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Designing a Garden

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Abstract: *Various designs of gardens when number of streets (9), number of trees (19) and the number of trees (5) on each street are presented. Some gardens with different number of streets, number of trees, and trees on each street are also given. Constructions and the traveling paths for all gardens are given.*

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I. INTRODUCTION

The following is an ancient puzzle posted in our Puzzles group on WhatsApp by Laad [1].

ऐ बागवां एक बाग लगा,
जिसमे शजर उन्नीस हो।
नौ कतारे जिसमें होवे,
हर कतार मे पांच शजर हो।

Its English translation is the following. O gardener, prepare such a garden that has 19 trees distributed over 9 streets and each street has exactly 5 trees. One more condition is not explicitly mentioned, i.e., each tree must be located at a meeting point of minimum two streets. For convenience, we will represent such a garden by (streets, trees, trees on each street). Thus, it is a (9,19,5) garden.

1.1 Triangle Garden (TG)

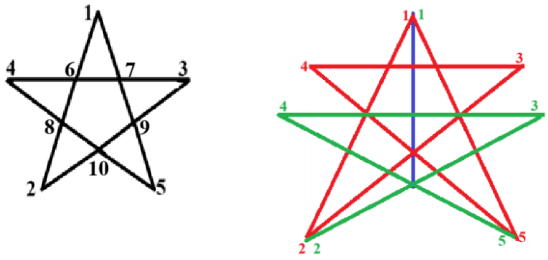
It is shown in Figure 1(a) [2]. Two triangles green and red are equilateral one placed inverted over the other, and their vertices are connected by 3 median lines, intersecting at the centroid 17. An alternative garden was obtained by replacing the equilateral triangles by two isosceles triangles [2] shown in Figure 1(b). Here the condition that centroid of the triangles overlap is relaxed. However, the three lines joining the vertices of two triangles should meet at a point (17).
In Section 2.1, we propose a generalised version of TG, and the gardens shown in Figures. 1(a) and (b) are the particular cases.

(a) (b)

Figure 1: TTGs (9,19,5) where the triangles are (a) equilateral (b) isosceles.

1.2 Star Garden (TSG)

A (5,10,4) star garden is shown in Figure 2(a). If another symmetrical star is overlapped on this, together there will be 10 streets and 20 trees. Therefore, the two stars are to be overlapped such that one street and 1 tree is common to both. The two stars are overlapped as shown in Figure 2(b), and adjusted the size of the green star such that tree 1, and two streets 1-2 and 1-5 of the two stars are common. Since two lines are overlapping, one



(a) (b)

Figure 2: (a) A star (5,10,4) garden, (b) TSG (9,19,5) garden.

more blue line is added to adjust 5 trees on each street. Trees are not marked so as to avoid clumsiness. Thus, the entire configuration is a (9,19,5) garden. This garden was proposed by Sharma [3].

Both the above TG and SG look beautiful being aesthetic and symmetrical. However, it is always beneficial to study the general configurations to have more insight. Sometimes they may be preferred when different sizes and shapes are required to fit into the available shape and size of the land for the garden. One can generate various sizes and shapes for a garden by

1. Enlarging along horizontal, vertical or both directions.
2. Reflection
3. Rotating.

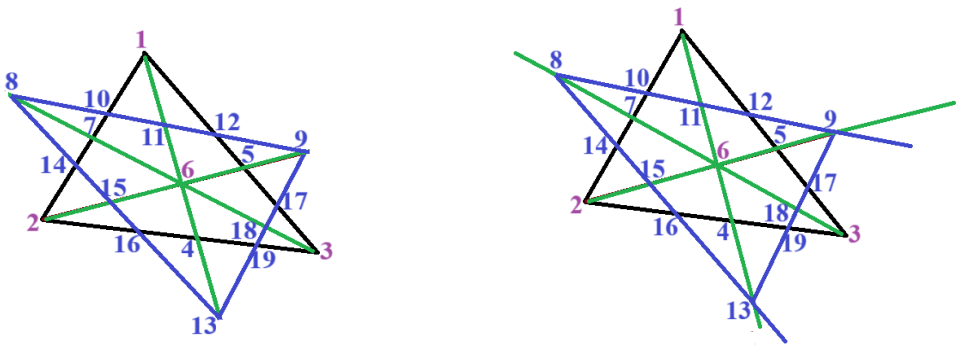
II. GENERALISED GARDENS

2.1 General TG (9,19,5)

A generalised (9,19,5) garden is shown in Figure 3. The two triangles are neither equilateral nor isosceles. However, the following conditions must be satisfied.

