International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 13, Issue 1 [January. 2024] PP: 70-89

Optimization of the compressive strength of Palm Kernel Shell lightweight Aggregate concrete Using Scheffe`s Modeling Theory

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ABSTRACT: *The environmental consequence of natural aggregate exploitation and Agro-base waste management especially open-air incineration is immense. In the construction industry, the role played by structural light weight concrete cannot be quantified enough especially in the case of high-rise buildings. The light weight concrete can be produced only by using light weight aggregates such as palm kernel shells in concrete mixture, and the introduction of this light weight aggregate in concrete is cost saving in construction. In this research, Henry Scheffe`s regression theory was used to develop a mathematical model to predict and optimize the compressive strength of palm kernel shells aggregate concrete.A total of seventy (70) cubes were produced for the experimental process for the thirty-five (35) points used to determine the coefficients of the model, while fifteen mix ratios consisting of a total of thirty (30) cubes were used for validation of the model. The mathematical model results obtained conform favourably with the experimental results. Validation and test of adequacy of the model was based on statistical analysis for the control points using ANOVA where the adequacy was tested using student t-test and fisher f-test at 95% confidence level and found to be adequate.A compressive strength of 5.45N/mm^2 corresponding to mix ratio of 0.5875: 1.0: 1.73755: 2.4625 for water, cement, fine aggregates and PKS respectively, was predicted by the model and laboratory strength of 3.80Mpa all of which are less than the minimum of 17.2Mpa specified for structural lightweight concrete. A wolframl computer program was coded to obtain the optimized compressive strength of the palm kernel shells concrete.*

Date of Submission: 02-01-2024 Date of acceptance: 14-01-2024

I. INTRODUCTION

Concrete has wide area of application in Civil Engineering and building works and due to urbanization especially in developing countries, the use of concrete product has continued to be on its increase; leading to increased depletion of natural resources and thereby distorting the ecological balance. [1]. Concrete is a very variable material having a wide range of strength and concrete generally increases its strength with age. Concrete is the most widely used construction material worldwide, which is due to its versatility, strength, durability and ease to place into forms and shapes [2,3,4]. Concrete is composed principally of aggregate, a Portland or blended cement, and water, and may contain other cementitious materials and/or chemical admixtures. Chemical admixtures used to accelerate, retard, improve workability, reduce mixing water requirements, increase strength, or alter other properties of the concrete. [5,6]. Concrete as a major construction material has a high demand leading to decrease in granite and gravel deposit hence their scarcity and expensive in cost where available[7]. To mitigate against the continuously increasing demand for low cost and environmentally friendly construction materials, while strengthening economic growth and competitiveness, agricultural waste can be used as replacement material in construction industry especially where this waste is in abundant. [8].

Professionals in the built environment has for long period made great efforts towards reducing the environmental impact of the construction process through the use of alternative construction materials such as Agro-based waste, industrial based waste, etc, in recycled formed of which Palm Kernel Shell is prominent.

The management and environmental menace associated with agricultural and industrial waste has been of challenges to engineers and experts in different engineering related fields especially the environmental nuisance caused by the open field deposit incineration of such waste as palm kernel shells among others [9].

Concrete mix design is the technique of aptlychoosing the proportions of constituent materials such as cement, water, fine aggregates, and coarse aggregate and admixtures where conceivable so as to produce concrete satisfying all the required properties for minimum cost. Essentially, two treasured conditions to attain economy in mix design process are the use of locally available materials and adoption of less restraining specification requirements. [10, 11].

With the application of concrete in the construction industries, a workable design mix through optimization yielding conceivable mixture combination of components for the required maximum strength has become imperative. Optimization of the mix proportion in concrete production could beneficially impact on the construction project cost than when trial mix is unceasingly employed,which reduces the waste of individual component materials of concrete compared with experiential methods of trail mix.[12]. The task of concrete mix optimization is to estimate different concrete composition with different composition of aggregate, to choose the best alternatives of mix by comparing their economical and mechanical properties, includingmaterial durability [13].

Simplex is the structural depiction (shape) of lines or planes joining presumed positions of the constituent materials (atoms) of the mixture and are equidistant from each other. The atoms are the constituent components of the mixture and according to Henry Scheffe, the property studied in the mixture depends on the component proportions and not their quantities[14].The choice of the suitable mixture design entailstaking account of some points; such as the number of factors and interactions to be studied, the complexity of each design, the statistical validity and effectiveness of each design, and the ease of execution and cost, time constraints associated with each design. The most recurrently used mixture design types are the simplex lattice design and simplex- centroid design [15].

Scheffe`s optimizationtheory is used to optimize compressive strength of four componentfour-degree (4, 4) polynomial model at 28 days curing when granite is 100% replaced by PKS as coarse aggregate.

The compressive strength is the most common measure for judging the quality of concrete and the characteristics of concrete based on the 28day cube strength. It is what knowing that the concrete strength is normally specified in terms of characteristic strength at a particular given age of the concrete. (this is crushing strength of standard 150mm cubes at an age of 28 days after mixing) (Kong & Evans, 1986)[16]. Compressive and tensile strengths of concrete are important parameters utilized in the analysis and design of concrete members. (Mutiu *et`al,* 2017). [17]

The four-dimensional factor spaces for four componentsfour-degree polynomial regressions for the simplex design are presented in figure below.

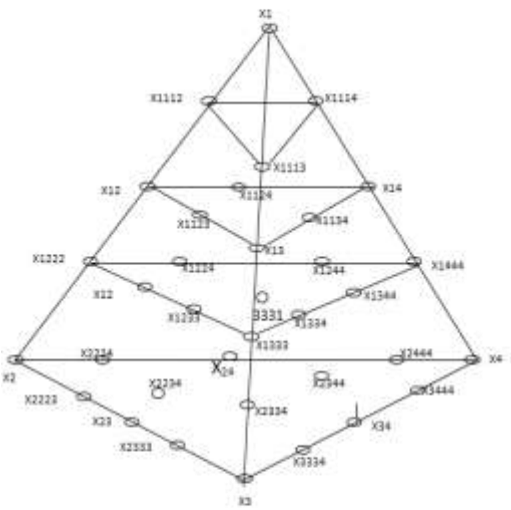

Fig.1: A four-dimensional factor space of four components

II. METHODOLOGY

MATHEMATICAL MODELING AND FORMATION

In concrete mix design, mathematical modelling has various applications with predictive models such as Henry Scheffe`s mixture design model to predict concrete properties like compressive, tensile and flexural strengths being the commonest in application. [**18, 19**]. Scheffe`s simplex model has been applied passably to develop mathematical models for concrete mix to predict and optimize properties of concrete such as the modulus of rupture and flexural strength etc, [**20, 21, 22**]. The recordfrequently used material in the construction industry is concrete and it is required that the strength at 28 days curing be tested for fulfilment of at least a minimum strength of 75% before being put to use. Optimization of the concrete mixture design is a course of

searching for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the obligatory performance of concrete, such as workability, strength and durability [**23**].
As the worth of palm kernel shells in concrete increases, the specific area increases requiring more cement paste for proper bonding because strength requires virtuous bonding of the aggregate and cement. Therefore, as bonding reduces with increase in replacement of palm kernel shells, the compressive strength reduces [7].

INTRODUCTION TO FACTOR SPACE IN SIMPLEX DESIGN

The Scheffe's simplex lattice method is a single step multiple comparison techniqueemploying the use of a single regression polynomial to compare all the constituents in a single step engendering the value of the objective function [**24]**. Scheffe's method stretchespure understanding of how proportioning the constituents of the concrete affect the engineering behaviours. In simplex lattice method to designing experiment to bout mixture problems regarding component property diagrams, the property studied is assumed a continuous function of certain opinions and with a sufficient accuracy it can be approximated with a polynomial. For multi-components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort [24][**25**]. Henry Scheffe developed a theory for experiments with mixture of which the property studied depends on the proportions of the components present and not on the quantity of the mixture. Scheffe showed that if q represents the number of constituent components of the mixture, the space of the variables known also as the factor space is a (q - 1) dimensional simplex lattice. The composition may be expressed as molar, weight, or volume fraction or percentage **[26**]. Simplex is the structural depiction (shape) of lines or planes joining assumed positions of the constituent materials (atoms) of the mixture and are equidistant from each other wherethe atoms are the constituent components of the mixture. **[27]**. The study of the relationship between the compressive strength of concrete and the proportion of water, cement, fine and a coarse aggregate for normal concrete mixture is a good example. Okafor***et'al***2009**[28],** has defined simplex as a convex polyhedron with (K+1) vertices produced by K intersecting hyper planes in k- dimensional space. The hyper planes refer to any co-ordinate system above 3- dimensions. This therefore gives a simplex of a mixture of four components and the simplex lattice of this four-component mixture is a 3- dimensional solid equilateral tetrahedron while 2- dimensional regular simplex is referred to as equilateral triangle. The factor space is a regular (q-1) dimensional simplex, and for the whole factor space of a mix design with evenly spaced distribution of points over the factor space is {q, m}**[25].** Kenneth2019**[29],** Scheffe`s lattice design provides a uniform scatter of points over a (q-1) simplex where the points form a (q-1) lattice on the simplex and q is the number of mixture components while `n` is the degree of the polynomial. Therefore, for binary system (q = 2), the required simplex is a straight line, for (q = 3, the required simplex is an equilateral triangle while for (q = 4), it is a regular tetrahedron.

