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Appling Poles Placement Method To Design The Controller For Longitudinal Motion Of Aircraft

Manh Hoan Vu[1], Xuan Tuyen Do[1], Quang Trung Doan[1], Anh Duong Vu[1], The Dong Dang[1]

[1] *Air Defense - Air Force Academy, Faculty of Aviation Technical, Vietnam*

Corresponding Author Email: Vuhoan.kthk@gmail.com

Summary: *This article presents a method to design a controller for a MIMO (multiple-input, multiple-Multiple-Output) linear system using the pole placement method. This method is applied to designing a controller for an object that is the edge motion channel of an aircraft. This is a very complex object; the mathematical model is represented as a system of equations with six degrees of freedom that operate under different conditions and are influenced by many impacts. The mathematical model of the aircraft is broken down into individual motions, namely edge motion and longitudinal motion, using the linear method with small deviations. Designing controllers for separate aircraft motion channels is very important to achieve aircraft control quality criteria. In this study, the pole point assignment method is used to design a controller for the object's edge motion channel based on the parameters in the object's mathematical model. Simulation results on MATLAB software show that the proposed design method produces results in accordance with the required quality criteria.*

Keywords: Airplane, method poles placement, longitudinal motion, coefficient gain.

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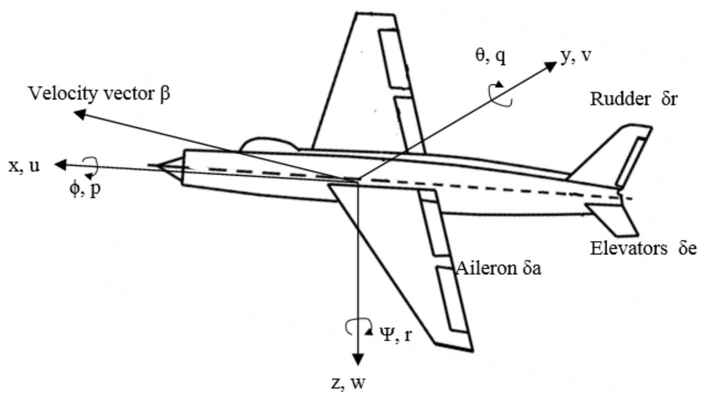
I. Introduction

The pole point assignment method is based on the principle that the quality of the automatic control system depends on the position of the system's pole points, so we can assign the system the given pole points and set stability standards for the system from which to build a controller for the system that fully meets the standards we set before designing. The ultimate goal of the design is to find the K coefficient matrix from which we can build a closed-loop circuit of the system that fully meets the standards we have set.

The selection of pole pairs for designing a controller for the aircraft's edge motion channel to fully meet the set standards is presented in Section 2.3 of this article. In Part 3, the calculation results are checked through simulation on Matlab software.

II. Content

2.1. Mathematical model of the vertical motion channel of an aircraft



x, y, z = Position coordinates; ϕ = Roll angle; u,v,w = Velocity coordinates; θ = Pictchi angle;
p = Roll rate; ψ = Yaw angle; q = Pitch rate; β = Slide-slip angle; r = Yaw rate; α = Angle of attack.

Figure 1. Aircraft motion parameters

Decoupled longitudinal motion is motion in response to a disturbance which is constrained to the longitudinal plane of symmetry, the *oxz* plane, only. The motion is therefore described by the axial force *X*, normal force *Z*, and the pitching moment *M* equations only. Since no lateral motion is involved the lateral motion variables *v*, *p* and *r* and their derivatives are all zero. Also, decoupled longitudinal - lateral motion means that the aerodynamic coupling derivatives are negligibly small and may be taken as zero. The parameters for calculation are shown in Figure 1.

