9International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 9, Issue 9 [September 2025] PP: 35-40

Geometrically Merging and Splitting of Areas

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Abstract: *With the help of Pythagoras theorem, it is shown that areas of two given polygons can be merged using a pencil and paper into a similar polygon. Conversely, a given polygon can be divided into two polygons such that the sum of their areas is equal to that of the given polygon. The methods of merging and splitting are also applicable to circles; however, they require geometrical instrument to draw the circle.*

Date of Submission: 01-08-2025 Date of acceptance: 04-08-2025

I. INTRODUCTION

A video on the WhatsApp is viral [1]. It is concerned with the geometrical merging of areas of two squares giving another square based on a Sanskrit script [1]. The script is correct but the video is not correct. The correct construction will be given in Section 3. We present a merging of two polygons in Section 2. The construction of merging two squares and two equilateral triangles are the special case of that of a polygon. We also deal with the merging of two circles. These merging is extended to any number of such figures. The converse method, i.e., to convert a polygon into two polygons is given in section 4.

II. MERGING

2.1. Merging of two polygons

The problem: Two polygons of areas A1 and A2 are to be geometrically merged into a single polygon of area A1 + A2 using paper and pencil.

Geometrical Construction

The construction is explained with the help of hexagons. Let the two hexagons H1 and H2 shown in Fig. 1(a) and (b) have the areas A1 and A2, respectively. Draw a vertical line AB and a horizontal line DB with the help of the hexagon. Put the two hexagons as shown in Fig 1(c). Join AG. Obviously,

$$a_1^2 + a_2^2 = a_3^2. \tag{1}$$

Multiplying both sides by

$$k = (3\sqrt{3})/2), \tag{2}$$

we get

$$ka_1^2 + ka_2^2 = ka_3^2 \tag{3}$$

$$\rightarrow A_1 + A_2 = A_3.$$

Thus, A_3 represent the sum of the areas of two hexagons. Therefore, we have to create a hexagon with side a_3. Cut the triangle ABG. This is shown in Fig. 1(d). Sides a_1, a_2 and a_3 will be used to draw lines of length and measure of angles. Attach the triangle as shown in Fig. 1(e). Angle LGH of the hexagon B is the same as AGM being opposite angles. Thus, the side GM represents the second side of the new hexagon. Now move the triangle to the new position shown in Fig. 1(f). This gives the third side MN of the hexagon. Next, with the help of the hexagon A draw a line NO and make it equal to *c* with the green triangle as shown in Fig. 1(g).

Repeat the same to get the next line OP. Lastly, Join PA. This completes the construction of the new hexagon.