SCHEFFE`S FACTOR SPACE

The strength of concrete depends on the adequate proportioning of its ingredients (components), and Scheffe developed an optimization theory that was used to optimize the strength of concrete. The Scheffe`s optimization theory can be used to analyse and predict possible mix proportions of concrete ingredient that can estimate/predict a desired concrete strength. The property studied in the mixture depends on the component proportions and not their quantities[**30**]. H. Scheffe 1958**[31**], stated that the property (response) of the mixture is assumed to be a real-value function on a simplex and introduced an appropriate form of polynomial regression model. The polynomial function of degree, n, in the q variables x_1, $x_{2,}$ ... x_q must subject to the constraint that;

$$\sum_{i=1}^{q} x_{i=1} \qquad (1)$$

$$\text{Let } n = 1: f(x) \sum_{i=1}^{q} \beta_i x_i \qquad (2)$$

$$\text{Let } n = 2: f(x) \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \le i \le j \le q}^{q} \beta_{ij} x_i x_j \qquad (3)$$

$$\text{Let } n = 3: f(x) \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \le i \le j \le q}^{q} \beta_{ij} x_i x_j + \sum_{1 \le i \le j \le k \le q} \left(\beta_{iij} x_i^2 + \beta_{iij} x_i x_i x_k\right) \qquad (4)$$

NUMBER OF COEFFICIENT (4, 4)

q = 4, m = 4

$$N = \frac{(q + m - 1)!}{m!\,(q + m - 1 - m)!}, \quad N = \frac{(4 + 4 - 1)!}{4!\,(4 - 1)!}, \quad N = \frac{7!}{4!\,3!} = 35 \qquad (5)$$

FOUR COMPONENT FACTOR SPACE

Infour-component factor space, there are thirty-five (35) design points with the first four pseudo components located at the vertices of the quartic simplex; while the remaining mix ratios are located at the mid-points of the lines joining the vertices of the simplex as presented below.

A1[1,0,0,0]	**A2**[0,1,0,0]	**A3**[0,0,1,0]	**A4**[0,0,0,1]	**A12**[0.5,0.5,0,0]
A13[0.5,0,0.5,0]	**A14**[0.5,0,0,0.5]	**A23**[0,0.5,0.5,0]	**A24**[0,0.5, 0, 0.5,]	**A34**[0,0,0.5,0.5]
A1112[0.75, 0.25, 0, 0]	**A1113** [0.75, 0, 0.25, 0]	A1114 [0.75, 0, 0, 0.25]	**A2223** [0, 0.75, 0.25, 0]	**A3334** [0, 0, 0.75, 0.25]
A1222 [0.25, 0.75,0,0]	**A3331** [0.25, 0, 0.75, 0]	**A1444** [0.25, 0, 0, 0.75]	A1233[0.25,0.25,0.5,0]	**A1344**[0.25,0,0.25,0.5]
A1123[0.5,0.25,0.25,0]	**A1134**[0.5,0,0.25,0.25]	**A2344**[0,0.25,0.25,0.5]	**A2234**[0,0.5,0.25,0.25]	**A1124**[0.5,0.25,0,0.25]
A1244[0.25,0.25,0,0.5]	**A2224**[0,0.75,0,0.25]	**A2333**[0,0.25,0.75,0]	**A2444**[0,0.25,0,0.75]	**A3444**[0,0,0.25,0.75]
A2334[0,0.25,0.5,0.25]	**A1223**[0.25,0.5,0.25,0]	**A1334**[0.25,0,0.5,0.25]	**A1224**[0.25,0.5,0,0.25]	**A1234**[0.25,0.25,0.25,0.25]

RESPONSES

The constituent elements for a normal concrete mixture are; water, cement, fine and coarse aggregates which thus give a simplex of a mixture of four components. The simplex lattice of this four-component mixture is a three- dimensional solid equilateral tetrahedron. The mixture components here are subjected to Scheffe`s constraint that the sum of all the components must be equal to unity (one). If q is the total components and xi is the proportion of the components of the i^{th} component in the mixture, is such that;

$$Xi \geq 0 \ (i = 1, 2 - - - q) \text{ and } 0 \leq Xi \leq 1 \quad (6)$$

Thus, the sum of the component constituent proportion is whole unity (one), and

$$X_1 + X_2 + X_3 + X_4 = 1 \text{ or, that } \sum Xi - 1 = 0 \quad (7)$$

$$Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \cdots + \sum b_{i1}, i2 \ldots i_n x_{i1} x_{i2} x_{in} \quad (8)$$

with, $1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i1 \leq i1 - - - \leq in \leq q$ and bo is the constant coefficient.

In a {4, 4} lattice which is a four components and four-degree polynomial lattice there exists five levels desired as follows: $X_4 = 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4},$ equally expressed as 0, 0.25, 0.5, 0.75, 1.0

The general form of the Polynomial is given as,

$$\acute{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 \quad (9)$$

While,

$$X1+X2+X3 = 1 \quad (10)$$

And,

$$b_0x_1 + b_0x_2 + b_0x_3 = b_0 \quad (11)$$

Multiplying eq. (10) in success by X1+X2+X3,

$$\left.\begin{aligned} x_1^2 &= x_1 - x_1x_2 - x_1x_3 \\ x_2^2 &= x_2 - x_1x_2 - x_2x_3 \\ x_3^2 &= x_3 - x_1x_3 - x_2x_3 \end{aligned}\right\} \quad (12)$$

Substituting eq. (11) and (12) into eq. (9) and after essential transformation will yield

$$\acute{y} = (b_0 + b_1 + b_{11})\, x_1 + (b_0 + b_2 + b_{22})\, x_2 + (b_0 + b_3 + b_{33})\, x_3 + (b_{12} - b_{11} - b_{22})\, x_1x_2 + (b_{13} - b_{11} - b_{33})\, x_1x_3 + (b_{23} - b_{22} - b_{33})\, x_2x_3 \quad (13)$$

Means that; $\alpha_i = b_0 + b_i + b_{ii}$, and $\alpha_{ij} = b_{ij} - b_{ii} - b_{jj}$ 14)

Giving the reduced second-degree Polynomial in three variables

$$\acute{y} = \alpha_1x_1 + \alpha_2x_2 + \alpha_3x_3 + \alpha_{12}x_1x_2 + \alpha_{13}x_1x_3 + \alpha_{23}x_3\, x_3 \quad (15)$$

The general form for the reduced Polynomial of (4, 4) model can be represented below **[32]**

$$\begin{aligned} \acute{y} &= \alpha_1x_1 + \alpha_2x_2 + \alpha_3x_3 + \alpha_4x_4 + \alpha_{12}x_1x_2 + \alpha_{13}x_1x_3 + \alpha_{14}x_1x_4 + \alpha_{23}x_2x_3 + \alpha_{24}x_2x_4 + \alpha_{34}x_3x_4 \\ &+ \lambda_{12}x_1x_2\,(x_1-x_2) + \lambda_{13}x_1x_3\,(x_1-x_3) + \lambda_{14}\,x_1x_4\,(x_1-x_4) + \lambda_{23}x_2x_3\,(x_2-x_3) + \lambda_{24}x_2x_4\,(x_2-x_4) \\ &+ \lambda_{34}x_3x_4\,(x_3-x_4) + \mu_{12}x_1x_2\,(x_1-x_2)^2 + \mu_{13}x_1x_3\,(x_1-x_3)^2 + \mu_{14}x_1x_4\,(x_1-x_4)^2 + \mu_{23}x_2x_3\,(x_2-x_3)^2 \\ &+ \mu_{24}x_2x_4\,(x_2-x_4)^2 + \mu_{34}x_3x_4\,(x_3-x_4)^2 + \alpha_{1123}x_1^2x_2x_3 + \alpha_{1124}x_1^2x_2x_4 + \alpha_{1134}x_1^2x_3x_4 + \alpha_{1223}x_1x_2^2x_3 \\ &+ \alpha_{1224}x_1x_2^2x_4 + \alpha_{2234}x_2^2x_3x_4 + \alpha_{1334}x_1x_3^2x_4 + \alpha_{2334}x_2x_3^2x_4 + \alpha_{1233}x_1x_2x_3^2 + \\ &\alpha_{1244}x_1x_2x_4^2 + \alpha_{1344}x_1x_3x_4^2 + \alpha_{2344}x_2x_3x_4^2 + \alpha_{1234}x_1x_2x_3x_4 \end{aligned} \quad (16)$$

To obtain the value of the coefficients, we substitute in succession the coordinates of all the thirty-five points of the design matrix in eq. (16)

