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Nonlinear Degradation Modeling and Reliability Assessment Method Based on Diffusion Process

Bo Sun*, Lixin Zhang, Xiangqin wu
School of Mechanical and Vehicle Engineering, Changchun University, Changchun, 130022, Changchun Province, People's Republic of China

Abstract
Aiming at the engineering challenges of scarce fault data for high-reliability and long-life products, and the difficulty of traditional degradation models in adapting to nonlinear and non-monotonic degradation processes, this paper proposes a nonlinear degradation modeling method based on the diffusion process. This method uses the diffusion process to describe the nonlinear degradation law of product performance, and simultaneously introduces a random effect term to characterize individual differences between samples, as well as a measurement error term to correct the systematic deviation of measured data. The probability density function of the failure time when the degradation process first reaches the fault threshold is derived, and the solution method for unknown model parameters based on maximum likelihood estimation is given. Taking the degradation data of CNC machine tool spindle positioning accuracy as an example, the fitting performance of the proposed model is compared with the Wiener process, Gamma process, and inverse Gaussian process models using the log-likelihood value, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The results show that the nonlinear degradation model based on the diffusion process has the optimal fitting performance, can accurately describe the nonlinear degradation characteristics of products under complex working conditions, and provides theoretical support for reliability assessment and remaining useful life prediction of key functional components.
Key words: *diffusion process; nonlinear degradation; reliability assessment; measurement error; remaining useful life prediction; CNC machine tool spindle*

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I. Introduction

Degradation refers to the process in which the performance of a system continuously deteriorates with the extension of service time under the action of working loads and environmental stresses. When the performance degradation reaches a preset fault threshold, the system is judged to have failed [1]. For systems with high reliability, long service life, and high test cost, such as CNC machine tools, aerospace equipment, and nuclear power equipment, very few fault failure data can be obtained during their full life cycle, making traditional reliability assessment methods based on large sample failure data difficult to apply. Therefore, reliability modeling and assessment technology based on performance degradation data has become the core technical approach to solve the problem of reliability quantification for such products.

At present, domestic and foreign scholars have developed a variety of degradation modeling methods for different degradation mechanisms, including random variable models, degradation distribution models, cumulative damage models, and stochastic process models. Among them, the stochastic process model can effectively describe the dynamic randomness in the degradation process, and has become the mainstream method in the field of degradation modeling. Commonly used stochastic processes include the Wiener process, Gamma process, and inverse Gaussian process. The Wiener process can describe non-monotonic degradation processes, but most existing studies assume that its drift coefficient is a linear function of time, or forcibly convert the nonlinear degradation process into a linear form through data transformation, which is difficult to adapt to strongly nonlinear degradation trajectories [2]. The Gamma process and inverse Gaussian process can only describe monotonic and irreversible degradation processes, and cannot characterize the non-monotonic degradation characteristics caused by fluctuating working conditions and environmental changes [3]. In addition, in engineering practice, random errors are inevitable in the measurement of degradation, and ignoring the influence of measurement errors will lead to significant deviations in reliability assessment results.

To address the above problems, Si et al. [2] proposed a nonlinear diffusion process degradation model with measurement errors, which provides a theoretical framework for the modeling of nonlinear and non-monotonic degradation processes. On this basis, this paper systematically constructs a nonlinear degradation modeling system based on the diffusion process, improves the model expression, failure time distribution derivation, and parameter estimation method considering random effects and measurement errors. Taking the degradation data of CNC machine tool spindle positioning accuracy as an engineering example, the model fitting performance verification and reliability assessment application are completed, which provides a practical technical method for degradation modeling and life prediction of key functional components under complex working conditions.

II. Theoretical Basis of Degradation Modeling

2.1 Basic Definition of Degradation Process

Let the performance degradation of the product be $X(t)$, where $t \geq 0$ is the service time, and $X(0)=x_0$ is the initial performance state of the product (usually $x_0=0$). The preset fault threshold for product performance degradation is D. When the degradation process $X(t)$ reaches the threshold D for the first time, the product is judged to have failed, and the corresponding time T is called the First Passage Time (FPT) of the product, which is expressed as:

$$T = int\{t: t \geq 0, X(t) \geq D | X(0) < D\} \tag{1}$$

The core of reliability assessment based on the degradation process is to construct a stochastic process model of $X(t)$ through measured degradation data, derive the probability distribution of the failure time T, and then complete the quantification of product reliability indicators and remaining useful life prediction.

