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Calculation of Dislocation Love Numbers Based on Earth Normal Mode Theory

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ABSTRACT: *Based on the spherically symmetric, non-rotating, elastic and isotropic Earth model, this paper presents a systematic method for calculating dislocation Love numbers using Earth normal mode theory, offering an alternative to conventional numerical integration approaches. First, the accuracy of normal mode-derived coseismic displacement calculations is verified by synthesizing fundamental-mode seismograms and comparing them with results from the Mineos software. With the validated method, seismic dislocations are represented as equivalent body forces, and normal mode expressions for coseismic displacement and gravitational potential perturbations induced by four basic fault types (vertical strike-slip, vertical dip-slip, vertical tensile, and horizontal tensile faults) are derived, along with the corresponding formulas for dislocation Love numbers. Numerical calculations of 2nd- to 5th-degree dislocation Love numbers for the Preliminary Reference Earth Model (PREM) are performed using Mineos, and the results show good agreement with reference values obtained from the reciprocal theorem. This method features a clear physical picture and helps clarify the relationship between Earth normal modes and global deformation responses, providing a practical theoretical basis for studying seismic dislocation-induced Earth deformation.*
Keywords: *Dislocation Love numbers; Earth's normal modes; SNREI Earth model; seismic dislocation; PREM.*

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I. INTRODUCTION

Earth normal modes represent the intrinsic free oscillations of the Earth under perturbation and constitute one of the fundamental theoretical frameworks in global geophysics and theoretical seismology. In the framework of continuum mechanics, the oscillatory response of a spherically symmetric elastic Earth can be expressed as a series of discrete eigenfunctions associated with specific eigenfrequencies. These eigenmodes provide essential constraints on the Earth's internal density distribution, elastic structure, and dynamic properties. The concept of Earth free oscillations can be traced back to Kelvin, who approximated the Earth as an elastic body and estimated its characteristic oscillation period using shear-wave propagation velocity. Lamb later established the mathematical basis of normal mode theory by deriving the governing equations for a homogeneous elastic sphere and demonstrating that the solutions can be divided into spheroidal and toroidal modes under stress-free boundary conditions.

The development of modern normal mode theory accelerated significantly during the mid-twentieth century. Alterman et al. [1] transformed the second-order governing equations into a first-order ordinary differential eigenvalue problem, thereby establishing the foundation for modern numerical integration techniques used in normal mode calculations. Subsequently, Benioff et al. [2] and Slichter [3] successfully observed Earth normal modes following large earthquakes, confirming the physical existence of global free oscillations.Since the 1960s, Earth normal modes have been extensively used to investigate the Earth's internal structure, mantle dynamics, and large-scale geodynamic processes. Gilbert and Dziewonski [4] used normal mode observations to invert the 1066A Earth model, while Dziewonski and Anderson [5] incorporated large numbers of normal-mode constraints into the construction of the Preliminary Reference Earth Model (PREM), which remains one of the most widely used global reference Earth models. Owing to the orthogonality and completeness of normal-mode eigenfunctions, normal mode theory has also been widely applied in synthetic seismogram calculations, seismic source inversion, and global deformation studies.

Love numbers are dimensionless parameters that characterize the deformation response of the Earth to external or internal forcing. Love [6] first introduced the parameters *h* and *k* in tidal deformation theory to describe radial displacement and gravitational potential perturbation, respectively, and Shida [7] later introduced the horizontal displacement parameter *l*. Longman [8,9] and Farrell [10] subsequently extended the concept to load Love numbers for surface loading problems, whereas Saito [11] proposed shear Love numbers to investigate deformations caused by surface shear stresses.Theoretical studies of seismic dislocation-induced deformation in a spherical Earth model were significantly advanced by Sun and Okubo [12,13], who established a comprehensive point-source dislocation theory and introduced dislocation Love numbers for describing global coseismic deformation. Subsequent studies further extended the theory to gravity-field variation, Earth-rotation excitation, and coseismic geodynamic responses associated with giant earthquakes [12–16]. Recent investigations have also emphasized the importance of normal-mode coupling and low-degree gravitational perturbations in understanding global seismic deformation processes [17–19].

Compared with conventional propagator-matrix or direct numerical integration methods, the normal-mode approach provides a clearer physical interpretation of global deformation processes because the coseismic response of the Earth can be explicitly represented as a weighted superposition of eigenmodes. In this framework, the excitation amplitude of each mode is directly controlled by the spatial distribution and symmetry of the seismic source, whereas the deformation characteristics are determined by the Earth's internal elastic-gravitational structure. Nevertheless, systematic derivations of dislocation Love numbers using Earth normal mode theory remain relatively limited.

