*International Journal of Engineering Inventions*
*e-ISSN: 2278-7461, p-ISSN: 2319-6491*
*Volume 13, Issue 3 [March. 2024] PP: 152-158*

# Free Convective Flow of A Visco-Elastic Fluid Bounded By An Oscillating Porous Flat Plate In Slip Flow Regime

**M/S SWAGATIKA DAS[1], Prof.SUSHILA CHAND PRADHAN[2]**
*1.TEACHER EDUCATOR,BIET,GUNUPUR,RAYGADA,ODISHA.*
*2.PRINCIPAL,SGI HIGHER SECONDARY SCHOOL,CUTTACK,ODISHA.*

Date of Submission: 13-03-2024 Date of acceptance: 27-03-2024

## I. INTRODUCTION

The study of fluctuating flow is important in the paper industry and many other technological fields. Also the study of flow and heat transfer in slip flow regime is of great interest due to its application in high speed flight in the upper atmosphere. Therefore, fluctuating flow problems are of great interest to many researchers.

Cookey *et al.* [1] have reported unsteady MHD free-convection and mass transfer flow past an infinite heated porous vertical plate with time dependant suction. Das *et al.* [2] have analyzed free convection flow and mass transfer of an elastico-viscous fluid past an infinite vertical porous plate in a rotating porous medium. Chaudhary and Chand [3] have studied hydromagnetic flow past a long vertical channel embedded in porous medium with transpiration cooling. Kurtcebe and Erim [4] have analyzed heat transfer of a visco-elastic fluid in a porous channel. Panda *et al.* [5] have studied free convection of conducting viscous fluid between two vertical walls filled with a porous material. Sharma and Yadav [6] have reported steady MHD boundary layer flow and heat transfer between two long vertical wavy walls. Dash[7] has studied the effects of radiation and chemical reaction in MHD flow past a stretched vertical permeable surface through a porous medium with constant suction. Sharma *et al.* [8] have analyaed the steady MHD flow and heat transfer between two rotating porous disk. Sharma and Sharma [9] have studied the effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium. Das and Panda [10] have reported the effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified fluid past a porous flat moving plate in the slip flow .

Panda and Das[11] have analyzed the MHD free convection flow of a particulate suspension past an infinite porous inclined flat plate with heat absorption.

The aim of the present problem is to study and bring out the effect of rarefaction parameter R, Hartman number M, Schmidt number Sc and Grashof number Gr, on velocity, concentration and skin friction of the MHD free convective flow of visco-elastic fluid (Walters B′Model)[12] through a porous medium bounded by an oscillating porous flat plate in slip flow regime.

**FORMULATION OF THE PROBLEM**

The physical configuration consists of an unsteady flow of an electrically conducting and incompressible elastico-viscous liquid of Walters B′ model [12] with simultaneous heat and mass transfer near an oscillating infinite porous flat plate in slip flow regime under the influence of a transverse magnetic field of uniform strength. The x-axis is taken along the flat plate in horizontal direction and y-axis is perpendicular to it. A uniform magnetic field of strength $B_0$ is applied in the direction of y – axis. For problems in aeronautical engineering the Reynolds number is usually small. Under the condition, the induced magnetic field may be neglected, with respect to the applied magnetic field. The pressure P in the fluid is assumed to be constant. The $V_0$ represents suction velocity. Initially the plate and fluid are at rest. Then the plate is set to an oscillatory motion. The plate is at constant temperature $T_w$ and concentration $C_w$.

Under the following assumptions the present problem has been studied.
The molecular transport properties are assumed to be constant. The density variation due to temperature and concentration difference is approximated by Boussinesq approximation.Mass fraction of diffusing species is low compared to the other species in the binary mixture.The viscous dissipation in energy equation is negligible.No chemical reaction takes place in the fluid.The permeability of the medium is uniform.