(1) The two triangles (black and indigo) should be such that their vertices are outside the triangles.

(2) The three (green) lines joining the opposite vertices of two triangles must intersect at one point.



(a) (b)

Figure 3: (a) A generalised (9,19,5) garden, (b) Construction

2.2 Construction of TG (9,19,5)

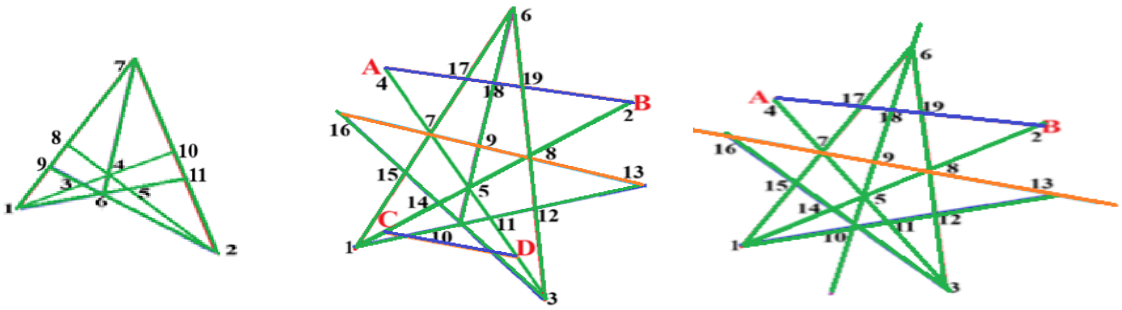
One possible construction is the following.

1. Draw a triangle 1-2-3.
2. Choose trees 13 and 9 outside the triangle. Join 1-13 and 2-9 and extend them. The crossing point is 6.
3. Draw line 3-6 and extend it.
4. Choose a tree 8 on the line 3-7 outside the triangle.
5. Draw the lines 8-9, 8-13 and 9-13.

From this general garden, the gardens of Figures 1(a) and (b), can easily be obtained.

2.3 (7,11,4) based Gardens

Consider a (7,11,4) garden shown in Figure 4(a). Two more lines are to be added suitably to convert it into a (9,19,5) garden as explained below.



(a) (b) (c)

Figure 4: (a) (7,11,4) garden, (b) (9,19,5) garden, (c) construction with line AB.

Type A Garden

The garden is shown in Figure 4(b). The part of the garden shown in green colour is the same as the garden of Figure 4(a). To make it a (9,19,5) garden, a line 13-16 is added. The other line could be either AB or CD. The garden of Figure 2 is the special case of Figure 4(b) with line AB.

This garden can be seen as a combination of two stars 6-1-2-4-3-6 and 6-1-13-16-3-6. Therefore, it is named as a star garden (SG). Four trees are out of the lines 1-6 and 3-6.

Construction

The construction is given in Figure 4(c). Steps are the following.