The general equations for the coefficients are generated follows:

$$\alpha_i = y_i \quad (17)$$

$$\alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (18)$$

$$\lambda_{ij} = (8/3)\,(-y_i + 2y_{iiij} - 2y_{ijjj} + y_j) \quad (19)$$

$$\mu_{ij} = (8/3)(-y_i + 4y_{iiij} - 6y_{ij} + 4y_{ijjj} - y_j) \quad (20)$$

$$\alpha_{iijk} = 32(3y_{iijk} - y_{ijjk} - y_{ijkk}) + (8/3)(6y_i - y_j - y_k) - 16(y_{ij} + y_{ik}) - (16/3)(5y_{iiij} + 5y_{iiik} + 3y_{ijjj} - 3y_{ikkk} - y_{jjjk} - y_{jkkk}) \quad (21)$$

$$\alpha_{ijjk} = 32(3y_{ijjk} - y_{iijk} - y_{ijkk}) - (8/3)(6y_j - y_i - y_k) - 16(y_{ij} + y_{jk}) - (16/3)(5y_{ijjj} + 5y_{jjjk} - 3y_{iiij} - 3y_{jkkk} - y_{iiik} - y_{ikkk} \quad (22)$$

$$\alpha_{ijkk} = 32(3y_{ijkk} - y_{iijk} - y_{ijjk}) + (8/3)(6y_k - y_i - y_j) - 16(y_{ik} + y_{jk}) - (16/3)(5y_{ikkk} + 5y_{jkkk} - 3y_{iiik} - 3y_{jjjk} - y_{iiij} - y_{ijjj}) \quad (23)$$

$$\alpha_{ijkl} = 256y_{ijkl} - 32(y_{iijk} + y_{iikl} + y_{ijjk} + y_{ijjl} + y_{jjkl} + y_{ijkk} + y_{ikkl} + y_{ijll} + y_{jkll} + y_{ijll} + y_{jkll} + y_{ikll}) + (32/3)(y_{iiij} + y_{iiik} + y_{iiil} + y_{ijjj} + y_{jjjk} + y_{jjjl} + y_{ikkk} + y_{jkkk} + y_{kkkl} + y_{illl} + y_{jlll} + y_{klll}) \quad (24)$$

ACTUAL COMPONENT AND PSEUDO COMPONENT

$$\text{Let } AZ = AX \quad (25)$$

Where; Z represent the actual components and X represents the pseudo components and A is a constant for a four-by-four matrix. The value of the matrix A is obtained from the first four mix ratios with the corresponding pseudo components as;

Z1 [0.65:1.0:2.0:2.85]; Z2 [0.60:1.0:1.75:2.5]; Z3 [0.55:1.0:1.55:2.2]; Z4 [0.70:1.0:2.3:3.25]

Corresponding mix ratios;

X1[1:0:0:0]; X2[0:1:0:0]; X3[0:0:1:0]; X4[0:0:0:1];

The actual mixture components can be determined using the corresponding pseudo components when xi and zi are substituted in eq. (17 - 24)

X1 = fraction of water-cement ratio

X2 = fraction of cement

X3 = fraction of fine aggregate

X4 = fraction of palm kernel shell

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix}$$

For the first run,

$$\begin{bmatrix} 0.65 \\ 1.0 \\ 2.0 \\ 2.85 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\mathbf{a_{11}}$= 0.65, $\mathbf{a_{21}}$ =1.0, $\mathbf{a_{31}}$ =2.0, $\mathbf{a_{41}}$ = 2.85

For the second run

$$\begin{bmatrix} 0.60 \\ 1.0 \\ 1.75 \\ 2.5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

a_{12} = 0.60, a_{22} =1.0,a_{32} = 1.75, a_{42} = 2.5

For the third run

$$\begin{bmatrix} 0.55 \\ 1.0 \\ 1.55 \\ 2.2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

a_{13} = 0.55, a_{23} = 1.0, a_{33} = 1.55, a_{43} = 2.2

For the fourth run

$$\begin{bmatrix} 0.70 \\ 1.0 \\ 2.3 \\ 3.25 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

a_{14} = 0.70, a_{24} = 1.0, a_{34} = 2.3, a_{44} = 3.25

We will have [A] matrix when the values of the constants are substituted.

$$\begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix}$$

For A_{12};

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.5 +0.60*0.5 = 0.625

Z2 = 1.0*0.5 + 1.0*0.5 = 1.0

Z3 = 2.0*0.5 +1.75*0.5 = 1.875

Z4 = 2.85*0.5 + 2.5*0.5 = 2.675

For A_{13}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.5 +0.55*0.5 = 0.60

Z2 = 1.0*0.5 + 1.0*0.5 = 1.0

Z3 = 2.0*0.5 +1.55*0.5 = 1.775

For A_{14}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

Z1 = 0.65*0.5 +0.70*0.5 = 0.675
Z2 = 1.0*0.5 + 1.0*0.5 = 1.0
Z3 = 2.0*0.5 +2.3*0.5 = 2.15
Z4 = 2.85*0.5 + 3.25*0.5 = 3.05

For A_{24}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0.5 \end{bmatrix}$$

Z1 = 0.60*0.5 +0.70*0.5 = 0.65
Z2 = 1.0*0.5 + 1.0*0.5 = 1.0
Z3 = 1.75*0.5 +2.3*0.5 = 2.025
Z4 = 2.5*0.5 + 3.25*0.5 = 2.875

For A_{1112}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.75 \\ 0.25 \\ 0 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.75 +0.60*0.25 = 0.6375
Z2 = 1.0*0.75 + 1.0*0.25 = 1.0
Z3 = 2.0*0.75 +1.75*0.25 = 1.9375
Z4 = 2.85*0.75 + 2.5*0.25 = 2.7625

For A_{1114}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.75 \\ 0 \\ 0 \\ 0.25 \end{bmatrix}$$

Z1 = 0.65*0.75 +0.70*0.25 = 0.6625
Z2 = 1.0*0.75 + 1.0*0.25 = 1.0
Z3 = 2.0*0.75 +2.3*0.25 = 2.075
Z4 = 2.85*0.75 + 3.25*0.25 = 2.95

For A_{3334}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0.75 \\ 0.25 \end{bmatrix}$$

Z1 = 0.55*0.75 +0.70*0.25 = 0.5875
Z2 = 1.0*0.75 + 1.0*0.25 = 1.0
Z3 = 1.55*0.75 +2.3*0.25 = 1.7375
Z4 = 2.2*0.75 + 3.25*0.25 = 2.4625

For A_{3331}

Z4 = 2.85*0.5 + 2.2*0.5 = 2.525

For A_{23}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix}$$

Z1 = 0.60*0.5 +0.55*0.5 = 0.575
Z2 = 1.0*0.5 + 1.0*0.5 = 1.0
Z3 = 1.75*0.5 +1.55*0.5 = 1.65
Z4 = 2. 5*0.5 + 2.2*0.5 = 2.35

For A_{34}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Z1 = 0.55*0.5 +0.70*0.5 = 0.625
Z2 = 1.0*0.5 + 1.0*0.5 = 1.0
Z3 = 1.55*0.5 +2.3*0.5 = 1.925
Z4 = 2.2*0.5 + 3.25*0.5 =2.725

For A_{1113}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.75 \\ 0 \\ 0.25 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.75 +0.55*0.25 = 0.625
Z2 = 1.0*0.75 + 1.0*0.25 = 1.0
Z3 = 2.0*0.75 +1.55*0.25 = 1.8875
Z4 = 2.85*0.75 + 2.2*0.25 = 2.6875

For A_{2223}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \\ 0 \end{bmatrix}$$

Z1 = 0.60*0.75 +0.55*0.25 = 0.5875
Z2 = 1.0*0.75 + 1.0*0.25 = 1.0
Z3 = 1.75*0.75 +1.55*0.25 = 1.70
Z4 = 2.5*0.75 + 2.2*0.25 = 2.425

For A_{1222}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.25 \\ 0.75 \\ 0 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.25 +0.60*0.75 =0.6125
Z2 = 1.0*0.25 + 1.0*0.75 = 1.0
Z3 = 2.0*0.25 +1.75*0.75 = 1.8125
Z4 = 2.85*0.25 + 2.5*0.75 = 2.5875

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.25\\0\\0.75\\0\end{bmatrix}$$

Z1 = 0.65*0.25 +0.55*0.75 = 0.575
Z2 = 1.0*0.25 + 1.0*0.75 = 1.0
Z3 = 2.0*0.25 +1.55*0.75 = 1.6625
Z4 = 2.85*0.25 + 2.2*0.75 =2.3625

For A_{1444}

For A_{1233}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.25\\0.25\\0.5\\0\end{bmatrix}$$

Z1 = 0.65*0.25 +0.60*0.25 + 0.55*0.5 = 0.5875
Z2 = 1.0*0.25 + 1.0*0.25 + 1.0*0.5 = 1.0
Z3 = 2.0*0.25 +1.75*0.25 + 1.55*0.5 = 1.7125
Z4 = 2.85*0.25 + 2.5*0.25 + 2.2*0.5 =2.4375

For A_{1344}

For A_{1123}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.5\\0.25\\0.25\\0\end{bmatrix}$$

Z1 = 0.65*0.5 +0.60*0.25 + 0.55 +0.25 = 0.6125
Z2 = 1.0*0.5 + 1.0*0.25 + 1.0*0.25 = 1.0
Z3 = 2.0*0.5 +1.75*0.25 + 1.55*0.25 = 1.825
Z4 = 2.85*0.5 + 2.5*0.25 + 2.2*0.25 = 2.60

For A2344

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0\\0.25\\0.25\\0.5\end{bmatrix}$$

Z1 = 0.60*0.25 +0.55*0.25 + 0.70 +0.5 = 0.6375
Z2 = 1.0*0.25 + 1.0*0.25 + 1.0*0.5 = 1.0
Z3 = 1.75*0.25 +1.55*0.25 + 2.3*0.5 = 1.975
Z4 = 2.5*0.25 + 2.2*0.25 + 3.25*0.5 = 2.80

For A_{1124}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.5\\0.25\\0\\0.25\end{bmatrix}$$

Z1 = 0.65*0.5 +0.60*0.25 + 0.70 +0.25 = 0.65
Z2 = 1.0*0.5 + 1.0*0.25 + 1.0*0.25 = 1.0
Z3 = 2.0*0.5 +1.75*0.25 + 2.3*0.25 = 2.0125
Z4 = 2.85*0.5 + 2.5*0.25 + 3.25*0.25 =2.8625