Thus, the equation of longitudinal motion referred to the body axes, taking into account zero initial conditions and since small perturbation only is considered possible to write:

$$\begin{cases} m\dot{u} - \overset{o}{X}_u u - \overset{o}{X}_{\dot{w}} \dot{w} - \overset{o}{X}_w w - \left(\overset{o}{X}_q - mW_e \right) q + mg\theta \cos\theta_e = \overset{o}{X}_\eta \eta + \overset{o}{X}_\tau \tau \\ -\overset{o}{Z}_u u + \left(m - \overset{o}{Z}_{\dot{w}} \right) \dot{w} - \overset{o}{Z}_w w - \left(\overset{o}{Z}_q + mU_e \right) q + mg\theta \sin\theta_e = \overset{o}{Z}_\eta \eta + \overset{o}{Z}_\tau \tau \\ -\overset{o}{M}_u u - \overset{o}{M}_{\dot{w}} \dot{w} - \overset{o}{M}_w w + I_y \dot{q} - \overset{o}{M}_q q = \overset{o}{M}_\eta \eta + \overset{o}{M}_\tau \tau \end{cases} \tag{1a}$$

Equations (1a) are the most general form of the dimensional decoupled equations of longitudinal symmetric motion referred to aeroplane body axes. In (1a) the dimensional variables are denoted by $\overset{o}{X}$.
If it is assumed that the aeroplane is in level flight and the reference axes are wind or stability axes then

$$\theta_e = W_e = 0 \rightarrow \cos\theta_e = 1 \text{ and } \sin\theta_e = 0, \text{ and } Z_q = Z_{\dot{w}} = 0$$

And the equations simplify further to

$$\begin{cases} m\dot{u} - \overset{o}{X}_u u - \overset{o}{X}_{\dot{w}} \dot{w} - \overset{o}{X}_w w - \overset{o}{X}_q q + mg\cos\theta = \overset{o}{X}_\eta \eta + \overset{o}{X}_\tau \tau \\ -\overset{o}{Z}_u u + \left(m - \overset{o}{Z}_{\dot{w}} \right) \dot{w} - \overset{o}{Z}_w w - \left(\overset{o}{Z}_q + mU_e \right) q = \overset{o}{Z}_\eta \eta + \overset{o}{Z}_\tau \tau \\ -\overset{o}{M}_u u - \overset{o}{M}_{\dot{w}} \dot{w} - \overset{o}{M}_w w + I_y \dot{q} - \overset{o}{M}_q q = \overset{o}{M}_\eta \eta + \overset{o}{M}_\tau \tau \end{cases} \tag{2a}$$

Variables η , τ in equation (2a) stand for elevator and throttle deflection, respectively. Since the longitudinal motion of aeroplane is described by four state variables u, w, q and θ and four differential equations are required. Thus the additional equation is the auxiliary equation relating pitch rate to attitude rate, which small perturbation is $\dot{\theta} = q$.

The motion, or state, of any linear dynamic system may be described by a minimum set of variables called the state variables. The number of state variables required to completely describe the motion of the system is dependent on the number of degrees of freedom the system has. Thus the motion of the system is described in a multidimensional vector space called the state space, thee number of state variables being equal to the number of dimensions. The equation of motion, or state equation, of the linear time invariant (LTI) multi-variable system is written:

$$\dot{x}(t) = Ax(t) + B\eta(t)$$
$$y(t) = Cx(t) + D\eta(t)$$

Hence, in terms of dimensionless derivatives the equations of longitudinal motion in the state space form is written as follows [1,2]:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & U_0 & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} U_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} +$$

$$+\begin{bmatrix} X_{\delta_e} & X_{\delta_{thl}} \\ Z_{\delta_e} & Z_{\delta_{thl}} \\ M_{\delta_e}+M_{\dot{w}}Z_{\delta_e} & M_{\delta_{thl}}+M_{\dot{w}}Z_{\delta_{thl}} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta\delta_e \\ \Delta\delta_{thl} \end{bmatrix},$$

where the state space vector *x* and control vector η are given by

$$x=\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} \quad \eta=\begin{bmatrix} \Delta\delta_e \\ \Delta\delta_{thl} \end{bmatrix}$$

2.2. Design the control system using the pole point assignment method.
Consider the control system in the form of state variables as follows:

$$\begin{aligned} \dot{x} &= Ax+Bu \\ y &= Cx \end{aligned} \qquad (1)$$

Here A is the state matrix of size $(n\times n)$, B is the input matrix of size $(n\times r)$, and C is the output matrix of the system.
The important thing in state control is the controllability and observability of the object. To check, we compare the states of the following two matrices with the rank of the system's state matrix A.

