# Vehicle Routing Planning Based on Hopfield Neural Network

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**Abstract:** In this paper, taking the shortest path as the optimization goal, vehicle routing problem is studied based on continuous Hopfield neural network. The energy function is designed according to the optimization goal and the constraint condition. Based on the energy function, the mathematical model of Hopfield neural network is established and its stability is analyzed. The feasibility of the path planning method applying Hopfield neural network is verified by simulation.

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# I. INTRODUCTION

Hopfield neural network is a kind of recurrent neural network with full feedback structure. And the structure is consistent with the neural feedback loops in the biological neural systems. Hopfield neural network has good nonlinear dynamics, associative memory and optimization calculation function, and also has the characteristics of easy realization in hardware. Hopfield neural network is used to store the initial models in the network as stable points of the system. Through the operation of the network, the system runs down to the minimum value along the direction of the energy function. When solving the specific optimization problem, we select a appropriate objective function and map it to the network. The objective function can be minimized through the operation of the network [1-2].

The traveling salesman problem (TSP) is a classical combination optimization problem. It describes that one salesman wants to visit a set of cities and needs to find a shortest traveling road that can travel back to the starting point, meanwhile he can't travel those cities repeatedly[3-5]. In 1984-1985, Hopfield used analog electronic circuits to achieve the Hopfield network, and successfully solved the TSP problem[6-7].TSP problem is everywhere in our daily life such as people arrange a tour between several cities with the shortest distance or how to arrange the machine to drill holes on the circuit board, technically TSP can also be used in the robotics' path planning.

In this paper, taking the shortest path as the optimization goal, by designing the appropriate energy function, vehicle routing planning problem is studied based on continuous Hopfield neural network. The simulation results show that the energy function converges quickly with the proposed method, and high-precision optimization results can be obtained.

# **II. STRUCTURE AND MODELING OF HOPFIELD NEURAL NETWORK**

#### A. Network Structure

Continuous Hopfield neural network (CHNN) is different from the discrete Hopfield neural network(DHNN), all the neurons in CHNN work synchronously and they are dealing with information parallel. CHNN is described by using a constant coefficient differential equation. It is more direct and easy to understand to describe it by using analog electronic circuits[3].

The output of each neuron in Hopfield neural network will be transferred to other neurons by the connection weights. And also each neuron will receive the information from other neurons to form a recursive structure. When inputting the testing samples, the network begins to run from the initial states, and gradually converges to a stable point of the network, that is, the corresponding target state[1]. Fig.1 shows a network topology map of continuous Hopfield neural network.



Fig.1 Schematic diagram of the Hopfield neural network

## B. Network Mathematical Model

In Fig.1,  $u_i$  and  $v_i$  are the output and input voltage of neuron *i* respectively,  $R_i$  and  $C_i$  is the input resistance and capacitance of neuron *i*,  $I_i$  is bias current, the element  $w_{ij}$  can be viewed as a description of the synaptic interconnection strength from neuron *j* to neuron *i*.

According to the Kirchhoff's law in the electronic circuit, the network equations can be written as:

$$\begin{vmatrix} C_i \frac{du_i}{dt} = \sum_{j=1}^n w_{ij} v_j - \frac{u_i}{R_i} + I_i \\ v_i = g(u_i) \end{vmatrix}$$
(1)

The transfer function g in CHNN is a hyperbolic function, which is expressed as follows:

$$g(s) = \rho \frac{1 - e^{-s}}{1 + e^{-s}}$$
(2)

C. Network Stability

The network's state in neurons has constantly updated and always iterated. With the change of time, the network will eventually converge to a stable state and the output of the network will be stable, it can be explained by the energy function.

$$E_{N} = -\frac{1}{2} \sum_{i} \sum_{j} w_{ij} v_{i} v_{j} + \sum_{i} \frac{1}{R_{i}} \int_{0}^{v_{i}} g_{i}^{-1}(v) dv - \sum_{i} I_{i} v_{i}$$
(3)

The energy function (3) takes the derivative of time, we have

$$\frac{dE_N}{dt} = \sum_{i=1}^n \frac{\partial E_N}{\partial v_i} \cdot \frac{dv_i}{dt}$$

where

$$\frac{\partial E_N}{\partial v_i} = -\frac{1}{2} \sum_{j=1}^n w_{ij} v_j - \frac{1}{2} \sum_{j=1}^n w_{ji} v_j - I_i + \frac{g_i^{-1}(v_i)}{R_i}$$

$$= -\frac{1}{2} \sum_{j=1}^{n} w_{ij} v_j - \frac{1}{2} \sum_{j=1}^{n} w_{ji} v_j - I_i + \frac{u_i}{R_i}$$