For A_{2224}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0\\0.75\\0\\0.25\end{bmatrix}$$

Z1 = 0.60*0.75 +0.70 +0.25 = 0.625
Z2 = 1.0*0.75 + 1.0*0.25 = 1.0

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.25\\0\\0\\0.75\end{bmatrix}$$

Z1 = 0.65*0.25 +0.70*0.75 = 0.6875
Z2 = 1.0*0.25 + 1.0*0.75 = 1.0
Z3 = 2.0*0.25 +2.3*0.75 = 2.225
Z4 = 2.85*0.25 + 3.25*0.75 = 3.15

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.25\\0\\0.25\\0.5\end{bmatrix}$$

Z1 = 0.65*0.25 +0.55*0.25 + 0.70 +0.5 = 0.65
Z2 = 1.0*0.25 + 1.0*0.25 + 1.0*0.5 = 1.0
Z3 = 2.0*0.25 +1.55*0.25 + 2.3*0.5 = 2.0375
Z4 = 2.85*0.25 + 2.2*0.25 + 3.25*0.5 = 2.887

For A_{1123}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.5\\0\\0.25\\0.25\end{bmatrix}$$

Z1 = 0.65*0.5 +0.55*0.25 + 0.70 +0.25 = 0.6375
Z2 = 1.0*0.5 + 1.0*0.25 + 1.0*0.25 = 1.0
Z3 = 2.0*0.5 +1.55*0.25 + 2.3*0.25 = 1.9625
Z4 = 2.85*0.5 + 2.2*0.25 + 3.25*0.25 = 2.7875

For A_{2234}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0\\0.5\\0.25\\0.25\end{bmatrix}$$

Z1 = 0.60*0.5 +0.55*0.25 + 0.70 +0.25 = 0.6125
Z2 = 1.0*0.5 + 1.0*0.25 + 1.0*0.25 = 1.0
Z3 = 1.75*0.5 +1.55*0.25 + 2.3*0.25 = 1.8375
Z4 = 2.5*0.5 + 2.2*0.25 + 3.25*0.25 = 2.6125

For A_{1244}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0.25\\0.25\\0\\0.5\end{bmatrix}$$

Z1 = 0.65*0.25 +0.60*0.25 + 0.70 +0.5 = 0.6625
Z2 = 1.0*0.25 + 1.0*0.25 + 1.0*0.5 = 1.0
Z3 = 2.0*0.25 +1.75*0.25 + 2.3*0.5 = 2.0875
Z4 = 2.85*0.25 + 2.5*0.25 + 3.25*0.5 = 2.9625

Z3 = 1.75*0.75 + 2.3*0.25 = 1.8875
Z4 = 2.5*0.75 + 3.25*0.25 = 2.6875

For A_{2333}

$$\begin{bmatrix}Z1\\Z2\\Z3\\Z4\end{bmatrix} = \begin{bmatrix}0.65 & 0.60 & 0.55 & 0.70\\ 1.0 & 1.0 & 1.0 & 1.0\\ 2.0 & 1.75 & 1.55 & 2.3\\ 2.85 & 2.5 & 2.2 & 3.25\end{bmatrix} * \begin{bmatrix}0\\0.25\\0.75\\0\end{bmatrix}$$

Z1 = 0.60*0.25 +0.55 +0.75 = 0.5625
Z2 = 1.0*0.25 + 1.0*0.75 = 1.0
Z3 = 1.75*0.25 + 1.55*0.75 = 1.60
Z4 = 2.5*0.25 + 2.2*0.75 = 2.27

For A2444

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.25 \\ 0 \\ 0.75 \end{bmatrix}$$

Z1 = 0.60*0.25 +0.70 +0.75 = 0.675
Z2 = 1.0*0.25 + 1.0*0.75 = 1.0
Z3 = 1.75*0.25 + 2.3*0.75 = 2.1625
Z4 = 2.5*0.25 + 3.25*0.75 = 3.0625

For A_{3444}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ 0.75 \end{bmatrix}$$

Z1 = 0.55*0.25 +0.70+0.75 = 0.6625
Z2 = 1.0*0.25 + 1.0*0.75 = 1.0
Z3 = 1.55*0.25 + 2.3*0.75 = 2.1125
Z4 = 2.2*0.25 + 3.25*0.75 = 2.9375

For A_{2334}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.25 \end{bmatrix}$$

Z1 = 0.60*0.25 +0.55*0.5+ 0.70 +0.25 = 0.60
Z2 = 1.0*0.25 + 1.0*0.5 + 1.0*0.25 = 1.0
Z3 = 1.75*0.25 +1.55*0.5 + 2.3*0.25 = 1.7875
Z4 = 2.5*0.25 + 2.2*0.5 + 3.25*0.25 = 2.5375

For A_{1223}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.25 \\ 0.5 \\ 0.25 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.25 +0.60*0.5 + 0.55 +0.25 = 0.60
Z2 = 1.0*0.25 + 1.0*0.5 + 1.0*0.25 = 1.0
Z3 = 2.0*0.25 +1.75*0.5 + 1.55*0.25 = 1.7625
Z4 = 2.85*0.25 + 2.5*0.5 + 2.2*0.25 = 2.445

Z3 = 2.0*0.25 +1.55*0.5 + 2.3*0.25 = 1.85
Z4 = 2.85*0.25 + 2.2*0.5 + 3.25*0.25 =2.625

For A_{1224}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.25 \\ 0.5 \\ 0 \\ 0.25 \end{bmatrix}$$

Z1 = 0.65*0.25 +0.60*0.5 + 0.70 +0.25 = 0.6375
Z2 = 1.0*0.25 + 1.0*0.5 + 1.0*0.25 = 1.0
Z3 = 2.0*0.25 +1.75*0.5 + 2.3*0.25 = 1.95
Z4 = 2.85*0.25 + 2.5*0.5 + 3.25*0.25 = 2.775

For A_{1334}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.25 \\ 0 \\ 0.5 \\ 0.25 \end{bmatrix}$$

Z1 = 0.65*0.25 +0.55*0.5 + 0.70 +0.25 = 0.6063
Z2 = 1.0*0.25 + 1.0*0.5 + 1.0*0.25 = 1.0

For A_{1234}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

Z1 = 0.65*0.25 +0.60*0.25 + 0.55*0.25 +0.70+0.25 = 0.625
Z2 = 1.0*0.25 + 1.0*0.25 + 1.0*0.5 + 1.0*0.25 = 1.0
Z3 = 2.0*0.25 +1.75*0.25 + 1.55*0.25+ 2.3*0.25 = 1.90
Z4 = 2.85*0.25 + 2.5*0.25 + 2.2*0.25 +3.25*0.25 =2.70

MIXTURE PROPORTIONS FOR CONTROL POINTS SHOWING ACTUAL AND PSEUDO COMPONENTS

Control points for A_1

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.2 +0.60*0.4+0.55*0.4 = 0.590
Z2 = 1.0*0.2+1.0*0.4+1.0*0.4 = 1.0
Z3 = 2.0*0.2+1.75*0.4+1.55*0.4 = 1.720
Z4 = 2.85*0.2+2.5*0.4+2.2*0.4 =2.450

Control points for A_2

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.2 \\ 0.4 \\ 0 \\ 0.4 \end{bmatrix}$$

Z1 = 0.65*0.2 +0.60*0.4+0.70*0.4 = 0.650

Control points A_3

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.2 \\ 0 \\ 0.4 \\ 0.4 \end{bmatrix}$$

Z1 = 0.65*0.2+0.55*0.4 +0.70*0.4 = 0.634
Z2 = 1.0*0.2+1.0*0.4+1.0*0.4 = 1.0
Z3 = 2.0*0.2+1.55*0.4 +2.3*0.4 = 1.940
Z4 = 2.85*0.2+2.2*0.4 +3.25*0.4= 2.750

Control points A_4

Control points A_5

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.4 \\ 0.4 \\ 0.2 \end{bmatrix}$$

Z1 = 0.60*0.4+0.55*0.4 +0.70*0.2= 0.600
Z2 = 1.0*0.4+1.0*0.4 +1.0*0.2 = 1.0
Z3 = 1.75*0.4+1.55*0.4+2.3*0.2 = 1.78
Z4 = 2.5*0.4+2.2*0.4 +3.25*0.2 = 2.530

Control points A_7

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.4 \\ 0.4 \\ 0 \\ 0.2 \end{bmatrix}$$

Z1 = 0.65*0.4+0.60*0.4 +0.70*0.2= 0.640
Z2 = 1.0*0.4+1.0*0.4 +1.0*0.2 = 1.0
Z3 = 2.0*0.4+1.75*0.4+ 2.3*0.2 = 1.960
Z4 = 2.85*0.4+2.5*0.4 +3.25*0.2 = 2.790

Control points A_9

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.4+0.60*0.4 +0.55*0.2= 0.610
Z2 = 1.0*0.4+1.0*0.4 +1.0*0.2 = 1.0
Z3 = 2.0*0.4+1.75*0.4+ 1.55*0.2 = 1.810
Z4 = 2.85*0.4+2.5*0.4 +2.2*0.2 = 2.580

Control pointsA_{10}

Control points A_{11}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.35 \\ 0.35 \\ 0.3 \end{bmatrix}$$

Z1 = 0.60*0.35+0.55*0.35 +0.70*0.3 = 0.6125
Z2 = 1.0*0.35+1.0*0.35 +1.0*0.3 = 1.0
Z3 = 1.75*0.35+1.55*0.35+ 2.3*0.3 = 1.8450
Z4 = 2.5*0.35+2.2*0.35 +3.25*0.3 = 2.620