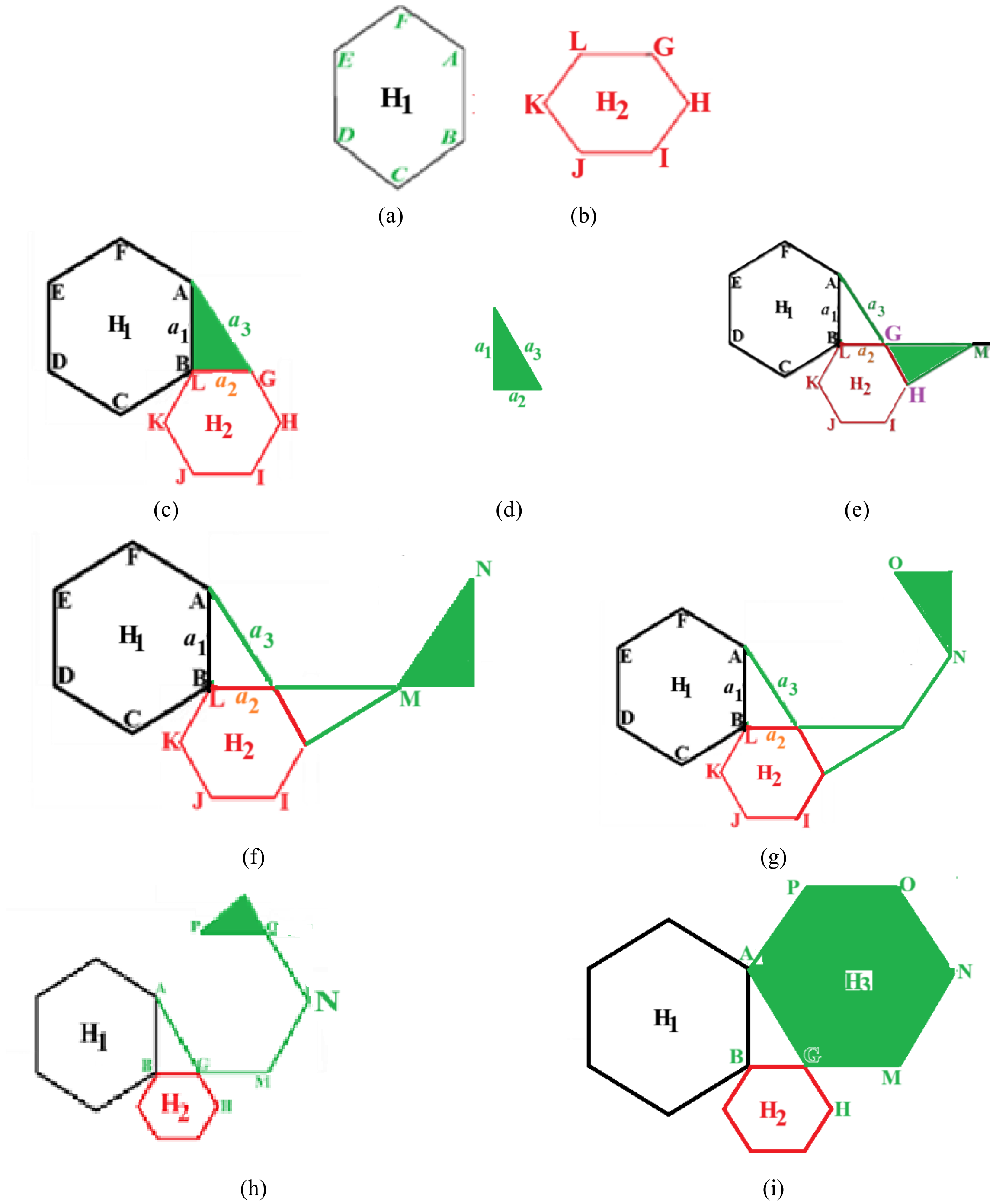

Fig. 1. Merging of two polygons.

2.2 Merging of two squares

In this case $k = 1$. Fig. 2(a) and (b) show the two squares given. Rest of the procedure is similar to that for the hexagon described above. CE represents the line of length c and at an angle of 90° as proved below.

$$AK = A + B \quad (4)$$

$$AK = AI + IK = B + IK \quad (5)$$

From (4) and (5),

$$IK = A$$

Now in triangle IEK, IK = A, KE = B, angle EKJ = 90º by construction. Hence $IE^2 = IK^2 + EK^2 = c^2$. Thus, three sides of triangle EIK and ACI are equal. Therefore, the triangles are congruent. Therefore, angle ACI = angle EIK and angle CIA = IEK. But CIA + EIK = 90º. Therefore, angle CIE = 180º – (angle CIA + EIK). = 180º – 90º = 90º. Rest of the construction is self-explanatory.

