e_2, e_5, e_4\}$. Hence the circuit matrix is given by

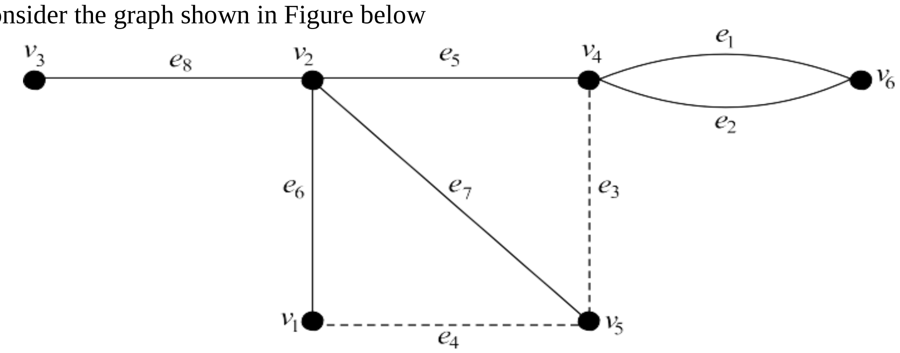
$$C\ (G) = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

2.5 Path matrix: Path is an open walk in which no vertex /edge can appear twice. Let G be a graph with m edges, and u and v be any two vertices in G. The path matrix for vertices u and v denoted by $P\ (u,v) = [p_{ij}]_{q\times m}$ where q is the number of different paths between u and v, is defined as

$$p_{ij} = \begin{cases} 1, \text{if jth edge lies in theith path} \\ 0, \text{otherwise} \end{cases}$$

Clearly, a path matrix is defined for a particular pair of vertices, the rows in $P(u,v)$ correspond to different paths between u and v, and the columns correspond to different edges in G.

Example: Consider the graph shown in Figure below

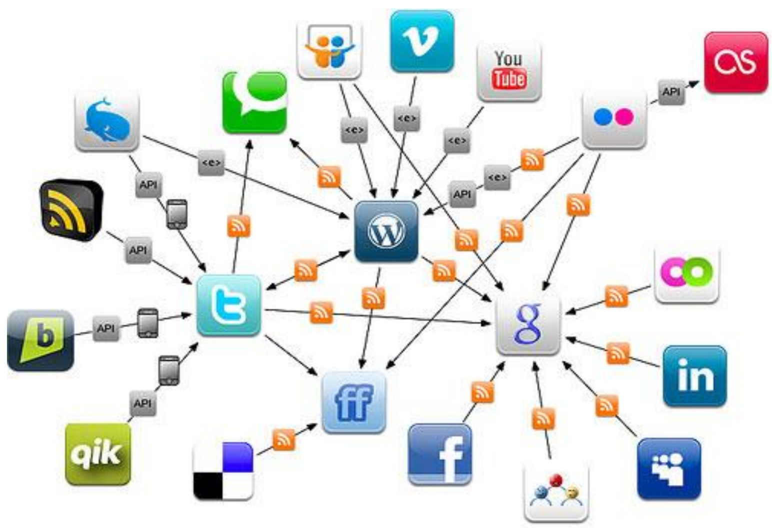


The different paths between the vertices v_3 and v_4 are $p_1 = \{e_8, e_5\}$, $p_2 = \{e_8, e_7, e_3\}$ and $p_3 = \{e_8, e_6, e_4, e_3\}$.The path matrix for v_3, v_4 is given by

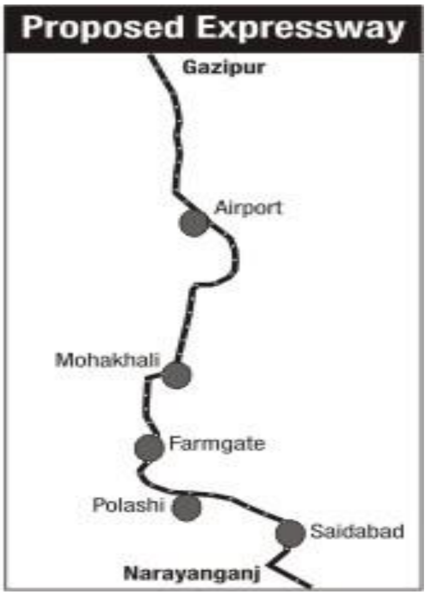
$$P\ (v_3, v_4) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