$$P_B = [B\ AB\ \dots\ A^{n-1}B];\ P_C = [\,C^T\ \ A^TC^T \dots \left(A^T\right)^{n-1}C^T\,]$$

If the rank of matrix P_B is equal to the rank of matrix A, then we say that the object is controllable, and if the rank of matrix P_C is equal to the rank of matrix A, then the object is completely observable.
The characteristic equation of the open system (1) is

$$a(s)=\det[sI-A]=s^n+a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+\dots++a_1s+a_0 \qquad (2)$$

Here I is the unit matrix of size $(n\times n)$ and a_i coefficients of the original characteristic polynomial. If the dynamical system defined by matrices of A, B, and C is controllable, characteristic polynomial equation of the closed loop control system can be set using state feedback matrix of K.

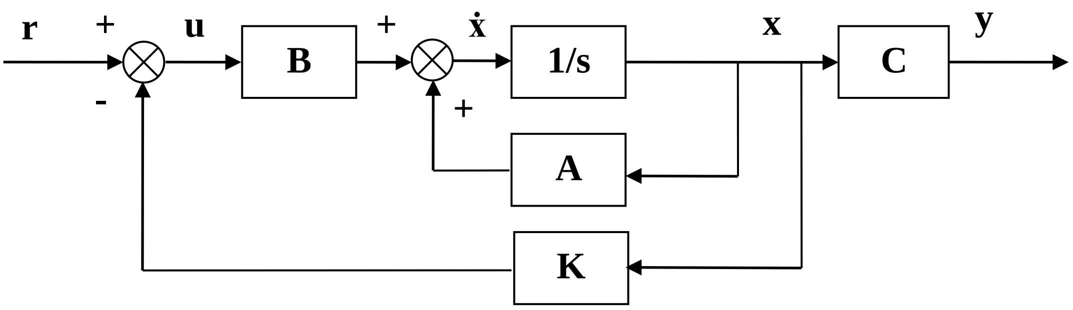


Figure 2. Closed-loop circuit structure diagram

Here K is a constant feedback gain vector.
Starting from the structure diagram of the closed-loop control system (Figure 2), we can determine

$$u=r-Kx \qquad (3)$$

The characteristic polynomial of the closed loop control system defined by equation (3) is as follows by can be expressed in the controllable canonical form, i.e.:

$$\begin{aligned} \dot{x} &= A_Cx+B_Cu \\ y &= C_Cx \end{aligned}$$

The state matrix A and matrix B of the open-loop state equation system (2) can be written as follows:

$$A_C = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_{n-1} & -a_{n-2} & -a_{n-3} & -a_{n-4} & \cdots & -a_0 \end{bmatrix}; \; B_C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Consider the block diagram of the system with state feedback control below:

Here r is the input signal of the closed system, and K is the feedback amplification factor.

$$K = [k_{n-1} \dots . k_0]$$

With the control structure diagram of the closed loop circuit and from (1), (3), the state equation system has the following form:

$$\begin{aligned} \dot{x} &= \left[A_C - B_C K\right] x + B_C r \\ y &= C_C x \end{aligned} \tag{4}$$

The A-BK state matrix of the closed-loop state equation system (4) can be written as follows:

$$A_C - B_C K = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_{n-1} - k_{n-1} & -a_{n-2} - k_{n-2} & -a_{n-3} - k_{n-3} & -a_{n-4} - -k_{n-4} & \cdots & -a_0 - k_0 \end{bmatrix}$$

The characteristic equation of the closed system (4) is

$$\begin{aligned} a(s) &= \det[sI - A_C + B_C K] \\ &= s^n + (a_{n-1} + k_{n-1}) s^{n-1} + (a_{n-2} + k_{n-2}) s^{n-2} + \dots + (a_1 + k_1) s + (a_0 + k_0) \end{aligned} \tag{5}$$

With closed-loop pole positions $p_1, p_2, p_3, \cdots, p_n$, the desired characteristic equation will be:

$$\begin{aligned} a(s) = \det[sI - A_C + B_C K] &= (s - p_1)(s - p_2)(s - p_3) \cdots (s - p_n) \\ &= s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \dots + b_1 s + b_0 \end{aligned} \tag{6}$$