As the plate is infinite length all the variables in the problem are function of y and t. As it is common in Rayligh's problem, convective terms and pressure gradient term in momentum and energy equation are neglected. The dissipation number corresponding to viscous dissipation is small for most common experiment with fluids for the gravitational field strength of the earth. For fluid like water and gases at ordinary temperature viscous dissipation will reject in turbulent flow. Viscous dissipation is neglected because of small velocity is encountered in free convection laminar flows. The relative effect of pressure gradient is evaluated by comparing pressure force with the viscous force in the boundary layer. The ratio is a function only of Prandtl number and varies monotonically from 20 to 0.2 over the Prandtl ranges from $10^{-2}$ to $10^{2}$. Hence, according to Gebhart[13] the pressure gradient for free convection laminar flow neglected with usual boundary layer approximation. The governing equations for visco-elastic liquid of Walters B′ model is given by

$$\frac{\partial v}{\partial y} = 0 \Rightarrow \quad v=\text{constant}( = -V_0 \text{ at } y=0) \tag{1}$$

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B_0^2 u}{\rho} + g\beta^* (C - C_\infty) - \frac{\nu u}{K} - \frac{K_0 V_0}{\rho}\frac{\partial^3 u}{\partial y^3} - \frac{K_0}{\rho}\frac{\partial^3 u}{\partial t \partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t} - V_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t} - V_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \tag{4}$$

First order velocity slip boundary condition of the problem when the plate executes linear harmonic oscillation in its own plane are given by

$$u = U_0 e^{int} + L_1 \frac{\partial u}{\partial y}, \; T = T_\infty, C = C_\infty \; at \; y=0$$

$$u \to 0, T \to T_\infty, C \to C_\infty \quad as \; y \to \infty \tag{5}$$

where $L_1 = (2 - m_1)\dfrac{L}{m_1}$ and $L = \mu\left[\dfrac{\pi}{2P\rho}\right]^{1/2}$ is the mean free path and $m_1$ is the Maxwell's reflection coefficient.

On introducing the following non-dimensional quantities

$$y^* = U_0 \frac{y}{\nu}, \; u^* = \frac{u}{U_0}, \theta^* = \frac{T - T_\infty}{T_\omega - T_\infty}$$

$$t^* = U_0^2 \frac{t}{\nu}, \; V_0^* = \frac{V_0}{U_0}, \phi^* = \frac{C - C_\infty}{C_\omega - C_\infty}, n^* = \frac{\nu n}{U_0^2}, \; K_p^* = \frac{KU_0^2}{\nu}$$

$R = U_0 \dfrac{L_1}{\nu}$ (Rarefaction parameter), $Rc = \dfrac{\rho K_0 U_0^2}{\mu^2}$ (Elasticity parameter)

$Sc = \dfrac{\nu}{D}$ (Schmidt number), $Pr = \dfrac{\nu}{\alpha}$ (Prandtl number)

M = $\frac{B_0}{U_0}\left(\frac{\sigma\nu}{\rho}\right)^{1/2}$ (Hartman number), Gr = $\frac{g\nu\beta\left(T_\omega - T_\infty\right)}{U_0^3}$ (Thermal Grashof number)

Gm = $\frac{g\nu\beta^*\left(C_\omega - C_\infty\right)}{U_0^3}$ (Mass Grashof number), Q = $M^2 + \frac{1}{K_p}$

in equation (2), (3) and (4), after dropping the asterisks (*)

$$\frac{\partial u}{\partial t} - V_0\frac{\partial u}{\partial y} = Gr\theta + Gm\Phi + \frac{\partial^2 u}{\partial y^2} - Qu - V_0 Rc\frac{\partial^3 u}{\partial y^3} + \frac{\partial^3 u}{\partial t\partial y^2}$$

(6)

$$Pr\left(\frac{\partial\theta}{\partial t} - V_0\frac{\partial\theta}{\partial y}\right) = \frac{\partial^2\theta}{\partial y^2}$$

(7)

$$Sc\left(\frac{\partial\phi}{\partial t} - V_0\frac{\partial\theta}{\partial y}\right) = \frac{\partial^2\phi}{\partial y^2}$$

(8)

With the boundary conditions

$$u = e^{int} + R\frac{\partial u}{\partial y},\quad \theta = 1, \phi = 1 \text{ at } y = 0$$

$$u \to 0,\ \theta \to 0\ ,\phi \to 0\ \text{ as } y \to \infty$$

(9)