Control points A_{13}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.35 \\ 0.35 \\ 0 \\ 0.3 \end{bmatrix}$$

Z1 = 0.65*0.35+0.60*0.35 +0.70*0.3= 0.6475
Z2 = 1.0*0.35+1.0*0.35 +1.0*0.3= 1.0

Z2 = 1.0*0.2+1.0*0.4+1.0*0.4 = 1.0
Z3 = 2.0*0.2+1.75*0.4+2.3*0.4 = 2.020
Z4 = 2.85*0.2+2.5*0.4+3.25*0.4 = 2.870

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$$

Z1 = 0.60*0.2+0.55*0.4 +0.70*0.4 = 0.620
Z2 = 1.0*0.2+1.0*0.4+1.0*0.4 = 1.0
Z3 = 1.75*0.2+1.55*0.4 + 2.3*0.4 = 1.890
Z4 = 2.5*0.2+2.2*0.4 +3.25*0.4 = 2.680

Control points A_6

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.4 \\ 0 \\ 0.4 \\ 0.2 \end{bmatrix}$$

Z1 = 0.65*0.4+0.55*0.4 +0.70*0.2= 0.620
Z2 = 1.0*0.4+1.0*0.4 +1.0*0.2 = 1.0
Z3 = 2.0*0.4+1.55*0.4+2.3*0.2 = 1.880
Z4 = 2.85*0.4+2.2*0.4 +3.25*0.2 =2.670

Control points A_8

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.4+0.60*0.4 +0.55*0.2= 0.610
Z2 = 1.0*0.4+1.0*0.4 +1.0*0.2 = 1.0
Z3 = 2.0*0.4+1.75*0.4+ 1.55*0.2 = 1.810
Z4 = 2.85*0.4+2.5*0.4 +2.2*0.2 = 2.580

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

Z1 = 0.65*0.25+0.60*0.25+0.55*0.25+0.70*0.25= 0.6250
Z2 = 1.0*0.25+1.0*0.25 +1.0*0.25+1.0*0.25 = 1.0
Z3 = 2.0*0.25+1.75*0.25+1.55*0.25+ 2.3*0.25 = 1.90
Z4 = 2.85*0.25+2.5*0.25 +2.280.25 +3.25*0.25 = 2.70

Control points A_{12}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.3 \\ 0 \\ 0.35 \\ 0.35 \end{bmatrix}$$

Z1 = 0.65*0.3+0.55*0.35 +0.70*0.35= 0.6325
Z2 = 1.0*0.3+1.0*0.35 +1.0*0.35= 1.0
Z3 = 2.0*0.3+1.55*0.35+ 2.3*0.35 = 1.9475
Z4 = 2.85*0.3+2.2*0.35+3.25*0.35 = 2.7625
Z3 = 2.0*0.35+1.75*0.35+ 2.3*0.3 = 2.0025
Z4 = 2.85*0.35+2.5*0.35+3.25*0.3 = 2.8475

Control points A_{14}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0.3 \\ 0.35 \\ 0.35 \\ 0 \end{bmatrix}$$

Z1 = 0.65*0.3+0.60*0.35 +0.55*0.35= 0.5975
Z2 = 1.0*0.3+1.0*0.35 +1.0*0.35= 1.0
Z3 = 2.0*0.3+1.75*0.35+ 1.55*0.35 = 1.755
Z4 = 2.85*0.3+2.5*0.35+2.2*0.35 = 2.50

Control points A_{15}

$$\begin{bmatrix} Z1 \\ Z2 \\ Z3 \\ Z4 \end{bmatrix} = \begin{bmatrix} 0.65 & 0.60 & 0.55 & 0.70 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 \\ 2.85 & 2.5 & 2.2 & 3.25 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.3 \\ 0.35 \\ 0.35 \end{bmatrix}$$

Z1 = 0.60*0.3+0.55*0.35 +0.70*0.35= 0.6175
Z2 = 1.0*0.3+1.0*0.35 +1.0*0.35= 1.0
Z3 = 1.75*0.3+1.55*0.35+ 2.3*0.35 = 1.8726
Z4 = 2.5*0.3+2.2*0.35+3.25*0.35 = 2.6575

III. MATERIALS AND METHOD

MATERIALS

Dangote3x Ordinary Portland cement product conforming to BS and ASTM standards with minimum rate of hardening, andnatural river sharp sand used as fine aggregate were obtained in kaura Namoda, Zamfara state, Nigeria; with maximum size fine aggregate being 4.75mm and treated to be free from impurities. Palm kernel shell used as 100% replacement for granite, was obtained from palm oil producing areas of Edo state, Nigeria. The crushed shells were dried and sieved to get rid of impurities. The water used for mixing and curing is potable drinking water from bored hole at the Federal Polytechnic Kaura Namoda, Zamfara state,and tested to be suitable for concrete work.

METHODS

Required quantity of the constituent materials were first thoroughly dry mixed in a manually operated concrete mixer before water is added for wet mixing and casting of cubes for the strength test.

The compressive strength test was performed in accordance with BS 1881- 116 and ACI 311.6 – 18 specifications using Magnus Compression Testing Machine and concrete cubes of sizes of (150x150x150) mm. A total of seventy (70) cubes were produced for the experimental process for the thirty-five (35) points, while fifteen points of control was considered and a total of thirty cubes were produced as control.

Two specimen of each mix was crushed at the curing regime of 28 days and the average recorded as the strength achieved.

The compressive strength was determined using the equation; Cs = $\frac{P}{A}$ where,

Table 1: Matrix for Scheffe`s (4,4) Lattice Polynomial

	PSEUDO					ACTUAL			
POINTS	W/C	CEMENT	SAND	PKS	RESPONSE	W/C	CEMENT	SAND	PKS
	X1	X2	X3	X4	Y	Z1	Z2	Z3	Z4
1	1	0	0	0		0.65	1	2	2.85
	0	1	0	0		0.6	1	1.75	2.5
3	0	0	1	0		0.55	1	1.55	2.2
4	0	0	0	1		0.7	1	2.3	3.25
5	0.5	0.5	0	0		0.625	1	1.875	2.675
6	0.5	0	0.5	0		0.6	1	1.775	2.525
7	0.5	0	0	0.5		0.675	1	2.15	3.05
8	0	0.5	0.5	0		0.575	1	1.65	2.35
9	0	0.5	0	0.5		0.65	1	2.025	2.875
10	0	0	0.5	0.5		0.625	1	1.925	2.725
11	0.75	0.25	0	0		0.6375	1	1.9375	2.7625
12	0.75	0	0.25	0		0.625	1	1.8875	2.6875
13	0.75	0	0	0.25		0.6625	1	2.075	2.95
14	0	0.75	0.25	0		0.5875	1	1.7	2.425
15	0	0	0.75	0.25		0.5875	1	1.7375	2.4625

16	0.25	0.75	0	0		0.6125	1	1.8125	2.5875
17	0.25	0	0.75	0		0.575	1	1.6625	2.3625
18	0.25	0	0	0.75		0.6875	1	2.225	3.15
19	0.25	0.25	0.5	0		0.5875	1	1.7125	2.4375
20	0.25	0	0.25	0.5		0.65	1	2.0375	2.8875
21	0.5	0.25	0.25	0		0.6125	1	1.825	2.6
22	0.5	0	0.25	0.25		0.6375	1	1.9625	2.7875
23	0	0.25	0.25	0.5		0.6375	1	1.975	2.8
24	0	0.5	0.25	0.25		0.6125	1	1.8375	2.6125
25	0.5	0.25	0	0.25		0.65	1	2.0125	2.8625
26	0.25	0.25	0	0.5		0.6625	1	2.0875	2.9625
27	0	0.75	0	0.25		0.625	1	1.8875	2.6875
28	0	0.25	0.75	0		0.5625	1	1.6	2.275
29	0	0.25	0	0.75		0.675	1	2.1625	3.0625
30	0	0	0.25	0.75		0.6625	1	2.1125	2.9875
31	0	0.25	0.5	0.25		0.6	1	1.7875	2.5375
32	0.25	0.5	0.25	0		0.6	1	1.7625	2.5125
33	0.25	0	0.5	0.25		0.6063	1	1.85	2.625
34	0.25	0.5	0	0.25		0.6375	1	1.95	2.775
35	0.25	0.25	0.25	0.25		0.625	1	1.9	2.7

Table 2: Mixture Proportion of Control points showing pseudo and actual

Points	**Pseudo**					**Actual**			
	w/c	Cement	Sand	PKS	Response	w/c	Cement	Sand	PKS
	X1	X2	X3	X4	Yc	Z1	Z2	Z3	Z4
1	0.2	0.4	0.4	0	Yc_1	0.59	1	1.72	2.45
2	0.2	0.4	0	0.4	Yc_2	0.65	1	2.02	2.87
3	0.2	0	0.4	0.4	Yc_3	0.63	1	1.94	2.75
4	0	0.2	0.4	0.4	Yc_4	0.62	1	1.89	2.68
5	0	0.4	0.4	0.2	Yc_5	0.6	1	1.78	2.53
6	0.4	0	0.4	0.2	Yc_6	0.62	1	1.88	2.67
7	0.4	0.4	0	0.2	Yc_7	0.64	1	1.96	2.79
8	0.4	0.4	0.2	0	Yc_8	0.61	1	1.81	2.58
9	0.4	0.4	0.2	0	Yc_9	0.61	1	1.81	2.58
10	0.25	0.25	0.25	0.25	Yc_{10}	0.625	1	1.9	2.7
11	0	0.35	0.35	0.3	Yc_{11}	0.6125	1	1.845	2.62
12	0.3	0	0.35	0.35	Yc_{12}	0.6325	1	1.9475	2.7625
13	0.35	0.35	0	0.3	Yc_{13}	0.6475	1	2.0025	2.8475
14	0.3	0.35	0.35	0	Yc_{14}	0.5975	1	1.755	2.5
15	0	0.3	0.35	0.35	Yc_{15}	0.6175	1	1.8726	2.6575

IV. RESULTS AND DISCUSSION

The results of the compressive strength for palm kernel shells concrete were obtained from laboratory test on 150x150x150 concrete cube specimens after 28 days curing using compression machine. The results are presented in tables 3 and 4, for the control points.