3. Applications of Graphs in real life

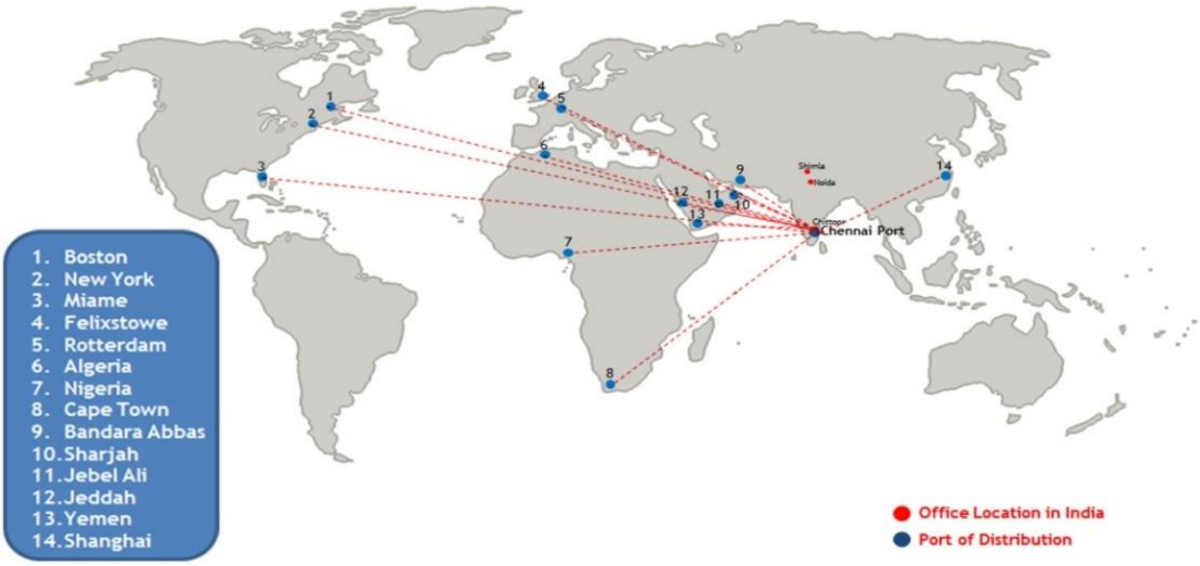
3.1 Communication networks (social networks)