## II. METHOD OF SOLUTION

Equation (6) is the third order and two boundary conditions are available. Due to inadequate condition a perturbation method has been applied with Rc<1 as the perturbation parameter. Let

$u = u_0 + Rc\, u_1 + Rc^2 u_2 + \ldots\ldots\ldots\ldots\ldots$

$\theta = \theta_0 + Rc\,\theta_1 + Rc^2\theta_1^2 + \ldots\ldots.$

$\phi = \phi_0 + Rc\,\phi_1 + R^2\phi_1^2 + \ldots\ldots.$

Substituting in equation(6)-(8) and equating the powers of Rc we get the following zeroth order and first order equations with the boundary conditions.

**Zeroth order**

$$\frac{\partial u_0}{\partial t} - V_0\frac{\partial u_0}{\partial y} = Gr\theta_0 + Gm\,\phi_0 + \frac{\partial^2 u_0}{\partial y^2} - Qu_0 \qquad (10)$$

$$Pr\left(\frac{\partial\theta_0}{\partial t} - V_0\frac{\partial\theta_0}{\partial y}\right) = \frac{\partial^2\theta_0}{\partial y^2} \qquad (11)$$

$$Sc\left(\frac{\partial\phi_0}{\partial t} - V_0\frac{\partial\phi_0}{\partial y}\right) = \frac{\partial^2\phi_0}{\partial y^2} \qquad (12)$$

**First order**

$$\frac{\partial u_1}{\partial t} - V_0\frac{\partial u_1}{\partial y} = Gr\theta_1 + Gm\phi_1 + \frac{\partial^2 u_1}{\partial y^2} - Qu_1 - V_0\frac{\partial^3 u_1}{\partial y^3} - \frac{\partial^3 u_0}{\partial t\partial y^2}$$

(13)

$$Pr\left(\frac{\partial\theta_1}{\partial t} - V_0\frac{\partial\theta_1}{\partial y}\right) = \frac{\partial^2\theta_1}{\partial y^2} \tag{14}$$

$$Sc\left(\frac{\partial\phi_1}{\partial t} - V_0\frac{\partial\phi_1}{\partial y}\right) = \frac{\partial^2\phi_1}{\partial y^2} \tag{15}$$

with boundary conditions

$$u_0 = e^{int} + R\frac{\partial u_0}{\partial y}, u_1 = 0, \theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 0 \ at\ y = 0$$

$$u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, \phi_0 \to 0, \phi_1 \to 0 \text{ as } y \to \infty \tag{16}$$

Further we introduce

$u_0 = u_{00} + u_{01}\, e^{int}$ (17)
$u_1 = u_{10} + u_{11}\, e^{int}$ (18)
$\theta_0 = \theta_{00} + \theta_{01}\, e^{int}$ (19)
$\theta_1 = \theta_{10} + \theta_{11}\, e^{int}$ (20)
$\phi_0 = \phi_{00} + \phi_{01}\, e^{int}$ (21)
$\phi_1 = \phi_{10} + \phi_{11}\, e^{int}$ (22)

in zeroth order and first order equations harmonic and non-harmonic considerations separately and obtain the the following equations .

$$u''_{00} + V_0 u'_{00} - Q u_{00} = -Gr\theta_{00} - Gm\phi_{00} \tag{23}$$

$$u''_{01} + V_0 u'_{01} - (Q + in)u_{01} = -Gr\theta_{01} - Gm\phi_{01} \tag{24}$$

$$u''_{10} + V_0 u'_{10} - Qu_{01} = -Gr\theta_{10} - Gm\phi_{10} + V_0 u'''_{00} \tag{25}$$

$$u''_{11} + V_0 u'_{11} - (Q + in)u_{11} = -Gr\theta_{11} - Gm\phi_{11} + V_0 u'''_{01} + inu''_{01} \tag{26}$$