Table 3: Uniaxial Compressive Strength of Concrete

Sample Points	Curing Age	Failure Load (KN) A	B	Area (mm^2)	Compressive Strength(Nmm^{-2}) MPa A	B	Average
N_1	28days	35	40	22500	1.556	1.778	1.667
N_2	28days	40	45	22500	1.778	2.000	1.889
N_3	28days	85	86	22500	3.778	3.822	3.800
N_4	28days	35	40	22500	1.556	1.778	1.667
N_5	28days	60	45	22500	2.667	2.000	2.334
N_6	28days	65	50	22500	2.889	2.222	2.556

N_7	28days	60	60	22500	2.667	2.667	2.667
N_8	28days	65	70	22500	2.889	3.111	3.000
N_9	28days	45	50	22500	2.000	2.222	2.111
N_{10}	28days	50	50	22500	2.222	2.222	2.222
N_{11}	28days	60	65	22500	2.667	2.889	2.778
N_{12}	28days	55	60	22500	2.444	2.667	2.556
N_{13}	28days	55	50	22500	2.444	2.222	2.333
N_{14}	28days	60	65	22500	2.667	2.889	2.778
N_{15}	28days	70	60	22500	3.111	2.667	2.889
N_{16}	28days	70	75	22500	3.111	3.333	3.222
N_{17}	28days	65	50	22500	2.889	2.222	2.556
N_{18}	28days	55	50	22500	2.444	2.222	2.333
N_{19}	28days	70	70	22500	3.111	3.111	3.111
N_{20}	28days	60	55	22500	2.667	2.444	2.556
N_{21}	28days	35	50	22500	1.556	2.222	1.889
N_{22}	28days	45	55	22500	2.000	2.444	2.222
N_{23}	28days	50	60	22500	2.222	2.667	2.445
N_{24}	28days	70	65	22500	3.111	2.889	3.000
N_{25}	28days	60	60	22500	2.667	2.667	2.667
N_{26}	28days	55	60	22500	2.444	2.667	2.556
N_{27}	28days	65	60	22500	2.889	2.667	2.778
N_{28}	28days	60	65	22500	2.667	2.889	2.778
N_{29}	28days	35	40	22500	1.556	1.778	1.667
N_{30}	28days	55	60	22500	2.444	2.667	2.556
N_{31}	28days	50	60	22500	2.222	2.667	2.445
N_{32}	28days	65	70	22500	2.889	3.111	3.000
N_{33}	28days	60	60	22500	2.667	2.667	2.667
N_{34}	28days	65	70	22500	2.889	3.111	3.000
N_{35}	28days	65	55	22500	2.889	2.444	2.667

Table 4: (4, 4) Lattice Control Mix. (28 days compressive Strength of PKS Concrete)

Sample Points	Curing age	Failure Loads (KN)		Area (mm^2)	Compressive Strength (N/mm^2) Mpa		
		A	B		A	B	Average
C_1	28 (days)	38	40	22500	1.689	1.778	1.733
C_2	28	86	86	22500	3.822	3.822	3.822
C_3	28	65	55	22500	2.889	2.444	2.667
C_4	28	60	60	22500	2.667	2.667	2.667
C_5	28	45	50	22500	2.000	2.222	2.111
C_6	28	60	65	22500	2.667	2.889	2.778
C_7	28	60	65	22500	2.667	2.889	2.778
C_8	28	65	50	22500	2.889	2.222	2.556
C_9	28	70	70	22500	3.111	3.111	3.111
C_{10}	28	46	55	22500	2.044	2.444	2.244
C_{11}	28	70	65	22500	3.111	2.889	3.000
C_{12}	28	50	60	22500	2.000	2.667	2.333
C_{13}	28	38	40	22500	1.689	1.778	1.733
C_{14}	28	60	60	22500	2.667	2.667	2.667
C_{15}	28	65	57	22500	2.889	2.533	2.711

REGRESSION EQUATION FOR COMPRESSIVE STRENGTH

The coefficients of the fourth-degree polynomial are determined as presented below

α1 = 1.667	α2 = 1.889	α3 = 3.8	α4 = 1.667	α12 =2.222	α13 = -1.156	α14 = -0.266
α23 = 0.622	α24 = -2.49	α34 =-2.046	λ12 =-1.776	λ13 = 5.688	λ14 = 5.921E-16	λ23 = 5.096
λ24 = 5.925	λ34 =23.349	μ12 = 17.181	μ13 = -0.949	μ14= -1.792	μ23 = 1.781	μ24 = 3.561
μ34 = 7.944	α1123=-101.085	α1124=-17.176	α1134=33.395	α1223=17.419	α1224 = 14.464	α1233 = 79.747
α1234=5.872	α1244 = 11.835	α1334 = 40.096	α1344 = 3.200	α2234 = 7.107	α2334 = -13.864	α2344= 10.325

Substituting thesevalues of coefficients into Scheffe`s model equation will give;

$$\hat{y} = 1.667x_1 + 1.889x_2 + 3.8x_3 + 1.667x4 + 2.222x_1x_2 - 1.156x_1x_3 - 0.266x_1x_4 + 0.622x_2x_3 - 2.49x_2x_4 - 2.046x_3x_4 - 1.776x_1x_2(x_1-x_2) + 5.688x_1x_3(x_1-x_3) + 5.921E\text{-}16x_1x_4(x_1-x_4) + 5.096x_2x_3(x_2-x_3) + 5.925x_2x_4(x_2-x_4) + 2\ 3.349x_3x_4(x_3-x_4) + 17.181x_1x_2(x_1-x_2)^2 - 0.949x_1x_3(x_1-x_3)^2 - 1.792x_1x_4(x_1-x_4)^2 + 1.781x_2x_3(x_2-x_3)^2 + 3.561x_2x_4(x_2-x_4)^2 + 7.944x_3x_4(x_3-x_4)^2 - 101.085x_1^2x_2x_3 - 17.176x_1^2x_2x_4 + 33.395x_1^2x_3x_4 + 17.419x_1x_2^2x_3 + 14.464x_1x_2^2x_4 + 7.107x_2^2x_3x_4 + 40.096x_1x_3^2x_4 - 13.864x_2x_3^2x_4 + 79.747x_1x_2x_3^2$$

$+ 11.835x_1x_2x_4^2 + 3.20x_1x_3x_4^2 + 10.325x_2x_3x_4^2 + 5.872x_1x_2x_3x_4$

This is the improved model for the optimization of the compressive strength of palm kernel shell concrete using Scheffe`s fourth degree polynomials.

REPLICATE VARIANCE

Table 5: Compressive Strength Test Results and Replication Variance of Response

No. of Expt. Pts (N)	Replicates	Response yi (N/mm²)	Response Symbol	$\sum_{i=1}^{m_i} y_i$	ỹ	$\sum_{i=1}^{m_i} y_i^2$	S_i^2
1	1P 1Q	1.556 1.778	y_1	3.334	1.667	5.5824	0.0246
2	2P 2Q	1.778 2	y_2	3.778	1.889	7.1613	0.0246
3	3P 3Q	3.778 3.822	y_3	7.6	3.8	28.8810	0.0010
4	4P 4Q	1.556 1.778	y_4	3.334	1.667	5.5824	0.0246
5	5P 5Q	2.667 2	y_{12}	4.667	2.3335	11.1129	0.2224
6	6P 6Q	2.889 2.222	y_{13}	5.111	2.5555	13.2836	0.2224
7	7P 7Q	2.667 2.667	y_{14}	5.334	2.667	14.2258	0.0000
8	8P 8Q	2.889 3.111	y_{23}	6	3	18.0246	0.0246
9	9P 9Q	2 2.222	y_{24}	4.222	2.111	8.9373	0.0246
10	10P 10Q	2.222 2.222	y_{34}	4.444	2.222	9.8746	0.0000
11	11P 11Q	2.667 2.889	y_{1112}	5.556	2.778	15.4592	0.0246
12	12P 12Q	2.444 2.667	y_{1113}	5.111	2.5555	13.0860	0.0249
13	13P 13Q	2.444 2.222	y_{1114}	4.666	2.333	10.9104	0.0246
14	14P 14Q	2.667 2.889	y_{2223}	5.556	2.778	15.4592	0.0246
15	15P 15Q	3.111 2.667	y_{3334}	5.778	2.889	16.7912	0.0986
16	16P 16Q	3.111 3.333	y_{1222}	6.444	3.222	20.7872	0.0246