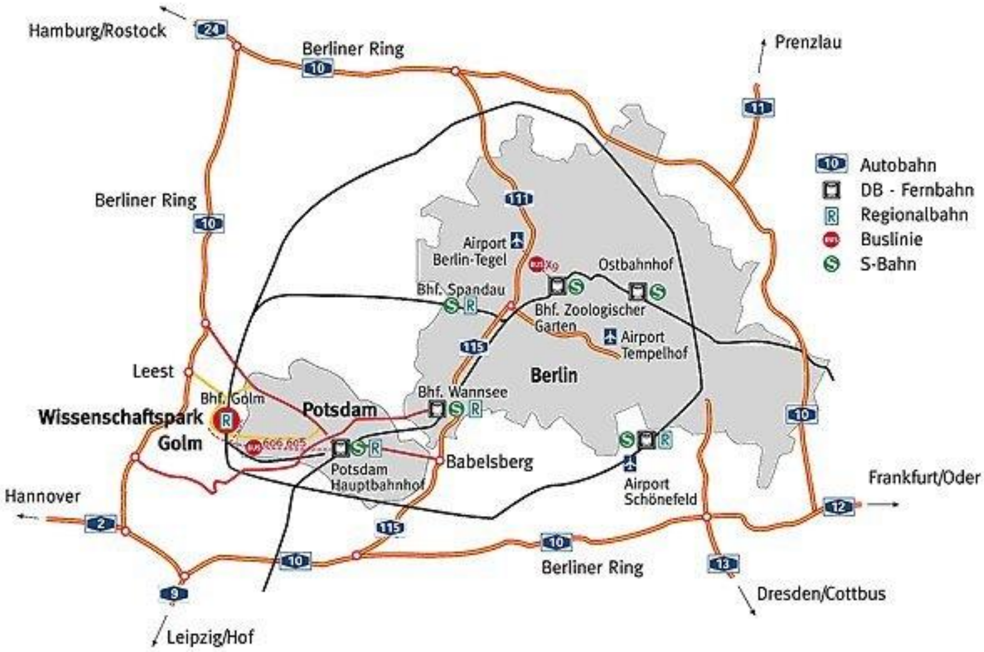
3.2 Routing and shortest path problems



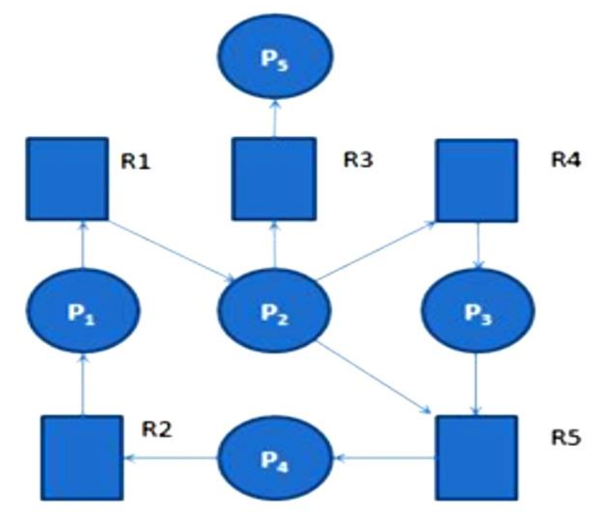
3.3 Commodity distribution (network flow)