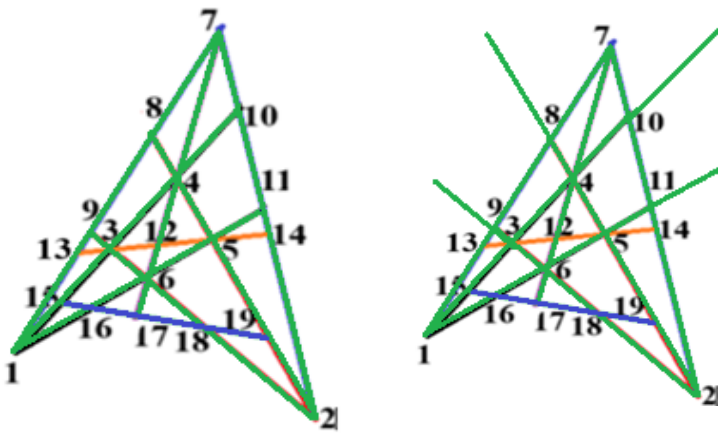
1. Draw green lines 1-2 and 3-4.
2. Choose tree 6 in between 2 and 4. Join 6-5, 6-1, and 6-3.
3. Join 7-8, 1-10, 3-10 and extend them on both sides.
4. Choose trees 4 and 2 on extended lines 3-5 and 1-8, respectively.
5. Join 2-4.

An alternative method is the following.

1. Draw two lines 1-2 and 1-13 at an acute angle.
2. Choose any tree 6 and draw lines 6-1, 6-2 and arbitrary line 6-10.
3. Join 3-5, 3-10 and 7-8.
4. Draw a line above 7,8,9 and below 6.

Type B Garden

Consider Figure 5(a). In the (7,10,4) green garden, line 3-5 is added. The other line is inserted below 6 and above 1. This garden has all the trees inside the lines 1-7 and 2-7.



(a) (b)

Figure 5: (a) Type B (9,19,5) garden. (b) Construction

The construction is given in Figure 5(b). The steps are

1. Choose trees 1 and 2 arbitrarily (not necessarily in the horizontal line).
2. Draw 2 black lines making an acute angle from 1, and 2 red lines making an acute angle from 2.
3. Draw 3-5 and extend it.
4. Choose a tree 7 on line 3-7.
5. Join 7-1 and 7-2.
6. Join 3-5 and extend it on both sides.
7. Draw a line 15-19 below 6 and above 1.

An alternative construction is as follows.

1. Draw two lines 1-10 and 1-11 making an acute angle.
2. Choose a tree 7. Draw any line 7-6.
3. Draw the line 7-1.
4. Choose a tree 2. Draw 7-2.
5. Draw the lines 2-8 through 4, and 2-9 through 6.
6. Draw a line 13-14 through the trees 3 and 5.
7. Draw a line below 6 and above 1.

One can see that there is a lot of choices to construct the gardens of various shapes and sizes.

III. DIFFERENT PATHS

Consider the problem: Which portions of the streets of a given garden are to be finished with cement so that a tourist can visit all the trees, driving only on cemented portions of the streets without going twice to any of the trees.

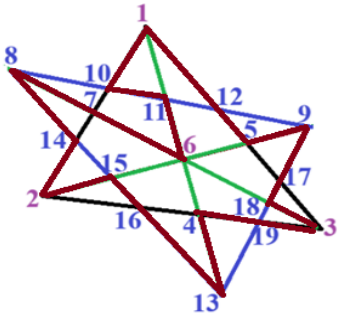
Starting from a tree, ending could be on (1) another tree, and (2) the same tree. We will attempt option (2) which is more challenging than (1), and leave the latter for the readers to find out.

Paths for TG: There are many possible paths for the generalized TG shown in Figure 3. Let us consider the tree 6 where there are 6 directions to go. We can take two neighbouring directions 6-7 and 6-11; one for going and the other for returning or vice versa. The path is shown by purple line in Figure 6(a). Similarly, taking the other 5 pairs of neighbouring lines, corresponding 5 paths are shown in Figure 6(b)-(f). It can be a good exercise to verify that no other combinations of directions give the closed path. There is total 14 pieces of streets to be travelled.

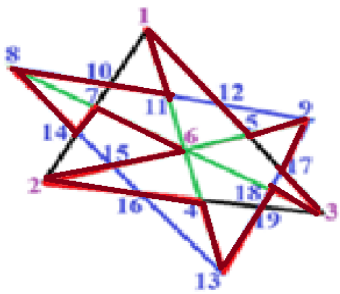
Note from these figures that there are two options for each of the corner trees (1,9,3,13,2,8). For example, from corner tree 9 to reach 17 (i) 12-5-9-17 and (ii) 12-9-5-17. Similar options will be available from all other corner trees except 8. Thus, there is a large number of paths possible for (a)-(f) gardens. Since complete path is a closed one, one can start from any one tree and come back to the same. Of course, one can travel in the reverse way also.

