International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 15, Issue 2 [February 2026] PP: 106-110

Application of Exponential - Logarith Functions in Financial Problems

Nguyen Quoc Thinh
The University of Danang, Vietnam-Korea University of Information and Communication Technology

Abstract: *In this article, I apply exponential and logarithmic functions in bank interest calculation problems such as simple interest problems, compound interest problems, and periodic deposit problems. Forecast the deposit amount and deposit time so that depositors can be proactive in their financial plans. As well as forecast the monthly payment amount and how long it will take to pay to help borrowers be proactive in their finances.*
Keywords: *Interest rate, simple interest, compound interest, compound interest by term...*

Date of Submission: 15-02-2026 Date of acceptance: 28-02-2026

I. PROBLEM STATEMENT

Common problems in interest rate problems are very diverse. Mathematics is an extremely effective tool to help analyze and solve such problems in a rigorous and reasonable way, bringing practical benefits. Applying mathematics to solve interest rate problems as well as analyzing and testing the results logically is always an urgent requirement for experts working in the fields of finance, banking and those who participate in saving deposits. Applying exponential and logarithmic functions in interest rate problems helps us determine the amount received as well as shape the initial deposit amount and choose a reasonable deposit method to meet the depositor's requirements.

THEORETICAL BASIS
Exponential function
Exponential function with base a (0 < a ≠ 1) has the form
Domain of D = R, value domain V = R^+
Variation table

$a > 1$

x	$-\infty$	0	$+\infty$
y	$+\infty$ 1 $-\infty$	↗	↗

$0<a<1$

x	$-\infty$	0	$+\infty$
y	$+\infty$ 1 $-\infty$	↘	↘

Logarithmic Function

The logarithmic function with base a (0 < a ≠ 1) has the form: $y = \log_a x \Leftrightarrow x = a^y$

Domain of D = $D = R^+ = (0, +\infty)$, value domain V = R

$$\log_a MN = \log_a M + \log_a N \qquad \log_a \frac{M}{N} = \log_a M - \log_a N$$

$$\log_a M^k = k.\log_a M \quad \log_a b.\log_b c = \log_a c$$

Variation table

$a>1$ $\qquad\qquad$ $0<a<1$

x	0	0	$+\infty$
y	$+\infty$ $-\infty$ ↗	1 ↗	

x	0	0	$+\infty$
y	$+\infty$ 1 $-\infty$	↘	↘

SOME INTEREST RATE PROBLEMS IN PRACTICE

Interest is a concept considered from two perspectives: the lender and the borrower. From the perspective of the lender or capital investor, interest is the amount of money increased on the initial investment capital in a certain period of time. When investors invest a capital, they expect to receive a value in the future, more than the value they initially spent and this difference is called interest. From the perspective of the borrower or capital user, interest is the amount of money that the borrower must pay to the borrower for the use of the capital for a certain period of time.[1]

Interest is the ratio of interest paid to the loan capital in a unit of time.

The unit of time can be year, quarter, month, day.

Interest is calculated as a percentage or decimal number.

Simple interest is the amount of interest calculated only on the principal and not on the amount of interest generated by the principal in a fixed period of time. (Only the principal generates interest).

Simple interest formula: capital P_0, interest rate r, number of periods n.

Formula $P_n = P_0(1 + nr)$.

Compound interest arises when interest is added to the original principal. Therefore, from that point onwards, interest is calculated based on the sum of the original principal and the interest just earned. The addition of interest to the original principal is called compounding or capitalization.[1] Compound interest formula: Assume the original principal P0 with interest rate r% per period in the form of compound interest for n periods. Calculate Pn the total value achieved (principal and interest) after n periods.

he time unit of each period can be year, quarter, month, day.