17	17P 17Q	2.889 2.222	y_{1333}	5.111	2.5555	13.2836	0.2224
18	18P 18Q	2.444 2.222	y_{1444}	4.666	2.333	10.9104	0.0246
19	19P 19Q	3.111 3.111	y_{1233}	6.222	3.111	19.3566	0.0000
20	20P 20Q	2.667 2.444	y_{1344}	5.111	2.5555	13.0860	0.0249
21	21P 21Q	1.556 2.222	y_{1123}	3.778	1.889	7.3584	0.2218
22	22P 22Q	2 2.444	y_{1134}	4.444	2.222	9.9731	0.0986
23	23P 23Q	2.222 2.667	y_{2344}	4.889	2.4445	12.0502	0.0990
24	24P 24Q	3.111 2.889	y_{2234}	6	3	18.0246	0.0246
25	25P 25Q	2.667 2.667	y_{1124}	5.334	2.667	14.2258	0.0000
26	26P 26Q	2.444 2.667	y_{1244}	5.111	2.5555	13.0860	0.0249
27	27P 27Q	2.889 2.667	y_{2224}	5.556	2.778	15.4592	0.0246
28	28P 28Q	2.667 2.889	y_{2333}	5.556	2.778	15.4592	0.0246
29	29P 29Q	1.556 1.778	y_{2444}	3.334	1.667	5.5824	0.0246
30	30P 30Q	2.444 2.667	y_{3444}	5.111	2.5555	13.0860	0.0249
31	31P 31Q	2.222 2.667	y_{2334}	4.889	2.4445	12.0502	0.0990
32	32P 32Q	2.889 3.111	y_{1223}	6	3	18.0246	0.0246
33	33P 33Q	2.667 2.667	y_{1334}	5.334	2.667	14.2258	0.0000
34	34P 34Q	2.889 3.111	y_{1224}	6	3	18.0246	0.0246
35	35P 35Q	2.889 2.444	y_{1234}	5.333	2.6665	14.3195	0.0990
36	36P 36Q	1.689 1.778	C1	3.467	1.7335	6.0140	0.0040

37	37P 37Q	3.822 3.822	C2	7.644	3.822	29.2154	0.0000
38	38P 38Q	2.889 2.444	C3	5.333	2.6665	14.3195	0.0990
39	39P 39Q	2.667 2.667	C4	5.334	2.667	14.2258	0.0000
40	40P 40Q	2 2.222	C5	4.222	2.111	8.9373	0.0246
41	41P 41Q	2.667 2.889	C6	5.556	2.778	15.4592	0.0246
42	42P 42Q	2.667 2.889	C7	5.556	2.778	15.4592	0.0246
43	43P 43Q	2.889 2.222	C8	5.111	2.5555	13.2836	0.2224
44	44P 44Q	3.111 3.111	C9	6.222	3.111	19.3566	0.0000
45	45P 45Q	2.044 2.444	C10	4.488	2.244	10.1511	0.0800
46	46P 46Q	3.111 2.889	C11	6	3	18.0246	0.0246
47	47P 47Q	2 2.667	C12	4.667	2.3335	11.1129	0.2224
48	48P 48Q	1.689 1.778	C13	3.467	1.7335	6.0140	0.0040
49	49P 49Q	2.667 2.667	C14	5.334	2.667	14.2258	0.0000
50	50P 50Q	2.889 2.533	C15	5.422	2.711	14.7624	0.0634
						□ S_i^2 =	**2.6717**

The mean response Y and variance of replicates S_i^2 presented in table 5 above are obtained as follow;

$$Y = \frac{\sum_{i=1}^{n} Y_i}{n}$$

$$S_i^2 = \left\lfloor \frac{1}{n-1} \right\rfloor \left[\sum Y_i^2 - \left[\frac{1}{n(\sum Y_i)^2} \right] \right]$$

Therefore,

$$S_i^2 = \left\lfloor \frac{1}{n-1} \right\rfloor \left[\sum_{i=1}^{n} [Y_i - Y]^2 \right]$$

$$V_e = \left(\sum N \right) - 2 \quad = 50 - 2 = 48$$

$$\therefore \text{Replicate variance,} \quad S_i^2 = \frac{2.6717}{48} = 0.0557$$

Replicate error $S_{i=\sqrt{0.0557}}$ = 0.236

The experimental results and Scheffe`s model test results for the concrete compressive strength are presented as shown in table 6

Table 6: Experimental and Predicted values of 28 days compressive Strength of PKS Concrete

Sample Points	Compressive strength Y_{exp}(N/mm^2)	Compressive strength Y_{Pred}(N/mm^2)
N_1	1.667	1.667
N_2	1.889	1.889
N_3	3.800	3.800
N_4	1.667	1.667
N_5	2.334	2.334
N_6	2.556	2.445
N_7	2.667	1.601
N_8	3.000	3.000
N_9	2.111	1.156
N_{10}	2.222	2.222
N_{11}	2.778	2.778
N_{12}	2.556	2.472
N_{13}	2.333	1.533
N_{14}	2.778	3.045
N_{15}	2.889	5.445
N_{16}	3.222	3.222
N_{17}	2.556	2.472
N_{18}	2.333	1.533
N_{19}	3.111	3.123
N_{20}	2.556	1.782
N_{21}	1.889	1.857
N_{22}	2.222	2.911
N_{23}	2.445	1.011
N_{24}	3.000	2.419
N_{25}	2.667	1.813
N_{26}	2.556	1.509
N_{27}	2.778	2.089
N_{28}	2.778	3.045
N_{29}	1.667	0.867
N_{30}	2.556	0.000
N_{31}	2.445	3.021
N_{32}	3.000	3.063
N_{33}	2.667	3.836
N_{34}	3.000	2.315
N_{35}	2.667	2.418
C_1	1.733	1.667
C_2	3.822	3.380
C_3	2.667	1.818
C_4	2.667	2.747
C_5	2.111	1.929
C_6	2.778	2.830
C_7	2.778	3.505
C_8	2.556	2.032
C_9	3.111	2.110
C_{10}	2.244	2.110
C_{11}	3.000	2.418
C_{12}	2.333	2.275
C_{13}	1.733	2.908
C_{14}	2.667	1.855
C_{15}	2.711	2.798

V. VALIDATION AND TEST OF ADEQUACY OF MODEL

Statistical analysis using student`s t-test and ANOVA was employed to analyse the improved Scheffe`s model where the adequacy of the model was tested against the experimental results of the control points. The predicted values (Y-predicted) for the test control points were determined after substituting the corresponding values of X_1, X_2, X_3, and X_4, into Scheffe`s model equation. The predicted values and the experimental values (Y- pred. and Y-expt), were compared. To test for adequacy of the model, student`s t-test and ANOVA were used at 95% confidence level on the compressive strength at the control points of C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}.

Table 7 : Student t-test for 28 days compressive strength of concrete

		Compressive Strength (N/mm^2)		Students t-test		
Points	**Curing Age**	y_{expt}	y_{pred}	$Y = y_{expt} - y_{pred}$	Y_a-Y	$(Y_a-Y)^2$
C_1	28 DAYS	1.733	3.380	-1.647	1.920	3.687
C_2	28 DAYS	3.822	1.818	2.004	-1.731	2.995
C_3	28 DAYS	2.667	2.747	-0.080	0.353	0.125
C_4	28 DAYS	2.667	1.929	0.738	-0.465	0.216
C_5	28 DAYS	2.111	2.830	-0.719	0.993	0.985
C_6	28 DAYS	2.778	3.505	-0.727	1.000	1.000
C_7	28 DAYS	2.778	2.032	0.746	-0.472	0.223
C_8	28 DAYS	2.556	2.110	0.446	-0.173	0.030
C_9	28 DAYS	3.111	2.110	1.001	-0.728	0.530
C_{10}	28 DAYS	2.244	2.418	-0.174	0.447	0.200
C_{11}	28 DAYS	3	2.275	0.725	-0.452	0.204
C_{12}	28 DAYS	2.333	2.908	-0.575	0.848	0.720
C_{13}	28 DAYS	1.733	1.855	-0.122	0.395	0.156
C_{14}	28 DAYS	2.667	2.798	-0.131	0.405	0.164
C_{15}	28 DAYS	2.711	2.010	0.701	-0.428	0.183
TOTAL				**2.186**		**11.418**
AVERAGE (Y_a)				**0.273**		

$$t_{stat} = \frac{\sum(\text{exp - pred})}{\sqrt{\frac{\left(15*\sum\left((\text{exp-pred})^2\right)-\left(\sum(\text{exp - pred})\right)\right)^2}{15-1}}} = \frac{(2.186)}{\sqrt{\frac{(15*11.418)-(2.186^2)}{(14)}}} = \mathbf{0.634}$$

At 95% confidence level the significant level is 0.05. For the two-tailed t-test, the significant level α = 0.05 and 0.025 and using table 7, t_{stat} is calculated. $t_{critical} = 2.145$
$t_{sat.}$ = 0.634 and $t_{crit.}$ = 2.145, implies that $t_{sat} < t_{crit.,}$ but between – 2.145 and 2.145, indication of a good correlation

ANALYSIS OF VARIANCE

From the result in table 9, F = 0.7293andF$_{crit}$ = 4.2582 (F- distribution table). This means that $F_{crit.} > F$ hence, there is no significant difference between the experimental and the model results. The model is therefore adequate to use in predicting the split tensile strength when the mix ratio is known and vice versa.