Paths for Type B Garden

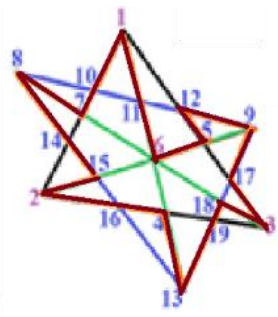
The path for the Type B Garden of Figure 5(a) is shown in Figure 7. Interestingly, it requires 13 pieces of streets only to be cemented.



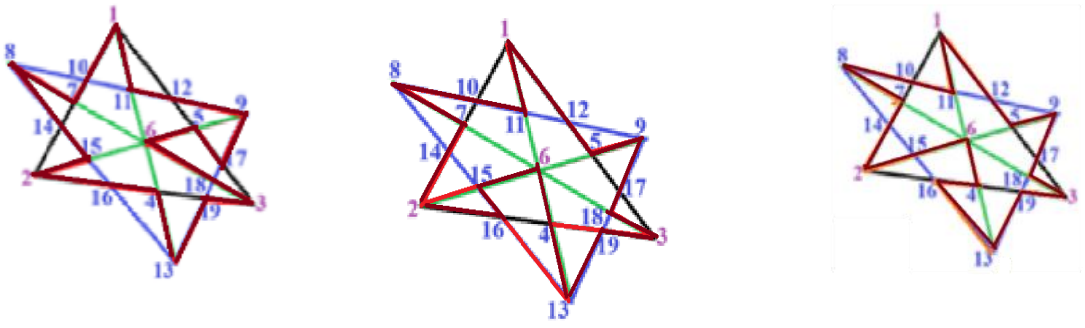
(a)



(b)



(c)



(d) (e) (f)

Figure 6: (a)-(f) Paths in purple colour.

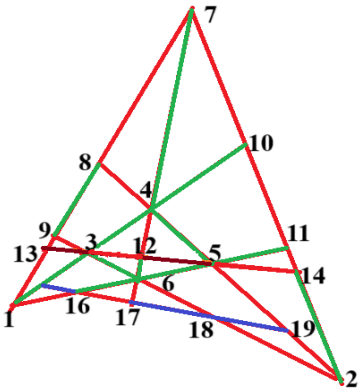


Figure 7: Path for Type B Garden (red)

IV. GARDENS WITH DIFFERENT STREETS AND TREES

(A) Pair of Triangles Garden (PTG)

A pair of triangles (red and green) is added instead of 3 medians in Figure 3(a) as shown in Figure 8.

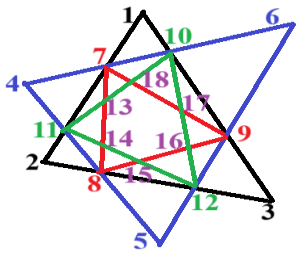
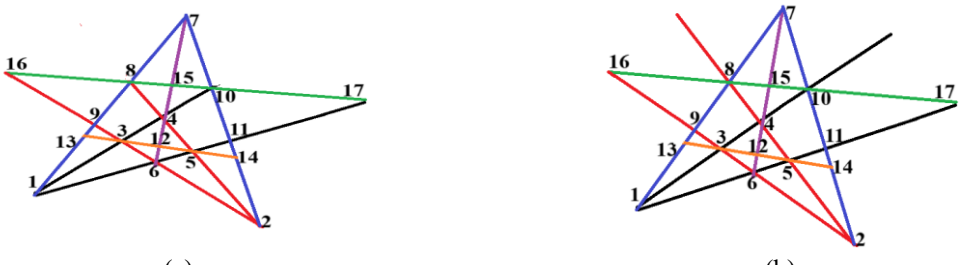