Therefore, this study presents a systematic derivation of dislocation Love numbers based on Earth normal mode theory. First, the theoretical framework of normal modes and their representation of static displacement and gravitational potential perturbations are introduced. Subsequently, analytical expressions for dislocation Love numbers corresponding to four fundamental fault types are derived. Finally, numerical calculations based on the PREM model are carried out using Mineos, and the obtained results are validated against reference solutions derived from the reciprocal theorem.

II. BASIC THEORY OF EARTH NORMAL MODES

2.1 Fundamental Concepts of Normal Modes

Taking the free vibration of a one-dimensional string fixed at both ends as an example, the transverse vibration of the string satisfies the wave equation, and eigenfrequencies and eigenfunctions can be obtained via the separation of variables method. Each eigenfrequency corresponds to a normal mode, and the displacement of the string can be expressed as a linear superposition of all normal modes. Similarly, the Earth, as an elastic sphere, possesses a series of inherent oscillation patterns—Earth normal modes. Under the SNREI Earth model, the displacement field of the Earth’s free oscillations can be decomposed into two categories: spheroidal modes and toroidal modes. The former contains both radial and horizontal displacement components and induces gravitational potential perturbations, while the latter has only horizontal displacement components and does not cause gravitational potential perturbations.

2.2 Static Displacement Field

According to Gilbert’s (1971) theory, when a large earthquake occurs, the Earth deforms, and the time-dependent displacement can be expressed as a linear superposition of normal modes. For static deformation (the limit case of zero frequency), the displacement field can be written as a sum over all normal modes:

$$\mathbf{u}(\mathbf{r}) = \sum_n \mathbf{s}_n(\mathbf{r})\left(\omega_n^{-2}\int_\Omega \mathbf{s}_n^*(\mathbf{r}')\cdot \boldsymbol{f}(\mathbf{r}')d\Omega'\right) = \sum_n C_f^n \mathbf{s}_n(\mathbf{r}) \tag{1}$$

where $\mathbf{u}(\mathbf{r})$ is the static displacement field, $\mathbf{s}_n(\mathbf{r})$ is the eigen-displacement function of the n-th normal mode (including both spheroidal and toroidal modes), ω_n is the eigenfrequency, the superscript * denotes complex conjugation, the integration domain Ω represents the entire Earth volume, and $\boldsymbol{f}(\mathbf{r}')$ is the body force density acting on the Earth. The coefficient $C_f^n=\omega_n^{-2}\int_\Omega \mathbf{s}_n^*(\mathbf{r}')\cdot\boldsymbol{f}(\mathbf{r}')d\Omega'$ is the projection of the force source onto the normal mode, called the normal mode combination coefficient.

2.3 Gravitational Potential Perturbation

The redistribution of Earth’s internal mass caused by earthquakes leads to changes in the external gravitational field. According to Newton’s law of universal gravitation, the gravitational potential perturbation $\Delta\varphi(\mathbf{r}_0)$ at an external point $\mathbf{r}_0$ can be expressed as:

$$\Delta\varphi(\mathbf{r}_0) = G\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\frac{1}{r_0^{l+1}}\Gamma_{lm}Y_{lm}^*(\theta_0,\phi_0) \tag{2}$$

where$G$is the gravitational constant, Y_{lm} is the fully normalized spherical harmonic function, and l and m are the degree and order of the spherical harmonic function, respectively. The coefficient Γ_{lm} is related to the displacement field by:

$$\Gamma_{lm} = \int_{\Omega}\rho(r)r^{l-1}\mathbf{u}(\mathbf{r})\cdot\left[lY_{lm}(\theta,\phi)\hat{\mathbf{r}} + \nabla_1 Y_{lm}(\theta,\phi)\right]d\Omega \tag{3}$$

where $\rho(r)$ is the Earth's density, and $\nabla_1=\hat{\boldsymbol{\theta}}\partial_\theta+\hat{\boldsymbol{\phi}}\csc\theta\partial_\phi$. Here, we use the Lagrangian description, calculating the integral using the density and positions before deformation.

III. CALCULATION OF DISLOCATION LOVE NUMBERS

3.1 Definition of Dislocation Love Numbers

Similar to tidal, load, and shear Love numbers, dislocation Love numbers are a set of dimensionless parameters introduced to describe the Earth's response to different dislocation force sources[13]. For the SNREI Earth model, due to spherical symmetry, the independent solutions of the equations reduce to four basic types, corresponding to: (1) vertical strike-slip fault rupture (12), (2) vertical dip-slip fault rupture (32), (3) vertical tensile fault rupture (22), and (4) horizontal tensile fault rupture (33). Schematic diagrams of the four independent sources are shown in Figure 1.