At the end of the first period, we have: Interest received: $P_0.r$

Total value achieved (principal and interest) at the end of the first period:

$$P_1 = P_0 + P_0.r = P_0\,(1 + r).$$

Because interest is added to the capital at the end of the second period, we have:

Interest received: $P_1.r$

Total value achieved (principal and interest) at the end of the second period is:

$$P_2 = P_1 + P_1.r = P_1(1 + r) = P_0(1 + r)(1 + r) = P_0(1 + r)^2$$

In general, after n periods, the total value achieved is $P_n = P_0(1 + r)^n$

In which: P_n is the total value achieved (principal and interest) after n periods.
P_0 is the principal.
r is the interest rate per period.

Problem 1: One-time compound interest

A person deposits an initial amount of P_0 VND in a bank with an interest rate of r% per period in the form of compound interest for n periods. Then, Pn is the total value achieved (principal and interest) after n periods. $P_n = P_0(1 + r\ \%)^n$. [2]

Situation 1: A person has a capital of A VND deposited in a bank with a compound interest rate of r per month. So after n years, how much money does that person want to receive?

For example: Mr. A deposits 10 million VND in a bank with a compound interest rate of 3 months with an interest rate of 1.65% per quarter, how much money will Mr. A receive after 2 years?

We have P_0 = 10,000,000 million, n = 2 years = 8 quarters, interest rate r = 1.65% per quarter.

Applying the formula, we can calculate the amount of money that person will receive after 2 years:

$P_2 = 10.000.000 \text{ x } (1 + 1{,}65\%)^8 \approx 11.399.000$ VND.

Remarks: The problem applies compound interest, giving the principal, interest rate per period and calculating the total amount received after n periods.

Situation 2: A person has a plan in the future to do something n years later with an amount of T VND. So at the present time, how much money does that person need to deposit in the bank so that after n years, he will have enough money as planned. Knowing that the bank interest rate is r per month.

Example 1: Mr. A plans to buy a motorbike worth 30 million VND in 3 years. So at this time, how much money does Mr. A need to deposit in the bank so that after 3 years, he can buy the motorbike. Knowing that the bank interest rate is 0.45% per month (unchanged interest rate)?
We have P_n = 30,000,000, n = 36 months, the interest rate in 1 month is r = 0.45% per month.

We have: $P_n = P_0(1 + r)^n \Rightarrow P_0 = \frac{P_n}{(1+r)^n} = \frac{30.000.000}{(1+0.45\%)^{36}} \approx 25.530.000$

So at the present time, Mr. A needs to deposit in the bank 25,530,000
Example 2: Mr. A has 4,150,000 VND deposited in the bank and plans to buy a phone worth 9 million VND in 2 years. So what is the bank's monthly compound interest rate with a 1-month term (knowing that the monthly interest rate remains unchanged) so that 2 years later Mr. A has enough money to buy a phone worth 9 million VND

We have: : $P_n = P_0(1 + r)^n \Rightarrow r = \sqrt[n]{\frac{p_n}{P_0}} - 1 = \sqrt[24]{\frac{9.000.000}{4.150.000}} - 1 \approx 0.03279$

So the bank interest rate is 3.279% per month
Situation 3: A person has a capital of A VND, and plans to do something in the future with an amount of T VND. So how long does that person need to deposit in the bank to have enough money as required? Knowing that the bank interest rate is r per month.
For example: Mr. A has 7000,000 deposited in the bank with a compound interest rate of 0.52% per month. So how many months does Mr. A need to deposit to receive 9 million VND (knowing that the monthly interest rate remains unchanged)?

We have: : $P_n = P_0(1 + r)^n \Rightarrow n = \frac{\ln \frac{P_n}{P_0}}{\ln(1+r)} = \frac{\ln \frac{9000.000}{7000.000}}{\ln(1+0.52\%)} \approx 49.5$ months

So Mr. A needs to deposit in the bank for 50 months.