2.2 Limitations of Traditional Stochastic Process Degradation Models

The commonly used stochastic process degradation models in engineering have different applicable boundaries and limitations, as follows:

Wiener Process Model: The Wiener process (Brownian motion) is a Gaussian process with independent increments, which can describe non-monotonic degradation processes. However, its classical form assumes that the drift coefficient is constant and can only characterize linear degradation processes. Although nonlinear fitting can be achieved through time scale transformation, the transformation process will lose the physical characteristics of the original degradation data, resulting in limited fitting accuracy [2].

Gamma Process Model: The increment of the Gamma process follows an independent Gamma distribution, which has non-negative and monotonically increasing characteristics. It is only suitable for describing monotonic and irreversible degradation processes, and cannot adapt to non-monotonic fluctuation scenarios of performance parameters caused by working condition fluctuations [3].

Inverse Gaussian Process Model: The inverse Gaussian process uses the inverse Gaussian distribution to describe degradation increments, and can only characterize monotonic degradation processes, with insufficient fitting ability for nonlinear and non-monotonic degradation trajectories [3].

III. Nonlinear Degradation Model of Diffusion Process with Measurement Errors

3.1 Basic Degradation Model of Diffusion Process

For the nonlinear degradation process, the diffusion process is used to describe the evolution law of the product performance degradation $X(t)$, and its stochastic differential equation is expressed as:

$$dX(t) = \mu(t; \boldsymbol{\theta})dt + \sigma dB(t) \tag{2}$$

where $\mu(t;\boldsymbol{\theta})$ is the time-dependent nonlinear drift coefficient used to characterize the nonlinear trend of the degradation process; $\boldsymbol{\theta}$is the unknown parameter vector corresponding to the drift coefficient; $\sigma$is the diffusion coefficient, which characterizes the random fluctuation intensity of the degradation process; $B(t)$ is the standard Brownian motion, which satisfies the Gaussian distribution with independent increments and zero mean.

To characterize the individual differences between different samples of the same batch of products, the parameter vector $\boldsymbol{\theta}$ is decomposed into two parts:

$$\theta = \varphi + \eta \tag{3}$$

where $\boldsymbol{\varphi}$ is the fixed effect parameter, which characterizes the common characteristics of the degradation process of the same batch of products; $\boldsymbol{\eta}$ is the random effect term, which characterizes the individual differences of different samples. It is assumed that $\boldsymbol{\eta}$ follows a normal distribution with a mean of 0 and a covariance matrix of $\boldsymbol{\tau}^2$, that is, $\boldsymbol{\eta} \sim N(0,\boldsymbol{\tau}^2)$.

3.2 Measured Degradation Model with Measurement Errors

In engineering practice, the measurement results of degradation inevitably have random errors affected by sensor accuracy, measurement environment, personnel operation and other factors. Therefore, the measured degradation $Y(t_i)$ at time t_i can be expressed as [2]:

$$Y(t_i)=X(t_i)+\varepsilon_i \tag{4}$$

where ε_i is an independent and identically distributed measurement error term, which is assumed to follow a normal distribution with a mean of 0 and a variance of σ_ε^2, that is, $\varepsilon_i \sim N(0,\sigma_\varepsilon^2)$; and the measurement error ε_i is independent of the degradation process $X(t)$.

3.3 Probability Density Function of Failure Time

Based on the definition of first passage time, combined with the diffusion process degradation model in Equation (1), the probability density function $f_T(t)$ of the product failure time T can be derived. For the nonlinear drift diffusion process with random effects, the probability density function of the failure time can be expressed as [2]:

$$f_T(t) = \frac{D - \mu(t;\boldsymbol{\varphi})}{\sqrt{2\pi t^3(\sigma^2 + \boldsymbol{\tau}^2 t)}} \exp\left[-\frac{(D - \mu(t;\boldsymbol{\varphi}))^2}{2t(\sigma^2 + \boldsymbol{\tau}^2 t)}\right] \tag{5}$$

Equation (5) is the analytical expression of the product failure time under the nonlinear diffusion process. Based on this equation, reliability indicators such as the reliability function and mean time to failure of the product can be further derived.