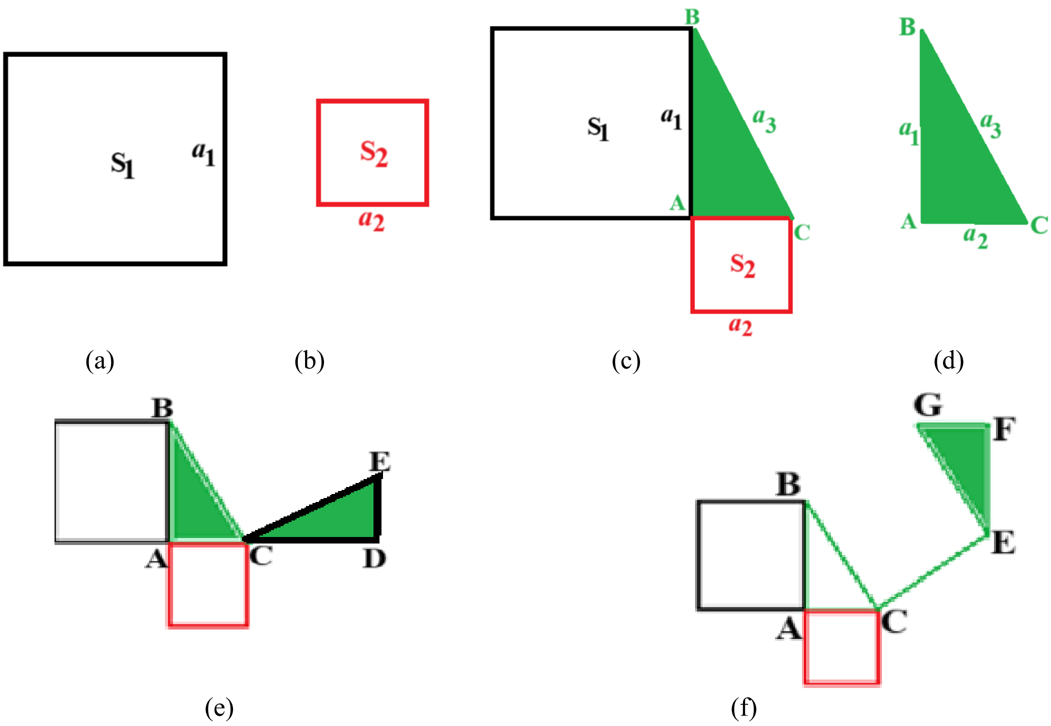

Fig. 2: Merging of two squares

The method outlined in [1] needs the following corrections. Refer to the correct construction given in Fig. 3.

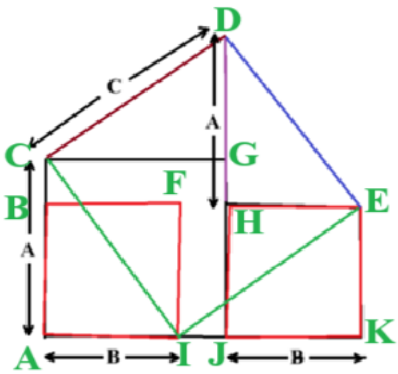

Fig. 3. Correct construction.

1. The smaller square should be placed on the left side of the bigger square instead of the right one.
2. It is not necessary to draw the complete horizontal line. Only a point C is to be marked. No need to draw the line CE with the help of a rope by rotating it. Just join C and E.
3. Next two lines are just drawn without any explanation. They are to be drawn as follow. Put a bigger square over the smaller square. Mark point D. Join DE. Finally join CD.

2.3 Merging of equilateral triangles

The given triangles are as shown in 4(a) and (b). In this case k = ½. The construction is self-explanatory.