Problem 2: Compound interest on periodic deposits (deposited at the beginning of the period)
A person deposits T dong in the bank every month (one period) with a monthly interest rate of r. How much money will that person receive after n months?[2]
Let P_n be the amount of money multiplied (including principal and interest) at the end of the nth month.
At the end of the first month, the amount received is: P_1=T+ T.r
At the beginning of the second month, the person deposits more T dong, so the principal amount at the beginning of the second month is:
P_1+T = (T+ T.r)+T = (2+r).T
So at the end of the second month, the amount received is

$$(2+r).T + (2+r).T.r = (2+r)T(1+r)$$
$$= [(1+r)+1)].T.(1+r)$$
$$= \frac{\left[(1+r)^2 - 1\right].T.(1+r)}{(1+r)-1} = \frac{\left[(1+r)^2 - 1\right].T.(1+r)}{r}$$

In general, at the end of the nth month, we receive the amount of money (including principal and interest):
$$P_n = \frac{T}{r}\left[(1+r)^n - 1\right](1+r)$$

Periodic compound interest is often suitable for those who do not have initial capital, so they will deduct income from savings every month to have money to carry out future plans. Here are some common situations.
Situation 1: A person deducts income of A VND every month to deposit in a bank with an interest rate of r per month. So after n years, how much money will that person receive?
For example: Mr. B deposits 5 million VND in the bank every month with a compound interest rate of 0.41% per month. Ask how much money will Mr. A receive after 3 years (both principal and interest). Knowing that the monthly interest rate remains unchanged?

We have $P_n = \frac{T}{r}\left[(1+r)^n - 1\right](1+r) = \frac{5000.000}{0.41\%}\left[(1+0.41\%)^{36} - 1\right](1+0.41\%) = 194.329.457$

Situation 2: Implementing a plan in the future

Problem 1: A person has a plan in the future that n years from now he will do something with an amount of T VND. So how much income does that person need to deduct every month to deposit in the bank so that after n years he will have enough money as planned. Knowing that the bank interest rate is r per month.
For example: Mr. B wants to buy a car worth 650 million VND in 5 years. So from now on, how much does Mr. B need to deposit in the bank every month so that after 5 years he can buy a car. Knowing that the compound interest rate is 0.52% per month (the monthly interest rate is unchanged).
We have:

$$P_n = \frac{T}{r}\left[(1+r)^n - 1\right](1+r)$$

$$\Rightarrow T = \frac{P_n.r}{\left[(1+r)^n - 1\right](1+r)} = \frac{650000000.0,52\%}{\left[(1+0,52\%)^{60} - 1\right](1+0,52\%)} \approx 9211090$$

So every month Mr. B needs to deposit in the bank 9,211,090 VND
Problem 2: A person deposits A VND in the bank every month, and plans to do something with T VND in the future. How long does that person need to deposit in the bank to have enough money as required? Knowing that the bank interest rate is r per month.
For example: Mr. B deducts 3 million VND from his income every month to deposit in the bank with a compound interest rate of 0.46% per month. How many months does Mr. B need to deposit to have 200 million VND to buy a car? (monthly interest rate remains unchanged).

We have:

$$P_n = \frac{T}{r}\left[(1+r)^n - 1\right](1+r) \Rightarrow n = \frac{\ln\left(\frac{P_n.r}{T} + 1 + r\right)}{\ln(1+r)} - 1$$

$$= \frac{\ln\left(\frac{200000000.0,46\%}{3000000} + 1 + 0,46\%\right)}{\ln(1+0,46\%)} - 1 \approx 58.05$$

So Mr. B needs to deposit for 59 months.