The purpose of design is to find K so that the characteristic equation for the control system is identical to the desired characteristic equation. Therefore, vector K is obtained by unifying the coefficients of equations (5) and (6):

$$a_{i-1} + k_{i-1} = b_{i-1} \text{ với } i = 1 \dots n$$

Note that if the state model is not in normal form, we can use transformation techniques to transform the given state model into normal form by using the non-degenerate matrix T. The coefficient K obtained for this model is then back-transformed to fit the original model. This transformation is performed according to the Bass-Garu transformation [3].

$$x = T\widehat{x}$$

we have

$$\frac{d\widehat{x}}{dt} = T^{-1} \frac{dx}{dt} = T^{-1}(A_C x + B_C u) = T^{-1} A_C T \widehat{x} + T^{-1} B_C u$$

Use the Bass-Garu transformation to determine the transformation matrix T so that the matrix $T^{-1}AT$ has the same normal form. Then the T matrix will be calculated according to the following formula:

$$T = P_B W$$

where P_B is the matrix to check the controllability of the system and W is the matrix of the form

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & a_2 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Then the control law of the closed-loop circuit with r = 0 is

$$u = \hat{K}\hat{x} = -(\hat{K}T^{-1})x = -Kx$$

2.3. Method of selecting pole points

We want the system to achieve a certain quality in terms of overshoot and settling time. In terms of kinetics, the closed system will be equivalent to a certain sample stage [6, 7].

Consider a general object with a transfer function.

$$H(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0} \text{ với } n \ge m$$

The transfer function $H(s)$ can be written in the following approximate form:

$$H(s) \cong \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{1}{T^2 s^2 + 2\zeta T s + 1} \quad (7)$$

where $T^2 = \frac{a_2}{b_0}$ and $2\zeta T = \frac{a_1}{b_0}$.

The solution of the characteristic polynomial of (7) has the following form:

$$p_{1,2} = -\frac{\zeta}{T} \pm j\frac{1}{T}\sqrt{1-\zeta^2} \quad (8)$$

And the overcorrection degree and overcorrection time are calculated according to the formula.

$$\Delta h(\%) = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right);\ t_{xl(2\%)} = \frac{4T}{\zeta} \quad (9)$$

The pole points are selected as follows [5]:

A pair of poles with dominant characteristics determines the entire dynamic characteristics of the entire system.

All remaining poles are chosen far enough away so as not to affect the dominant poles.

From the requirements for the quality of the closed system, we can deduce the requirements for the characteristics of the second order stage. We can determine the parameters T, and ζ from these two parameters, we can find the pair of poles of the second order oscillation stage. is the dominant pair of poles in the system. The remaining pole points of the system (if any) will be chosen to be 5 to 10 times farther from the virtual axis than the dominant pair of pole points and have no imaginary component.

2.4. Calculation and simulation results

Specific research with light aircraft, parameters are referenced from document [2]. The flight condition assumed corresponds with Mach 2.0 at an altitude of 60,000 ft.

The parameters in the aircraft edge motion state equation system according to equation (1a) are

$$A = \begin{bmatrix} -0.00871 & -0.019 & -135 & -32.12 \\ -0.0117 & -0.311 & 1931 & -2.246 \\ 0.000471 & -0.00673 & -0.182 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix};\ B = \begin{bmatrix} 6.24 \\ -89.2 \\ -9.8 \\ 0 \end{bmatrix};\ C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ D = 0.$$

From the pole point distribution chart of the open system (Figure 3), we see that the system has 4 pole points with values (-0.245 ± 0.54i; -0.245 ± 0.54i; -0.00592+0.0215i; -0.00592+0.0215i). In which there is a pair of symmetric poles, we can find a pair of dominant solutions to replace this pair of solutions, and we see that the open system has one solution located close to the virtual axis, so the stability of the open system is not possible. stable.

