

A New Template for Magic Squares

Tejmal Rathore¹ and Rahul Kumar Sharma²

¹Independent Researcher,

G-803, Country Park, Dattapada Road, Borivali (E), Mumbai 400066, India

²School of Computer Science & Information Technology,

Devi Ahilya Vishwavidyalaya, Takshashila Campus,

Khandwa Road, Indore - 452020, India.

ABSTRACT - A new template for a magic square (MS) of order 4, which has four specified numbers located at the centre, and the sum (S) of the numbers of four 2×2 corner squares and the four rows is the same, is derived. It has two variables which provide the flexibility to obtain different MSs. Interestingly, it also gives the same S for the four columns, two diagonals, four corners and middle elements of rows 1 and 4, middle elements of columns 1 and 4. Conditions are derived so that all the cells have distinct non-negative numbers. It is shown how the MSs with any desired sum can be derived from a MS of known S. This method can be used to find MSs when only S is specified.

Date of Submission: 10-04-2026

Date of acceptance:21-04-2026

I. INTRODUCTION

Magic square (MS) of size 4 that deals with 4 numbers (each of 2 digits) of birthdate is popular [1]. The numbers are placed in the first row of the square. Sharma and Rathore [2] have suggested a template shown in Equation (1) in which two variables x and y are placed side by side in the first row.

$$M_S = \begin{bmatrix} x & y & (c+d) - y & (a+b) - x \\ (b+c+d) - (x+y) & a & b & (x+y) - b \\ (a+b-d) - (x-y) & c & d & d + (x-y) \\ x - (b-d) & (b+d) - y & y - (d-a) & (b+c) - x \end{bmatrix} \quad (1)$$

This paper presents a new template which has the variables x and y at the two corners of the first row in Section 2. There is no restriction on the number of digits and the digits can be 0. The restrictions are brought out so that the square has all the numbers distinct non-negative (DNN). It is shown how squares of different sums can be derived from the one of known S in Section III.

Consider a square matrix of order 4 shown in Equation (2).

$$M_S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (2)$$

Let us assume that the central square has *specified* distinct non-negative elements a, b, c, d , i.e.,

$$S_{22} = a; S_{23} = b; S_{32} = c; S_{33} = d, \quad (3)$$

such that

$$a + b + c + d = S \quad (4)$$

Let us assume

$$S_{11} = x \geq 0 \quad (5)$$

and

$$S_{14} = y \geq 0 \tag{6}$$

Then Equation (1) becomes

$$M_S = \begin{bmatrix} x & S_{12} & S_{13} & y \\ S_{21} & a & b & S_{24} \\ S_{31} & c & d & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}. \tag{7}$$

II. PROPOSED TEMPLATE

The proposed template is shown in Equation (8).

$$M_S = \begin{bmatrix} x & (c+d) - y & (a+b) - x & y \\ c - x & a & b & x + d \\ b + y & c & d & a - y \\ (a+d) - y & y + (b-c) & x - (b-c) & (b+c) - x \end{bmatrix} \tag{8}$$

It has the same specified sum for all the numbers in rows, columns, diagonals, corners, middle numbers of first and fourth rows, middle numbers of first and fourth columns irrespective of x and y ; hence the name MS. Since x and y are single variables, they cannot be negative and equal.

2.3 Constraints

All cells will be distinct non-negative, from Equation (8), if

$$x, y \neq 0, x \neq y, S_k \neq a, b, c, d, \quad k = 11, 14 \tag{9}$$

$$\max\{(b-c), -d, 0\} \leq x \leq \min\{(a+b), c\}, \tag{10}$$

$$\{(c-b) - b\} \leq y \leq \{c+d\}, (a+d), a\}. \tag{11}$$

$$S_k \neq x, y, a, b, c, d, \quad k = 12, 13, 21, 24, 31, 34, 41, 42, 43, 44 \tag{12}$$

In view of the conditions given by Equation (9), it is possible that there may not be any MS at all, or may be multiple MSs. It is advisable to check using a computer as the number may be large.

2.4 Procedure

1. Find the constraints on x and y from Equations (9)-(12).
2. Choose any one permissible pair $\{x, y\}$
3. Find MS form Equation (8).