Problem 3: Problem of paying off bank installments or buying goods in installments (paying at the end of the month)
Practical problem: A person borrows an amount of money a dong, after a one-month term with an interest rate for the unpaid amount of r% per month (called calculating interest on the decreasing balance), the number of months of borrowing is n months, after exactly one month from the date of borrowing, that person begins to repay the debt, two consecutive repayments are exactly one month apart, the repayment amount each time is the same, the amount paid to the bank regularly is x dong. Find the formula to calculate x? Knowing that the bank interest rate remains unchanged throughout the loan period.[3]

Construct the formula for the problem:
Let P be the remaining amount after the nth month.
After the first month, the principal and interest are:

$$a + \text{ar} = a(1+r) = a.d \quad (d = 1+r)$$

Pay x VND, then the remaining amount after the first month is:

$$P_1 = ad - x = ad - x\frac{d-1}{d-1}$$

After the second month, the principal and interest are:

$$ad - x + (ad-x)r = (ad-x)(1+r) = (ad-x)d$$

Pay x VND, then the remaining amount after the second month is:

$$P_2 = (ad-x)d - x = ad^2 - xd - x = ad^2 - x(d+1) = ad^2 - x\frac{d^2-1}{d-1}$$

After the third month, the principal and interest are:

$$ad^2 - x(d+1) + \left[ad^2 - x(d+1)\right]r = \left[ad^2 - x(d+1)\right](1+r) = \left[ad^2 - x(d+1)\right]d$$

Pay x VND, then the remaining amount after the third month is:

$$P_3 = \left[ad^2 - x(d+1)\right]d - x = ad^3 - xd^2 - xd - x = ad^3 - x\left(d^2 + d + 1\right) = ad^3 - x\frac{d^3 - 1}{d - 1}$$

The remaining amount after the nth month is:

$$P_n = ad^n - x\frac{d^n - 1}{d - 1} = a(1-r)^n - x\frac{(1+r)^n - 1}{r} \text{ with } d = 1 + r$$

Because after the nth month, the borrower has paid off the entire loan amount, so $P_n = 0$ =>

$$ad^n - x\frac{d^n - 1}{d - 1} = 0 \Leftrightarrow x = \frac{ad^n(d-1)}{d^n - 1} = \frac{a(1+r)^n r}{(1+r)^n - 1}$$

For example: Mr. C borrows 150 million VND from the bank to repair his house, the interest rate is 12% per year. Mr. C repays the bank loan in monthly installments (Exactly one month after the loan date, he starts repaying the loan, two consecutive repayments are exactly one month apart, the repayment amount each time is the same). So how much does Mr. C have to pay the bank each month so that after 2 years, he can pay off the loan? Knowing that the bank interest rate does not change during the time Mr. A repays the loan.
The interest rate is 12% per year, so the interest rate in one month is 1% per year.

Apply the formula $x = \frac{a(1+r)^n r}{(1+r)^n - 1}$

With a = 150,000,000 VND, Interest rate r = 1%, n = 24 months (2 years), P_{24} = 0. Find x?

$$x = \frac{a(1+r)^n r}{(1+r)^n - 1} = \frac{150.000.000(1+0,01)^{24}.0,01}{(1+0,01)^{24} - 1} = 7.060.000$$

So, the amount that Mr. C has to pay each month is 7,060,000 VND.
Through the above problems, the author wants to send readers the applications of exponential and logarithmic functions in real life and in the field of banking and finance. Know how to calculate the following factors: Initial capital P0, interest rate r, deposit term n, total amount of money after n periods T (these problems are often applied in practice, as well as in insurance packages). Depositors who understand the calculation formula will consider whether to buy insurance or not. How much to contribute monthly to have the required amount, thereby having certain knowledge and understanding in life.

II. Conclusion:

Through the article, with real situations, the author wants to convey to the readers that mathematics is very close to our daily lives. And perhaps in reality we still encounter many other situations. The remaining problem is whether we can recognize and apply mathematical knowledge to solve that problem or not. At this point, the author wants to affirm that: mathematics is also practical, it is not completely abstract and it has many applications in practice.

References

[1]. Hoang Dinh Tuan, Theory of Economic Mathematics Models, Science and Technology Publishing House, 2003.
[2]. Nguyen Thanh Hai, Applied Mathematics, Hanoi Pedagogical University, 2005.
[3]. Nguyen Thanh Hai, Methods of Economic Mathematics, Hanoi Pedagogical University, 2008.