$$\theta''_{00} + V_0 \Pr\theta'_{00} = 0 \tag{27}$$

$$\theta''_{01} + V_0\ Pr\ \theta''_{01} - in\,Pr\,\theta_{01} = 0 \tag{28}$$

$$\theta''_{10} + V_0\ Pr\ \theta'_{10} = 0 \tag{29}$$

$$\theta''_{11} + V_0 \Pr\theta'_{11} - in\Pr\theta_{11} = 0 \tag{30}$$

$$\phi''_{00} + V_0 Sc\phi'_{00} = 0 \tag{31}$$

$$\phi''_{01} + V_0 Sc\phi'_{01} - inSc\phi_{01} = 0 \tag{32}$$

$$\phi''_{10} + V_0 Sc\phi'_{10} = 0 \tag{33}$$

$$\phi''_{11} + V_0 Sc\phi'_{11} - inSc\phi_{11} = 0 \tag{34}$$

with boundary condition

$$u_{00} = R\frac{\partial u_{00}}{\partial y}, u_{01} = 1 + R\frac{\partial u_{01}}{\partial y}, u_{10} = 0, u_{11} = 0$$

$$\theta_{00} = 1, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0,$$

$$\phi_{00} = 1, \phi_{01} = 0, \phi_{10} = 0, \phi_{11} = 0 \quad \text{as} \quad y=0$$

$$u_{00} \to 0, u_{01} \to 0, u_{10} \to 0, u_{11} \to 0, \theta_{00} \to 0, \theta_{01} \to 0$$

$$\theta_{10} \to 0, \theta_{11} \to 0, \phi_{00} \to 0, \phi_{01} \to 0, \phi_{10} \to 0, \phi_{11} \to 0 \text{ as } y \to \infty \tag{35}$$

Hence the velocity, temperature and concentration field can be expressed in terms of the fluctuating part as

$$u = -A_1 e^{-\Pr V_0 y} - A_2 e^{-ScV_0 y} + A_3 e^{-\lambda_1 y}$$

$$+e^{-\lambda_{31}y}\left[A_{41}\cos(nt-\lambda_{32}y)-A_{42}\sin(nt-\lambda_{32}y)\right]$$
$$+Rc\Big[A_5\left(e^{-Pr V_0 y}-e^{-\lambda_1 y}\right)+A_6\left(e^{-ScV_0 y}-e^{-\lambda_1}y\right)$$
$$+A_7e^{-\lambda_1 y}y+V_0ye^{-\lambda_{31}y}\{M_1\cos(nt-\lambda_{32}y)$$
$$-M_2\sin(nt-\lambda_{32}y)\}+nye^{-\lambda_{31}y}\{M_3\cos(nt-\lambda_{32}y)$$
$$+M_4\sin(nt-\lambda_{32}y)\}\Big] \quad (36)$$
$$\theta=e^{-P_rV_0y} \quad (37)$$
$$\phi=e^{-ScV_0y} \quad (38)$$