2.5 Examples

Example 1: Let $\{a, b, c, d\} = \{29, 10, 19, 43\}, S = 101$

Substituting these values into Equation (8), we get

$$M_S = \begin{bmatrix} x & 62 - y & 39 - x & y \\ 19 - x & 29 & 10 & x + 24 \\ 10 + y & 19 & 43 & 13 - y \\ -y + 72 & y - 9 & x + 9 & 29 - x \end{bmatrix}.$$

From Equations (9)-(12), the constrains are

$$\begin{aligned} x, y \neq 0, x \neq y, S_k \neq 29, 10, 19, 43, \quad k = 11, 14 \\ 0 \leq x \leq 19, \\ 9 \leq y \leq 29. \end{aligned}$$

$$S_k \neq x, y, a, b, c, d, \quad k = 12,13,21,24,31,34,41,42,43,44$$

Choosing $\{x, y\} = \{15, 12\}$ and $\{14, 27\}$, we get from Equation (8),

$$M_{101} = \begin{matrix} \begin{bmatrix} 15 & 50 & 24 & 12 \\ 04 & 29 & 10 & 58 \\ 22 & 19 & 43 & 17 \\ 60 & 03 & 24 & 14 \end{bmatrix} \\ \text{(a)} \end{matrix}, \quad M_{101} = \begin{matrix} \begin{bmatrix} 14 & 35 & 25 & 27 \\ 05 & 29 & 10 & 57 \\ 37 & 19 & 43 & 02 \\ 45 & 18 & 23 & 15 \end{bmatrix} \\ \text{(b)} \end{matrix}, \quad (13)$$

Example 2: Let $\{a, b, c, d\} = \{100, 150, 200, 250\}$.

Constraints are

$$\begin{aligned} x, y \neq 0, x \neq y, S_k \neq 100,150,200,250, \quad k = 11,14 \\ 0 \leq x \leq 200, \\ 0 \leq y \leq 100. \\ S_k \neq x, y, a, b, c, d, \quad k = 12,13,21,24,31,34,41,42,43,44 \end{aligned}$$

Choosing $\{x, y\} = \{70, 90\}$,

$$M_{700} = \begin{bmatrix} 070 & 360 & 180 & 090 \\ 170 & 100 & 150 & 280 \\ 010 & 200 & 250 & 240 \\ 450 & 040 & 120 & 090 \end{bmatrix}. \quad (14)$$

Example 3: Let $\{a, b, c, d\} = \{12, 05, 09, 03\}$, $S = 29$.

The constraints are

$$\begin{aligned} x, y \neq 0, x \neq y, S_k \neq 12,05,09,03, \quad k = 11,14 & (15) \\ \max \{(-04, c), -d, 0\} \leq x \leq \min \{(a + b), c\}, & (16) \\ \{(c - b) - b\} \leq y \leq \{c + d\}, (a + d), a\}. & (17) \\ S_k \neq x, y, a, b, c, d, \quad k = 12,13,21,24,31,34,41,42,43,44 & (18) \end{aligned}$$

None of the $\{x, y\}$ values restricted by Equations (16)-(17) satisfy Equation (18). Therefore, there is no MS.

If

$$y = c - b \quad (19)$$

then all the peripheral 2×2 squares and both the bent diagonals will also have the sum S , however, there will be repetition of numbers.

III. APPLICATIONS

3.1 Extensions of the method

The method is general and is applicable to any set of 4 numbers, such as birthdate. One can choose a fixed but valid value for x (y), and get various squares by varying y (x) alone. However, a single square is good enough, unless one has some particular requirement. These squares can be used for decorating tiles, wall papers, screen, sending birthday wish, etc.

3.2 Deriving MSs for different S

(a) To get the MS for any integer $S + n$, add n to 4 numbers each one of which lies in a different row, different column and different diagonal. Let us assume that S_{41} is one of the numbers. Then no other number will lie on the fourth row, first column and the reverse diagonal. Now consider the forward diagonal. The other number can be either S_{22} or S_{33} . Case 1: Choose say S_{33} . Now the remaining two numbers have to be S_{12} and S_{24} . Thus, the 4 numbers are $\{S_{41}, S_{33}, S_{12}, S_{24}\}$. Case 2: Choose S_{22} .

In this case the numbers are {S41, S22, 13, S34}. Similar procedure can be used with each of the remaining 3 corner numbers S11, S14, S44. The n can take only those values which give DNN numbers. Thus, in all there will be 8 possible sets of 4 numbers.