3.4 Parameter Solution Based on Maximum Likelihood Estimation

For the measured degradation data, the Maximum Likelihood Estimation (MLE) method is used to solve the unknown parameters of the model. Assume that there are n test samples in the experiment, the measurement time of the i-th sample is $t_{i1},t_{i2},\ldots,t_{im_i}$, with a total of m_i measurement points, and the corresponding measured degradation is $y_{i1},y_{i2},\ldots,y_{im_i}$.

Combined with the diffusion process and the measurement error model , the measured degradation vector $\boldsymbol{Y}_i=[y_{i1},y_{i2},\ldots,y_{im_i}]^T$ follows a multivariate normal distribution, with its mean vector $\boldsymbol{\mu}_i=[\mu(t_{i1};\boldsymbol{\varphi}),\mu(t_{i2};\boldsymbol{\varphi}),\ldots,\mu(t_{im_i};\boldsymbol{\varphi})]^T$, and the covariance matrix $\boldsymbol{\Sigma}_i$, whose elements are jointly determined by the diffusion coefficient σ, the random effect variance $\boldsymbol{\tau}^2$, and the measurement error variance σ_ε^2.

Thus, the log-likelihood function of the unknown parameter set $\boldsymbol{\Theta}=[\boldsymbol{\varphi},\sigma,\boldsymbol{\tau}^2,\sigma_\varepsilon^2]$ of the model can be constructed:

$$\ln\ L(\boldsymbol{\Theta}) = -\frac{1}{2}\sum_{i=1}^{n}[\ln\ |\boldsymbol{\Sigma}_i| + (\boldsymbol{Y}_i - \boldsymbol{\mu}_i)^T\boldsymbol{\Sigma}_i^{-1}(\boldsymbol{Y}_i - \boldsymbol{\mu}_i)] \tag{6}$$

The maximum likelihood estimation of the unknown parameters of the model can be obtained by maximizing the log-likelihood function in Equation (6).

IV. Example Verification and Result Analysis

4.1 Test Data and Evaluation Criteria

To verify the engineering applicability of the proposed nonlinear degradation model based on the diffusion process, this paper uses the degradation data of CNC machine tool spindle positioning accuracy for example analysis [3]. The test objects are 5 CNC machine tool spindles of the same model, and the degradation data of the spindle positioning accuracy during service are collected, with a measurement duration of 500h. The trend of the degradation data is shown in Figure 1. To protect the privacy of the test data, the original data has been standardized, and only its statistical characteristics are retained [3].

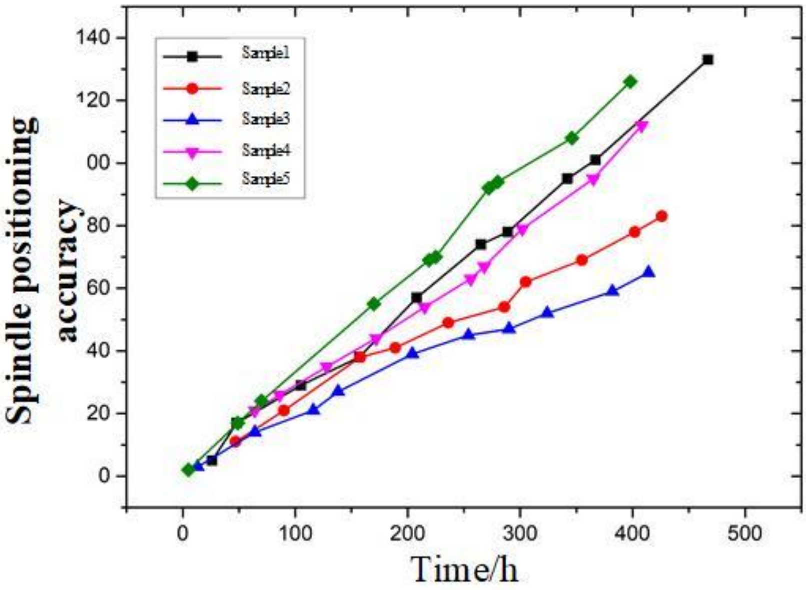

Figure 1 Trend chart of degradation data of CNC machine tool spindle positioning accuracy