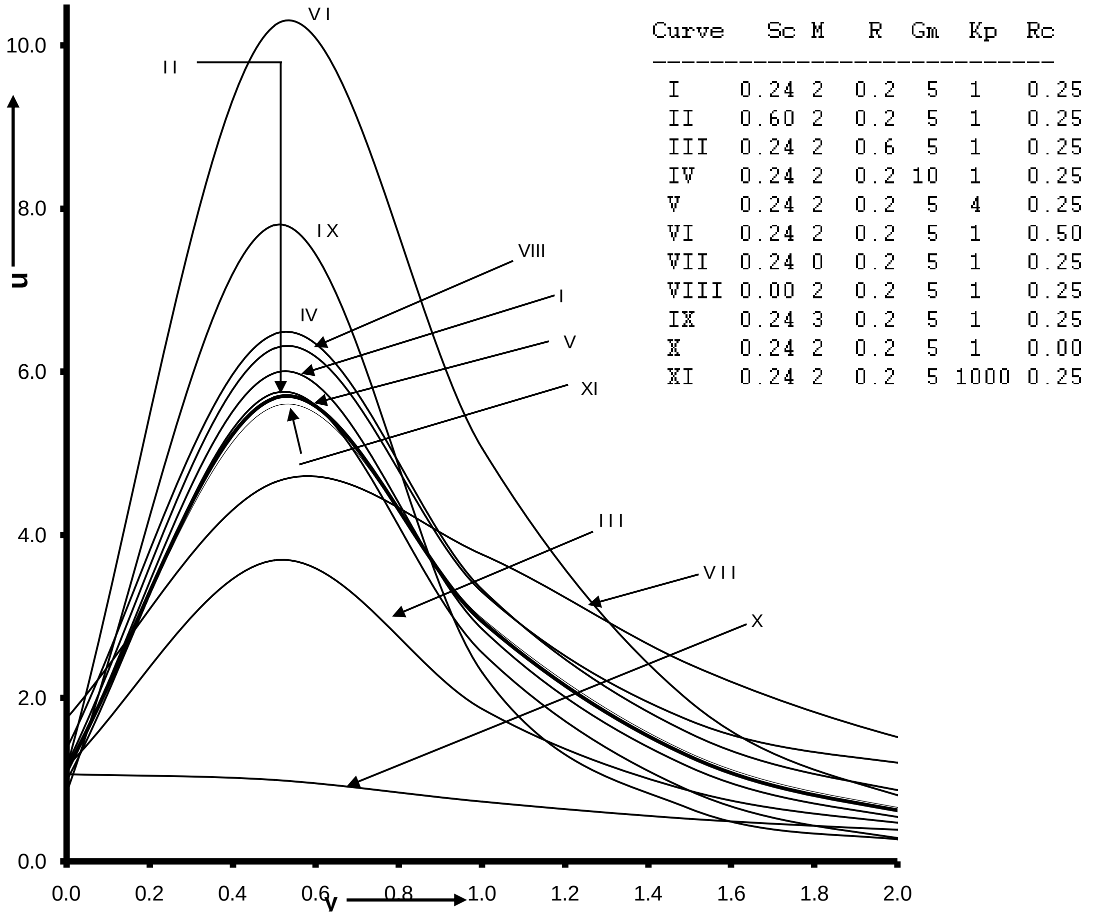
where $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $A_1$ to $A_{11}$ are constants given in Appendix

## III. RESULTS AND DISCUSSION

In order to get a clear insight to the physical problem considered here numerical results are displayed with the help of graphical illustrations. The dimensionless velocity profiles for different values of magnetic parameter (M), Rarefication parameter (R), porosity parameter (Kp), Elasticity parameter (Rc), modified Grashof number (Gm), schimdt number (Sc), Prandtl number (Pr) are presented with the help of graphs . For numerical computation the value of Gr is taken positive (i.e., Gr.=5.0).This indicates that the study has been carried out under the influence of the cooling of the plate. The interesting aspect of the problem is to study the combined effect of the flow parameters with that of first order velocity slip boundary condition when the plate executes linear harmonic oscillation in its own plane.

From Fig.1, it is observed that for the heavier species i.e, with increasing Sc the velocity decreases. This is in good agreement with the property of the Schmidt number presenting the ratio of Kinematic viscosity and diffusivity of the diffusing species. Further, with an increasing magnetic parameter, velocity increases when $y < 0.8$. Thereafter, reverse effect is observed. However, the effect of the permeability parameter is just opposite to that of magnetic parameter. Further, it is observed that under the influence of rarefication parameter, the velocity decreases at all points (Curves I and III). Moreover, buoyancy effect (Gm) due to mass transfer enhances the velocity (Curves I and IV). The sudden rise in the velocity is marked near the plate due to the presence of elasticity of the fluid, but when Rc = 0.0 i.e., in case of viscous liquid no sudden increase is marked instead, slow and uniform variation is noticed. Moreover, further increase in the elastic parameter leads to an increase in the velocity (Curves I and VI).