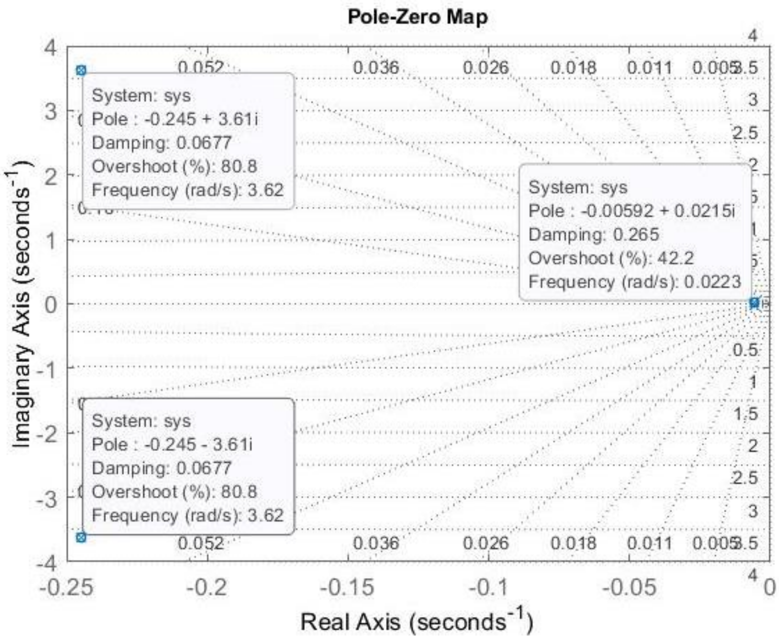


Figure 3. Polarity distribution diagram of an open-loop control circuit vertical motion channel of the aircraft

From the transient characteristics (Figure 4) of the open-loop circuit of the aircraft longitudinal motion channel, we see that the motion parameters do not meet the stability standards of an automatic control system.

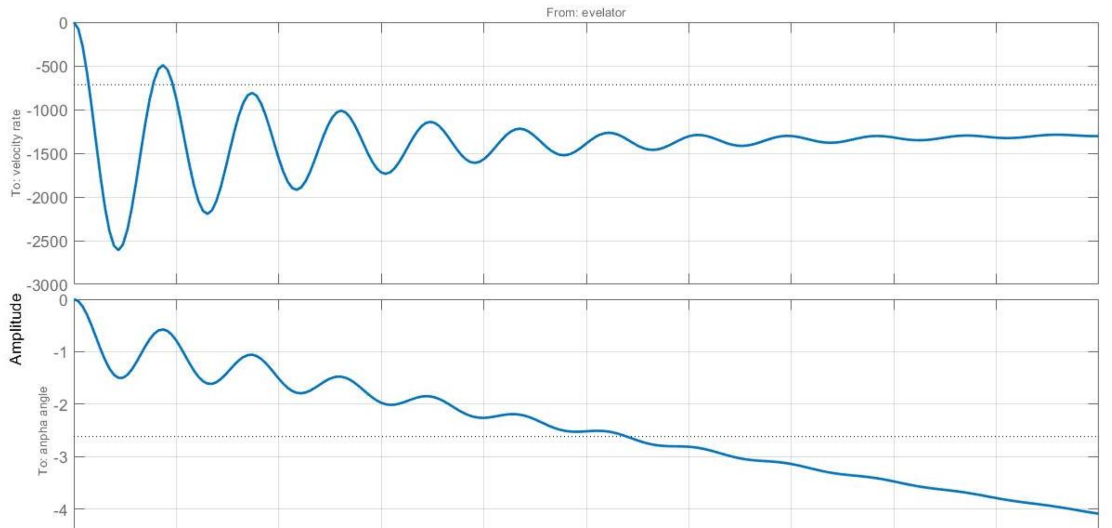


Figure 4. Transient characteristics of the open-loop longitudinal channel of an aircraft

The task is to design the system after having a quality state controller, as follows:
- Adjustment overshoot: 10%;
- Setup time: 10 s.

From formula (10), we can determine that the dominant pair of extreme points of the system is $p_{1,2} = -0,4 \pm i0,547$ and the remaining pair of valid solutions is $p_3 = -3{,}28$; $p_4 = -5{,}47$.