Example 4: In M_{101} given in (c) of Equation (14), let us take the case 1, and choose the four numbers {45,43,57,35}. It can be verified that n can be ≥ 3 for DNN numbers. Two magic squares are given in Equation (20), choosing $n = 3$ and 9.

$$M_{101+3=104} = \begin{bmatrix} 14 & \mathbf{38} & 25 & 27 \\ 05 & 29 & 10 & \mathbf{60} \\ 37 & 19 & \mathbf{46} & 02 \\ \mathbf{48} & 18 & 23 & 15 \end{bmatrix}, \quad M_{101+9=110} = \begin{bmatrix} 14 & \mathbf{44} & 25 & 27 \\ 05 & 29 & 10 & \mathbf{66} \\ 37 & 19 & \mathbf{52} & 02 \\ \mathbf{54} & 18 & 23 & 15 \end{bmatrix}, \quad (20)$$

and M_{90} is obtained by decreasing the 4 numbers by 11.

$$M_{101-11=90} = \begin{bmatrix} 14 & \mathbf{24} & 25 & 27 \\ 05 & 29 & 10 & \mathbf{46} \\ 37 & 19 & \mathbf{32} & 02 \\ \mathbf{34} & 18 & 23 & 15 \end{bmatrix}. \quad (21)$$

The lowest possible value of S is obtained by subtracting a number n such that the lowest of the highest 4 numbers (24) becomes 0. Thus, the lowest possible $S = 101 - 35 = 66$.

$$M_{66} = \begin{bmatrix} 14 & \mathbf{00} & 25 & 27 \\ 05 & 29 & 10 & \mathbf{22} \\ 37 & 19 & \mathbf{08} & 02 \\ \mathbf{10} & 18 & 23 & 14 \end{bmatrix}. \quad (22)$$

Let us take case 2 and choose the four numbers {45,29,25,02}. It can be verified that n can be ≥ 19 for DNN numbers. Choosing $n = 19$ and 39, we get

$$M_{101+19=120} = \begin{bmatrix} 14 & 35 & \mathbf{44} & 27 \\ 05 & \mathbf{48} & 10 & \mathbf{66} \\ 37 & 19 & 43 & 02 \\ \mathbf{64} & 18 & 23 & 15 \end{bmatrix}, \quad M_{101+39=140} = \begin{bmatrix} 14 & 35 & \mathbf{64} & 27 \\ 05 & \mathbf{68} & 10 & \mathbf{86} \\ 37 & 19 & 43 & 02 \\ \mathbf{84} & 18 & 23 & 15 \end{bmatrix} \quad (23)$$

Obviously, in above operations the central square changes to accommodate new value of S .

(b) If a sum S is specified, it can be expressed as a sum of 4 distinct non-negative numbers. There are many such tuples possible. One MS for $S = 257$ is shown as M_{257p} where $\{a, b, c, d\} = \{29, 10, 19, 199\}$. The one viral on social media shown as M_{257v} has 007 duplicated [3]. One may conclude from the demonstration of the video that it is restricted to 3 digits, none can be zero.

$$M_{257p} = \begin{bmatrix} 015 & 191 & 024 & 027 \\ 004 & 029 & 010 & 214 \\ 037 & 019 & 199 & 002 \\ 201 & 018 & 024 & 014 \end{bmatrix}, \quad M_{257v} = \begin{bmatrix} \mathbf{007} & 197 & 042 & 011 \\ 012 & 041 & 200 & 004 \\ 201 & 009 & \mathbf{007} & 040 \\ 037 & 010 & 008 & 203 \end{bmatrix}, \quad (24)$$

IV. CONCLUSION

A new template for a magic square (MS) of order 4, which has four specified numbers located at the centre, and the sum (S) of the numbers of four 2×2 corner squares and the four rows is the same, is derived. It has two independent variables which provides the flexibility to obtain different MSs. Interestingly, it also gives the same S for the four columns, two diagonals, four corners. Conditions are

derived so that all the cells have distinct non-negative numbers. MSs with any desired sum (within the restricted values) can be derived from that of a known S . This method can be used to find MSs when only S is specified without any restrictions of the number of digits specified.

REFERENCES

- [1] Patel Akshay Zaverbhai, 2023, Srinivasa Ramanujan Magic Square (formula), International Journal for Research Trends and Innovation, 8(12), 18-21
- [2] Sharma Rahul Kumar and Rathore Tejmal, 2025, Magic Squares with specified central square, 2025, submitted.
- [3] <https://youtu.be/rVBRICEgbXE?si=fr0ajuZlvjQQHV>