If the weight matrix **W** is symmetric, i.e,  $w_{ij} = w_{ji}$ , then from (1), we get,

$$\frac{\partial E_N}{\partial v_i} = -\left(\sum_j w_{ij}v_j - \frac{u_i}{R_i} + I_i\right) = -C_i \frac{du_i}{dt}$$

Because both the constant  $C_i$  and the derivative of the function  $g^{-1}(v_i)$  are all greater than 0, therefore, we obtain the following results:

$$\frac{dE_N}{dt} = -\sum_{i=1}^n C_i \frac{dg^{-1}(v_i)}{dv_i} \left(\frac{dv_i}{dt}\right)^2 \le 0$$
(4)

From (4), the network's energy is always reduced. When the state of all neurons in the network is no longer changed, the energy will be stable and at the global minimum point, which is corresponding to a steady state in a continuous network. In the optimization problem, the global minimum point will be the optimal solution of the network.

#### **III. ROUTING PLANNING**

## A. Energy Function

The multi-objective point path planning problem is mapped into a dynamic process of a neural network. It is clear and simple by creating a  $n \times n$  neurons matrix which is also called the transposition array. The output of a neuron is used to indicate whether the target point is accessed in a valid path and is expressed as  $v_{xi}$ . A objective point number is expressed by subscript x,  $x = 1, 2, \dots, n$ . The access order of the objective point in a valid path is expressed by subscript i,  $i = 1, 2, \dots, n$ . For example,  $v_{xi} = 1$  indicates that the objective point x is accessed in the order i. If  $v_{xi} = 0$ , that means in this access process the objective point x should not be accessed in the order i. Because it also has to meet the basic requirements, that is, each objective point can be only accessed once, and only one target location can be accessed at a time. Therefore in the transposition array each line or column only must have an element "1", and the remaining elements are "0". Taking the shortest path as the optimization goal, the energy function is designed as follows:

$$E = \frac{A}{2} \sum_{x=1}^{n} \left( \sum_{i=1}^{n} \upsilon_{xi} - 1 \right)^{2} + \frac{B}{2} \sum_{i=1}^{n} \left( \sum_{x=1}^{n} \upsilon_{xi} - 1 \right)^{2} + \frac{D}{2} \sum_{x=1}^{n} \sum_{y=1}^{n} \sum_{i=1}^{n} d_{xy} \upsilon_{xi} \upsilon_{y,i+1}$$
(5)

#### B. Network Mathematical Model

The energy function (5) takes the derivative of time, we have

$$\frac{dE}{dt} = \sum_{x}^{n} \sum_{i}^{n} \frac{\partial E}{\partial v_{xi}} \frac{\partial v_{xi}}{\partial u_{xi}} \frac{du_{i}}{dt}$$

If taking  $\frac{du_{xi}}{dt} = -\frac{\partial E}{\partial v_{xi}}$ , then we have

$$\frac{dE}{dt} = -\sum_{x}^{n} \sum_{i}^{n} \left(\frac{\partial E}{\partial v_{xi}}\right)^{2} \frac{\partial v_{xi}}{\partial u_{xi}} \le 0$$

So from (5), we get

$$\frac{\mathrm{d}u_{xi}}{\mathrm{d}t} = -\mathbf{A}\left(\sum_{i=1}^{n} v_{xi} - 1\right) - B\left(\sum_{x=1}^{n} v_{xi} - 1\right) - \mathbf{D}\sum_{y=1}^{n} d_{xy}v_{y,i+1}$$
(6)

# C. Simulation Results

By using the network model (6), simulation is carried out under the MATLAB environment with twelve vehicle target location points. The initial route is random in Fig.2(a), and the optimized route is displayed as shown in the Fig.2(b). According to the coordinates of the target points in the graph, the initial path length is 4.62, and the optimized path length is 2.95. So the optimization effect is obvious.



After the number of iteration increases to 600 times, the energy function decreases and becomes stable as shown in Fig.3.We can see from Fig.3 that the energy function is finally reached to 1.47 with 600 iterations. The changing of energy function is zero, and the states of neurons will be the stable points. So the system gets the best path. It can be seen from the graph that the rate of convergence is fast.



# **IV. CONCLUSION**

Hopfield neural network has good nonlinear dynamics, and optimization calculation function. In this paper, a continuous Hopfield neural network is applied to solve the multi-objective point path optimization problem. From the experimental results we can see that by using Hopfield neural network can obtain the optimal path quickly.

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