We can determine which match K

$$K = \begin{bmatrix} 0.1139 & -0.0146 & -0.7177 & 42.6209 \end{bmatrix}$$

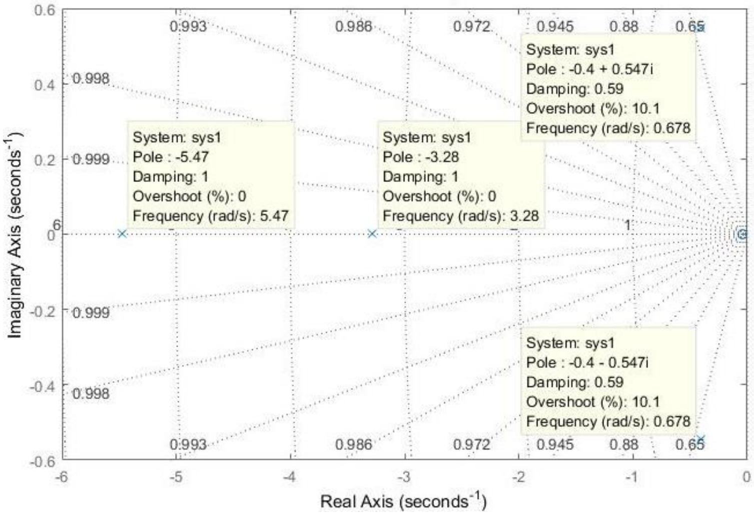


Figure 5. Polarity distribution diagram of a closed-loop control circuit vertical motion channel of the aircraft

With the received K value, we have a polar distribution chart of a closed system (Figure 5) with 4 pole points, including 1 pair of pole points that are symmetrical and the remaining 2 pole points located on the real axis and far away from the imaginary axis. These 4 poles are all located to the left of the virtual axis, ensuring the stable reserve of the closed system. The transient characteristics of the closed system (Figure 6) fully meet the stability standards required by the design.

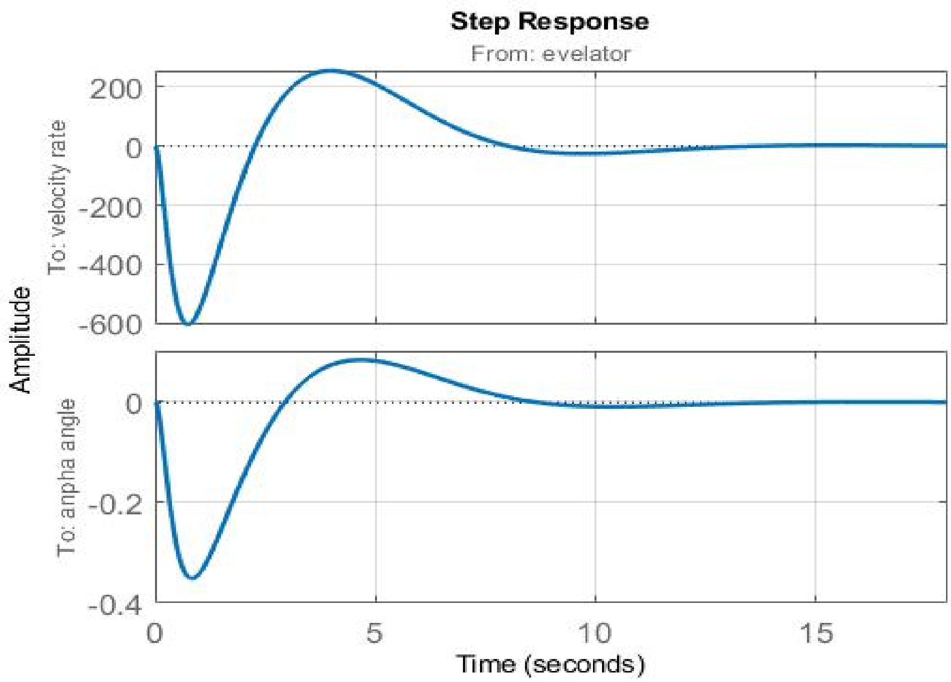


Figure 6. Transient characteristics of the closed-loop longitudinal channel of an aircraft

III. Conclusion

This article presents a calculation method to design a system controller that meets preset standards by using the pole point assignment method. This method is applied to calculate the gain response coefficient matrix of a closed system based on the mathematical model of an open system. This is a simple method to design a controller for a linear system that is highly effective.

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