Table 8: The Compressive Strength's ANOVA

Point	(Expt.)	(Pred.)	$(Expt.)^2$	$(Pred.)^2$
C1	1.733	3.380	3.003	11.424
C2	3.822	1.818	14.608	3.305
C3	2.667	2.747	7.113	7.546
C4	2.667	1.929	7.113	3.721
C5	2.111	2.830	2.456	8.009
C12	2.778	3.505	7.717	12.285
C13	2.778	2.032	7.717	4.129
C14	2.556	2.110	6.533	4.452
C15	3.111	2.110	9.678	4.452
C23	2.244	2.418	5.036	5.848
C24	3	2.275	9.000	5.176
C25	2.333	2.908	5.443	8.456

C34	1.733	1.855	3.003	3.441
C35	2.667	2.798	7.113	7.829
C45	2.711	2.010	7.350	4.040
Total	**38.911**	**36.725**	**102.883**	**94.113**

N = total scores =30; K=2; Df_b = K – 1= 1; Df_w =N – K =28

$$SS_b=\frac{(\sum(exp))^2}{15}+\frac{(\sum(pred))^2}{15}-\frac{((\sum(exp))+(\sum(pred)))^2}{30}$$

$$=\frac{(38.911)^2}{15}+\frac{(36.725)^2}{15}-\frac{((38.911)+(36.725))^2}{30}=190.853 - 190.693=\mathbf{0.16}$$

$$SS_w=\left(\sum(exp)^2\right)+\left(\sum(pred)^2\right)-\frac{(\sum(exp))^2}{15}+\frac{(\sum(pred))^2}{15}$$

$$=102.883+94.113-\frac{((38.911))^2}{15}+\frac{((36.725))^2}{15}=196.996-190.853 = \mathbf{6.143}$$

$$MS_b=\frac{SS_b}{Df_b}=\frac{0.16}{1}=\mathbf{0.16}$$

$$MS_w=\frac{SS_w}{Df_w}=\frac{6.143}{28}=\mathbf{0.2194}$$

$$F=\frac{MS_b}{MS_W}=\frac{0.16}{0.2194}=\mathbf{0.7293}$$

Fcrit = 4.2582 (F- distribution table).

Table 9: Summary of ANOVA

Groups	Count	Sum	Average
Expt.	15	38.911	2.5941
Predict	15	36.725	2.4483

ANOVA

Source of Variance	SS	df	MS	F	Fcrit
Between Groups	0.16	1	0.16	0.7293	4.2582
Within Groups	6.143	28	0.2194		
Total	**6.303**	**29**			

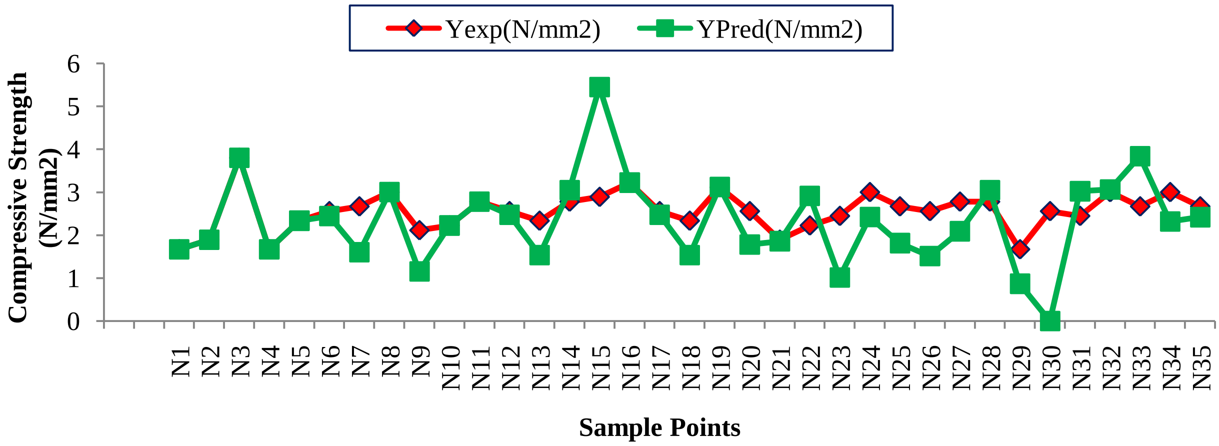

Figure 5.1 Experiment vs Model Compressive strength

VI. DISCUSSION OF RESULTS

Slump value averaged 0.00 mm for granite replacement rate of 100% using PKS. This low slump value may be due high water absorption capability hence taking up the required water content from the mix design, and more water will be required in such a situation in line with [29] The average density of concrete based on a 100% replacement was determined to be 1468 kg/m^3 hence; the concrete can be classified as lightweight concrete [4, 35, 37, 42]. Using the equation produced by Scheffe's simplex model, the strength of light-weight aggregate concrete including palm kernel shells was optimized. The strengths of different mix ratios may be predicted by the model and vice versa. The batch with point (N15) and mix ratio of (0.5875: 1.0: 1.7375: 2.4625) for water, cement, fine aggregates, and PKS, respectively, has the highest predicted compressive strength of 5.45N/mm^2 according to the model results in tables 3.1 and 4.1. The laboratory values for compressive strength were found to be between (1.67 N/mm^2 - 3.8 N/mm^2). Both the laboratory and model predicted compressive strengths are far lower than the minimum standard of 17.2Mpa specified for structural lightweight concrete in [35, 42].

The low compressive strength in this research may be attributed to the lightweight, shapes and semi-porous nature of PKS aggregate or breakdown of bonds between the aggregates and the paste, failure of shell aggregate and aggregate-paste interface as obtainable in [31]. The statistical tests used in this work demonstrated that Scheffe's model was sufficient for maximizing the compressive strengths of the palm kernel shell concrete. The value of $F_{crit.}$, being greater than $F_{cal.}$, Indicates a good relationship between the experimental and modeled values. The derived model equations were found to be adequate for forecasting the strengths. The statistical tests conducted in this work demonstrated that Scheffe's model was sufficient for optimizing the compressive strengths of the palm kernel shell concrete. A WOLFRAM MATHEMAICAL computer program was used to select the optimized compressive strength of the palm kernel shells concrete as in Appendix I and vice - versa.

The statistical tests used here, proved the adequacy of the Scheffe`s model for the optimization of the compressive strength of the palm kernel shells concrete and the compressive strength of all points in the simplex can be derived using this model. Because of the low compressive strength, the concrete produced in this research cannot be adopted for structural lightweight concrete construction.

VII. CONCLUSION:

The use of palm kernel shells in concrete production contributes in protecting our physical environment as it assists in preventing the depletion of the natural ground, means of waste disposal of the by-product to the areas of their production. Palm kernel shells are adequate for use in the production of light weight concrete. The responses (compressive strength) of the palm kernel shells aggregate concrete can be predicted by the mathematical model when the mix ratio is known or vice- versa.

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OPTIMIZATION MODEL

```
Out[-]= FittedModel[ «47» + «1» + 38 418.7 X3 («1»)^2 X4 + 9019.72 X1 X2 X4^2 + 164 110. X1 X3 X4^2 - 4748.66 X2 X3 X4^2 ]

Maximize[
 {eQnSt2, eQnSt2 == 3.8, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3, X4}]
Minimize[
 {eQnSt2, eQnSt2 == 1.667, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3, X4}]
Out[47]= {3.80001, {X1 → 0.0859379, X2 → 0.797634, X3 → 0.10659, X4 → 0.00983782}}
Out[48]= {1.66699, {X1 → 0.0992361, X2 → 0.782031, X3 → 0.116365, X4 → 0.00236768}}

Out[-]= FittedModel[ «47» + «1» + 38 418.7 X3 («1»)^2 X4 + 9019.72 X1 X2 X4^2 + 164 110. X1 X3 X4^2 - 4748.66 X2 X3 X4^2 ]
Maximize[
 {eQnSt2, eQnSt2 == 17.4, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3, X4}]
Minimize[
 {eQnSt2, eQnSt2 == 1.67, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3, X4}]
Out[23]= {17.4, {X1 → 0.0785538, X2 → 0.803111, X3 → 0.101787, X4 → 0.0165481}}
Out[24]= {1.66999, {X1 → 0.0992371, X2 → 0.782029, X3 → 0.116366, X4 → 0.00236785}}

Out[-]= FittedModel[ «47» + «1» + 38 418.7 X3 («1»)^2 X4 + 9019.72 X1 X2 X4^2 + 164 110. X1 X3 X4^2 - 4748.66 X2 X3 X4^2 ]

Maximize[
 {eQnSt2, eQnSt2 == 23, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3,
Minimize[
 {eQnSt2, eQnSt2 == 20, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3,
Out[29]= {23., {X1 → 0.0809488, X2 → 0.800047, X3 → 0.103504, X4 → 0.0155003}}
Out[30]= {20., {X1 → 0.0805991, X2 → 0.800469, X3 → 0.103195, X4 → 0.015737}}

Out[-]= FittedModel[ «47» + «1» + 38 418.7 X3 («1»)^2 X4 + 9019.72 X1 X2 X4^2 + 164 110. X1 X3 X4^2 - 4748.66 X2 X3 X4^2 ]

Maximize[
 {eQnSt2, eQnSt2 == 37, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3, X4}]
Minimize[
 {eQnSt2, eQnSt2 == 22, X1 + X2 + X3 + X4 == 1, X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0}, {X1, X2, X3, X4}]
Out[32]= {37.0002, {X1 → 0.0325433, X2 → 0.71593, X3 → 0.023632, X4 → 0.227894}}
Out[33]= {22., {X1 → 0.0805311, X2 → 0.800483, X3 → 0.103189, X4 → 0.0157972}}
```