To quantitatively evaluate the fitting performance of different degradation models, this paper adopts three mainstream model evaluation indicators: Log-Likelihood value (LL), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) . The calculation formula of each indicator is as follows:

AIC Criterion: Based on information entropy theory, it balances the goodness of fit and parameter complexity of the model, and the expression is:

$$AIC=-2\ln L+2k \tag{7}$$

where k is the number of unknown parameters of the model, and $\ln L$ is the log-likelihood value of the model. The smaller the AIC value, the better the comprehensive fitting performance of the model.

BIC Criterion: Based on the AIC criterion, a sample size penalty term is introduced, which has a stronger constraint on parameter complexity. The expression is:

$$BIC=-2\ln L+k\ln n \tag{8}$$

where n is the total number of sample observations of the degradation data. The smaller the BIC value, the better the generalization ability and fitting performance of the model.

4.2 Comparison of Model Fitting Performance

The Wiener process model, Gamma process model, inverse Gaussian process model, and the nonlinear degradation model based on the diffusion process proposed in this paper are used to fit the spindle positioning accuracy degradation data respectively. The calculation results of the evaluation indicators of each model are shown in Table 1.

Table 1 Comparison of fitting performance of different degradation models

Degradation Model	LnL	AIC	BIC
Wiener Process	-135.553	275.1064	278.9304
Gamma Process	-135.44	274.8802	277.9909
Inverse Gaussian Process Model	-137.571	279.1413	282.252
Nonlinear Diffusion Process	-135.281	274.5622	277.6729

It can be seen from the results in Table 1 that compared with the three traditional stochastic process models, the nonlinear degradation model based on the diffusion process proposed in this paper has the largest log-likelihood value, and the lowest AIC value and BIC value. This indicates that the model can better describe the nonlinear degradation trajectory of CNC machine tool spindle positioning accuracy, achieve the optimal balance between goodness of fit and model complexity, and its comprehensive fitting performance is significantly better than traditional models.

4.3 Reliability Assessment Results

The failure threshold of CNC machine tool spindle positioning accuracy is set as D=140. Based on the fitted nonlinear diffusion process degradation model, combined with the failure time distribution in Equation (3), the first passage failure times of the 5 spindles are calculated as: 894.83h, 1023.25h, 1071.52h, 917.48h, and 865.43h. This result can provide a quantitative reliability basis for the optimization of preventive maintenance cycle, spare parts management and life extension design of CNC machine tool spindles.

V. Conclusion

Aiming at the modeling problem of nonlinear and non-monotonic degradation processes of products under complex working conditions, this paper constructs a nonlinear degradation model of diffusion process with random effects and measurement errors, systematically improves the model expression, failure time distribution derivation and parameter estimation method, and completes the model verification through a CNC machine tool spindle degradation example. The main conclusions are as follows:

The nonlinear degradation model based on the diffusion process can effectively break through the limitation of traditional models that can only deal with linear or monotonic degradation processes, and simultaneously take into account the influence of sample individual differences and measurement errors, which is more in line with the actual engineering degradation scenarios.

The example analysis results show that the fitting effect of the model on the degradation data of CNC machine tool spindle positioning accuracy is significantly better than that of the Wiener process, Gamma process and inverse Gaussian process models, with higher fitting accuracy and engineering applicability.

Based on the established model, the failure time distribution of the product can be accurately derived, the reliability quantitative assessment and remaining useful life prediction of high-reliability products can be realized, which provides technical support for the full life cycle reliability management of key functional components.

Future research can further combine multi-source degradation information and accelerated degradation test data to improve the adaptability of the model in small sample and high-dimensional degradation scenarios. At the same time, the model can be combined with real-time monitoring data to construct an online dynamic prediction method for the remaining useful life of products.

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