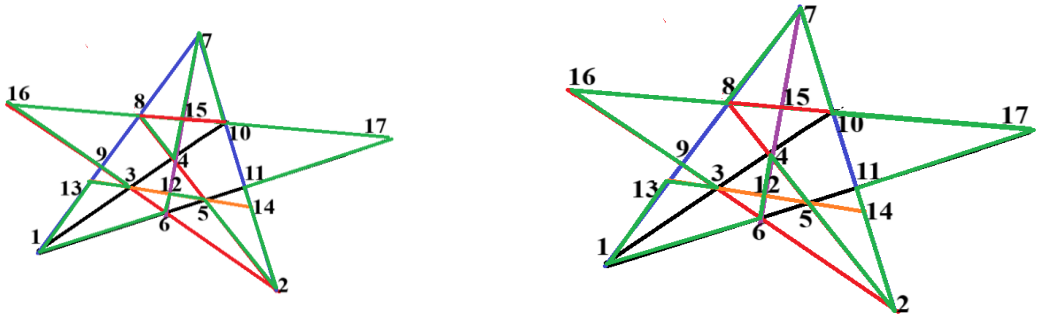
Figure 8: (12,18,4) garden.

(B) (9,17,5) garden

A (9,17,5) garden is shown in Figure 9(a). Its construction and path (green) are shown in Figure 9(b) and (c), respectively. An alternative path is shown in Figure 9(d). Paths (c) and (d) require, respectively, 14 and 12 pieces of streets to be cemented.



(a) (b)



(c) (d)

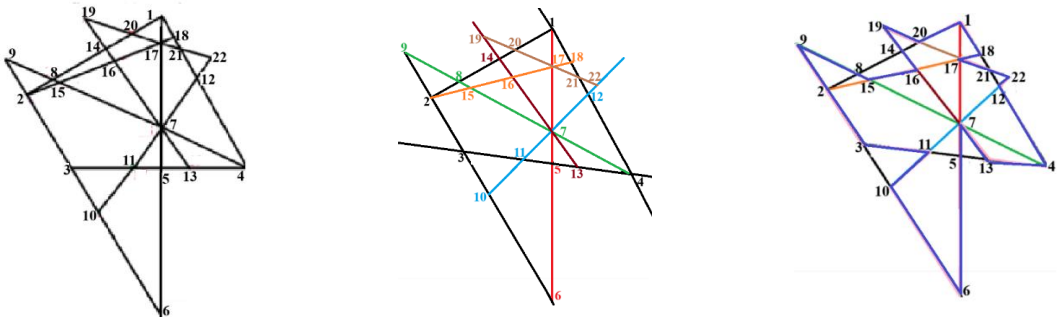
Figure 9: (a) (9,17,5) Garden, (b) Construction, (c) Path (green), (d) Alternative path

(C) (10,22,5) Garden

The garden, its construction, and the path (rose) are shown in Figures 10(a), (b) and (c), respectively. Its construction consists of three major steps:

1. To draw the rectangle 1-2-3-4 and extend its sides in all directions.
2. To locate the junction 7.
3. Draw lines 1-6, line 4-9. Draw a line 10-22.
4. Draw a line 13-14.
5. Locate the junction 17. Draw lines 2-18 and 19-22.

It requires 17 pieces of streets to be cemented.



(a) (b) (c)

Figure 10:(a) (10,22,5) garden, (b) Construction, (c) Path (blue).

V. CONCLUSION

Various designs of gardens, when number of streets (9), number of trees (19) and the number of trees (5) on each street are specified, are given. Constructions and the traveling paths are given. Earlier symmetrical gardens are the specials cases. Other types of gardens with different streets and trees and trees on each street are also given. These garden designs can also be used for decoration purposes, such as the tiles in buildings, making rangoli, etc.

REFERENCES

[1] Devendra Kumar Laad, Posted on Puzzles group of WhatsApp, May 18, 2025.
[2] T. S. Rathore, Posted on Puzzles group of WhatsApp, May 21, 2025
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