It is note worthy to record in respect of the representative curves such as X (Rc = 0.0),VII (M = 0.0) and XI ($K_p$ = 1000) representing the case of viscous fluid ,without magnetic field and without porous medium respectively. Curve VII characterizes the delay in attaining the free stream condition and thinning of boundary layer thickness when magnetic field is withdrawn.




| Curve | Sc | M | R | Gm | Kp | Rc |
|---|---|---|---|---|---|---|
| I | 0.24 | 2 | 0.2 | 5 | 1 | 0.25 |
| II | 0.60 | 2 | 0.2 | 5 | 1 | 0.25 |
| III | 0.24 | 2 | 0.6 | 5 | 1 | 0.25 |
| IV | 0.24 | 2 | 0.2 | 10 | 1 | 0.25 |
| V | 0.24 | 2 | 0.2 | 5 | 4 | 0.25 |
| VI | 0.24 | 2 | 0.2 | 5 | 1 | 0.50 |
| VII | 0.24 | 0 | 0.2 | 5 | 1 | 0.25 |
| VIII | 0.00 | 2 | 0.2 | 5 | 1 | 0.25 |
| IX | 0.24 | 3 | 0.2 | 5 | 1 | 0.25 |
| X | 0.24 | 2 | 0.2 | 5 | 1 | 0.00 |
| XI | 0.24 | 2 | 0.2 | 5 | 1000 | 0.25 |

**Fig - 1 Velocity profiles when Pr = 0.71, n = 2, Gr = 5, t = 0.2**

The curve X (Rc = 0.0) shows that the velocity is almost linear in the absence of elastic property of the fluid. Curve XI ($K_p$ =1000) has no special feature in comparision with its counterpart. i.e. $K_p$=1.0, 4.0

## IV. CONCLUSION

1. The effect of the permeability parameter is just opposite to that of magnetic parameter.
2. Rarefaction parameter contributes to decelerate the fluid particles in the flow domain. Elastic elements contributes to sudden rise of the velocity near the plate.
3. The buoyancy effect due to mass transfer enhances the velocity.
4. Magnetic force enhances the skin friction as the time elapses.

## Reference


[1]. Israel-Cookey C. and Sigalo,F.B., 2003, “Unsteady MHD free-convection and mass transfer flow past an infinite heated porous vertical plate with time dependant suction”, AMSE.Mod.,Meas. Cont., B, 72, 25-38.

[2]. Das, S.S.,Panda J.P. and Dash G.C.,2004 , “Free convection flow and mass transfer of an elastico-viscous fluid past an infinite vertical porous plate in a rotating porous medium” , AMSE. Mod. Meas. Cont., B, 73,.37-52.

[3]. Chaudhary R.C. and Chand,T.,2005 , “Hydromagnetic flow past a long vertical channel embedded in porous medium with transpiration cooling”, AMSE.Mod.,Meas. Cont.,B, 74,43-54.

[4]. Kurtcebe C.and Erim M.J.,2005, "Heat transfer of a visco-elastic fluid in a porous channel" , Int. J. of Heat and Mass Transfer, 48, 5072-5077.

[5]. Panda J.P., Patnaik A.B. and. Acharya A.,2006, "Free convection of conducting viscous fluid between two vertical walls filled with a porous material", 75,31-44 .

[6]. Sharma P. R. and Yadav G. R.,2006, "Steady MHD boundary layer flow and heat transfer between two long vertical wavy walls", 75, 21-36.

[7]. Dash Shreekanta,,2007,"Effects of radiation and chemical reaction in MHD flow past a stretched vertical permeable surface through a porous medium with constant suction ",76,83.

[8]. Sharma P. R. , Yadav G. R.,and Sharma M.K.,2007, "Steady MHD flow and heat transfer between two rotating porous disk", 76, 35-54.

[9]. Sharma P. R. and Sharma .K.,2007 ,"Effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium".,76,34-60.

[10]. 10. Das S.S. and Panda J.P.,2008 , "Effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified fluid past a porous flat moving plate in the slip flow regime",Int. J. Adv. Appl. Fluid Mechanics,4, 2, pp.187-203.

[11]. Panda J.P and Das S.S.,2009, "MHD free convection flow of a particulate suspension past an infinite porous inclined flat plate with heat absorption", AMSE. Mod. Meas. Cont., B, 78, 3 , pp.20-31.