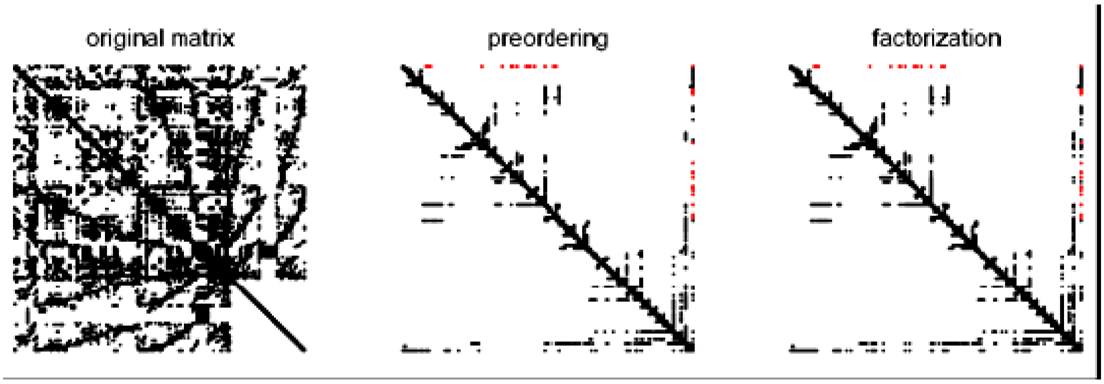
3.4 Traffic control



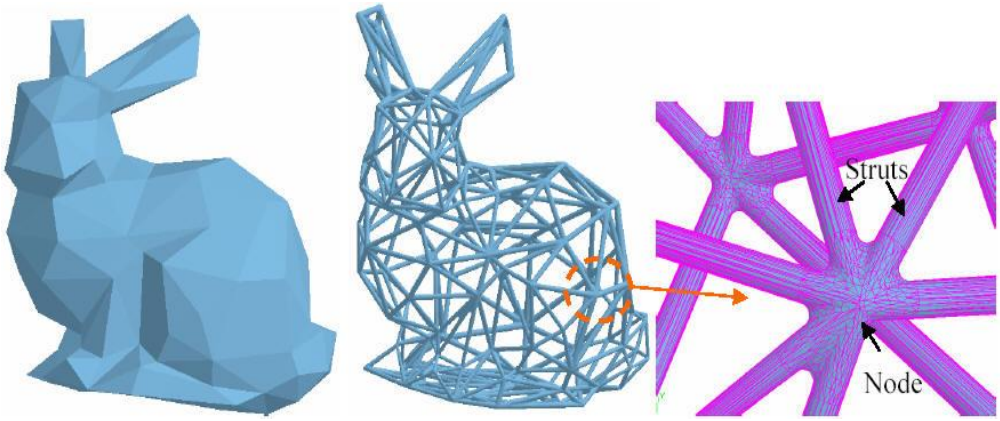
3.5 Resource allocation



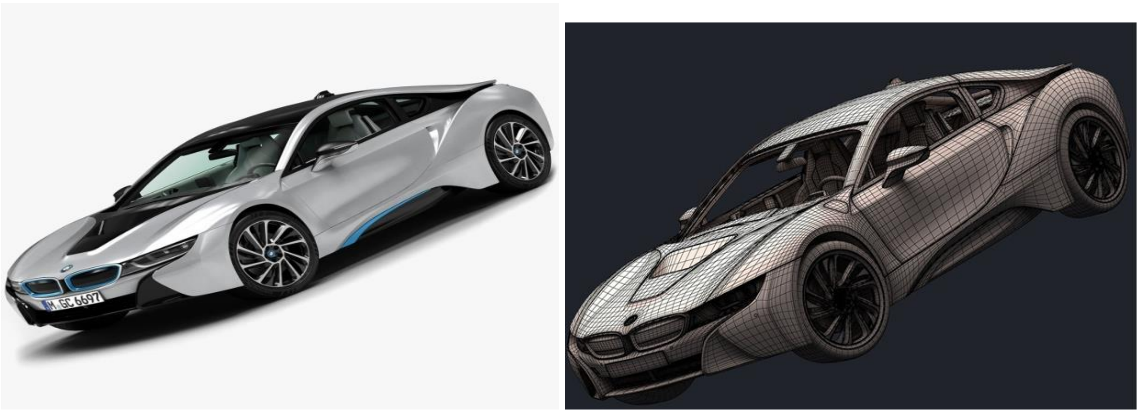
3.6 Numerical linear algebra (sparse matrices)



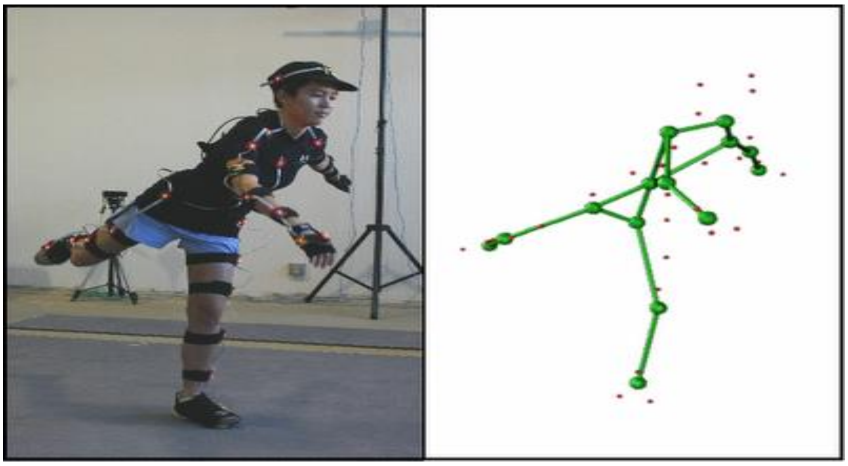
3.7 Geometric modeling (meshes, topology)



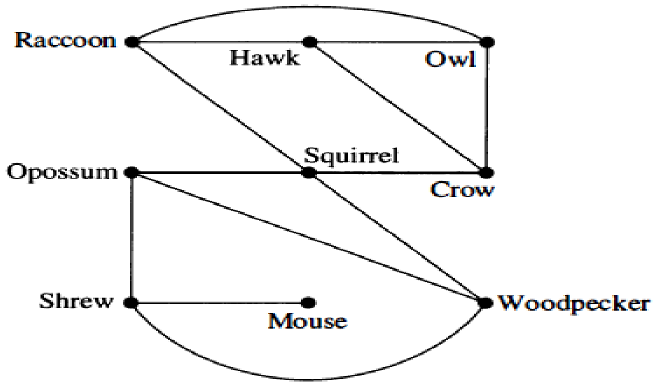
3.8 Image processing (graph cuts)



3.9 Computer animation (motion graphs)



3.10 Biological systems



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