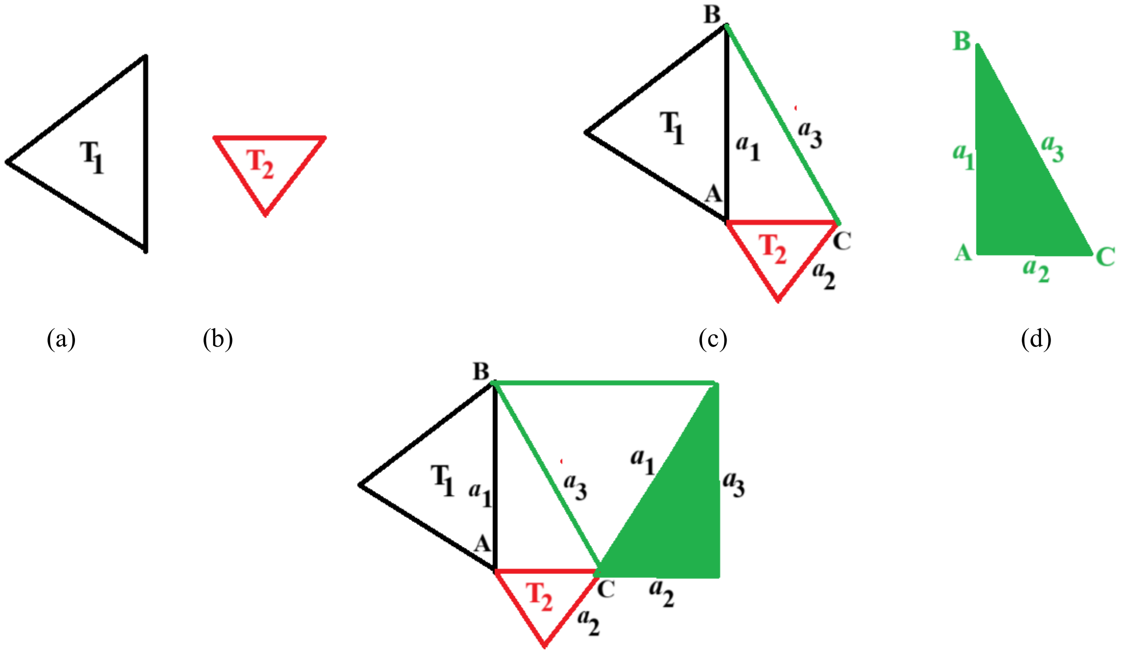

Fig. 4. Merging of two equilateral triangles

2.4 Merging of circles

Let the given circles be C1 with diameter AB, C2 with diameter AC. In this case $k = \pi/4$. The construction shown in Fig. 5 is self-explanatory. Though it is simple, it requires an instrument to draw a circle, because the circle C3 cannot be drawn with the help of the given circles.

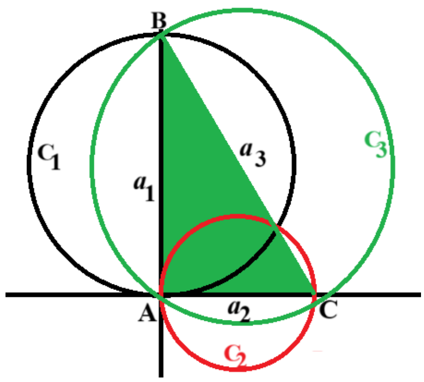

Fig. 5: Merging of two given circles

The following should be clear from above examples.

1. The area A should ka_1^2 and B should be proportional to ka_2^2. Then only $A_1 + A_2 = A_3$.
2. The new figure on side a_3 can be merged into another similar type of figure of area A_3. Thus, we can write

$$A_4 = k(a_1^2 + a_2^2 + a_3^2) \tag{6}$$

The result can be generalized as

$$A_n = k\sum_{1}^{n-1} A_i \tag{7}$$

III. SPLITTING THE GIVEN POLYGON AREA INTO TWO POLYGON AREAS

We will explain the spitting process with a square. Draw two vertical and horizontal lines (black) with the help of the square S (purple). Now place the square S touching its two ends to the two black lines at A and D as shown in Fig. 6. Then the sides a and b of the triangle AOD gives the sides of two required squares. Since line AD of the given square can have any negative slope; there are many possible squares of sides *a* and *b*. Square S can be replaced by a polygon or an equilateral triangle. In case of a circle, it will be the diameter of the given circle, and OA and OB will be the diameters of the two resulting split-circles.

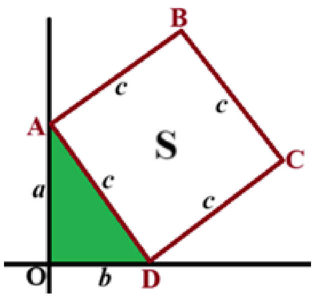

Fig. 6: Splitting of the square C into two squares.

IV. CONCLUSION

Methods using only a pencil for merging and splitting of areas of polygons based on Pythagoras theorem have been presented. For circles, an instrument is required for drawing the circles.

REFERENCES

[1]

WhatsApp Video 2025-07-08 at 19.29.0