[12]. Walters,K., Quart. J. Mech. appl. Math., pp.15-63,1962


**.APPENDIX**

$$\lambda_1 = \frac{1}{2}\left[V_0 + \sqrt{V_0^2 + 4Q}\right], \quad \lambda_2 = \frac{1}{2}\left[-V_0 + \sqrt{V_0^2 + 4Q}\right]$$

$$\lambda_{31} = \frac{V_0}{2} + \frac{1}{2\sqrt{2}}\sqrt{\sqrt{\left(V_0^2 + 4Q\right)^2 + 16n^2} + V_0^2 + 4Q}$$

$$\lambda_{32} = \frac{1}{2\sqrt{2}}\sqrt{\sqrt{\left(V_0^2 + 4Q\right)^2 + 16n^2} - V_0^2 - 4Q}$$

$$\lambda_{41} = -\frac{V_0}{2} + \frac{1}{2\sqrt{2}}\sqrt{\sqrt{\left(V_0^2 + 4Q\right)^2 + 16n^2} + V_0^2 + 4Q}$$

$$\lambda_{42} = \frac{1}{2\sqrt{2}}\sqrt{\sqrt{\left(V_0^2 + 4Q\right)^2 + 16n^2} - V_0^2 - 4Q}, \quad A_1 = \frac{Gr}{\left(\lambda_1 - P_r V_0\right)\left(-\lambda_2 - Pr\, V_0\right)}$$

$$A_2 = \frac{Gm}{\left(\lambda_1 - ScV_0\right)\left(-\lambda_2 - ScV_0\right)}, \quad A_3 = \frac{A_1\left(1 + R\,Pr\,V_0\right) + A_2\left(1 + RScV_0\right)}{1 + R\lambda_1}$$

$$A_{41} = \frac{1 + R\lambda_{31}}{\left(1 + R\lambda_{31}\right)^2 + R^2\lambda_{32}^2}, \quad A_{42} = \frac{-R\lambda_{31}}{\left(1 + R\lambda_{31}\right)^2 + \lambda^2\lambda_{32}^2}, \quad A_7 = \frac{A_3\lambda_1^3}{\lambda_1 + \lambda_2}$$

$$A_5 = \frac{A_1\, Pr^3\, V_0^4}{\left(\lambda_1 - Pr\, V_0\right)\left(-Pr\, V_0 - \lambda_2\right)}, \quad A_6 = \frac{A_2 S_c^3 V_0^4}{\left(\lambda_1 - ScV_0\right)\left(-ScV_0 - \lambda_2\right)}$$

$$A_8 = A_{41}\left(\lambda_{31}^3 - 3\lambda_{31}\lambda_{32}^2\right) - A_{42}\left(3\lambda_{31}^2\lambda_{32} - \lambda_{32}^3\right)$$

$$A_9 = A_{42}\left(\lambda_{31}^3 - 3\lambda_{31}\lambda_{32}^2\right) + A_{41}\left(3\lambda_{31}^2\lambda_{32} - \lambda_{32}^3\right)$$

$$A_{10} = A_{42}\left(\lambda_{31}^2 - \lambda_{32}^2\right) + 2A_{41}\lambda_{31}\lambda_{32}, \quad A_{11} = A_{41}\left(\lambda_{31}^2 - \lambda_{32}^2\right) - 2A_{42}\lambda_{31}\lambda_{32}$$

$$M_1 = A_8\left(\lambda_{31} + \lambda_{41}\right) + A_9(\lambda_{32} + \lambda_{42}), \quad M_2 = A_9\left(\lambda_{31} + \lambda_{41}\right) - A_8(\lambda_{32} + \lambda_{42})$$

$$M_3 = \frac{A_{10}\left(\lambda_{31} + \lambda_{41}\right) + A_{11}(\lambda_{32} + \lambda_{42})}{\left(\lambda_{31} + \lambda_{41}\right)^2 + (\lambda_{32} + \lambda_{42})^2}, \quad M_4 = \frac{A_{11}\left(\lambda_{31} + \lambda_{41}\right) + A_{10}(\lambda_{32} + \lambda_{42})}{\left(\lambda_{31} + \lambda_{41}\right)^2 + (\lambda_{32} + \lambda_{42})^2}$$

$$\lambda_3 = \lambda_{31} + i\lambda_{32}, \quad \lambda_4 = \lambda_{41} + i\lambda_{42}$$