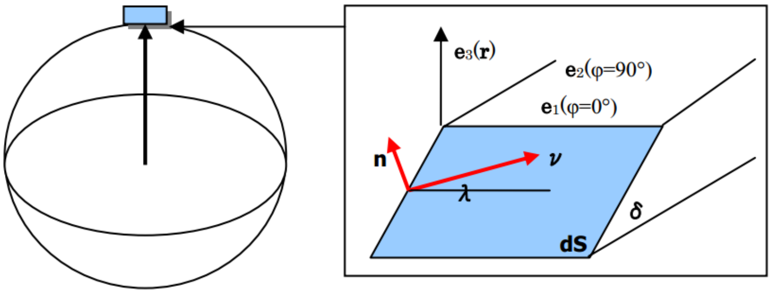

Fig. 1 Dislocation model of local coordinate system

At the Earth's surface, the dislocation Love numbers $h_{lm}^{ij}, l_{lm}^{ij}, l_{lm}^{t,ij}$, and k_{lm}^{ij} are defined from the spherical harmonic coefficients of the displacement and gravitational perturbation fields induced by fault dislocations, as expressed in Equation (4):

$$\left.\begin{aligned} h_{lm}^{ij} &= y_{1,m}^{l,ij}(a)\cdot a^2 \\ l_{lm}^{ij} &= y_{3,m}^{l,ij}(a)\cdot a^2 \\ l_{lm}^{t,ij} &= y_{1,m}^{t,l,ij}(a)\cdot a^2 \\ k_{lm}^{ij} &= y_{5,m}^{l,ij}(a)\cdot\frac{a^2}{g_0} \end{aligned}\right\} \tag{4}$$

Here, l and m denote the spherical harmonic degree and order, respectively, which describe the spatial variability of the dislocation-induced responses. The superscript ij identifies the four fundamental dislocation source types associated with different faulting mechanisms.The quantities $y_1^{l,ij}(a), y_2^{l,ij}(a), y_3^{l,ij}(a)$, and $y_5^{l,ij}(a)$ represent the spherical harmonic coefficients of radial displacement, tangential displacement, and gravitational potential perturbation, evaluated at the Earth's surface (r=a). The factor a^2 is introduced to nondimensionalize the displacement-related coefficients, while$g_0$the mean surface gravitational acceleration, is used in the definition of k_{lm}^{ij} to ensure consistency with standard dimensionless Love number conventions.

The four fundamental source types—vertical strike-slip, vertical dip-slip, vertical tensile, and horizontal tensile faulting—are illustrated in Figure 2, which provides schematic representations of the corresponding idealized rupture geometries.

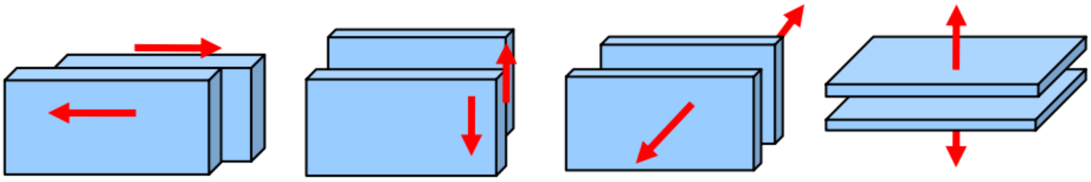

Fig. 2 Four independent dislocation model

The coseismic displacement can be expressed as:

$$u_r^{ij}(a,\theta,\phi)=\sum_{i,j}\left[u_r^{ij}e_r+u_\theta^{ij}e_\theta+u_\phi^{ij}e_\phi\right]\cdot v_i n_j\frac{UdS}{a^2} \tag{5}$$

where v_i and n_i are the slip vector and normal vector, respectively, U denotes the dislocation magnitude on surface element dS, a is the mean Earth radius, u_r^{ij}, u_θ^{ij}, and u_ϕ^{ij} are the radial and horizontal displacement components, e_r, e_θ, and e_ϕ are the unit basis vectors in radial, colatitude, and longitude directions, respectively. The components in the above equation are:

$$\left.\begin{aligned}
u_r^{ij}(a,\theta,\phi)&=\sum_{l,m,i,j}h_{lm}^{ij}Y_{lm}(\theta,\phi)\\
u_\theta^{ij}(a,\theta,\phi)&=\sum_{l,m,i,j}l_{lm}^{ij}\frac{\partial Y_{lm}(\theta,\phi)}{\partial\theta}+\sum_{l,m,i,j}l_{lm}^{t,ij}\frac{\partial Y_{lm}(\theta,\phi)}{\partial\phi}\\
u_\phi^{ij}(a,\theta,\phi)&=\sum_{l,m,i,j}l_{lm}^{ij}\frac{1}{\sin\theta}\frac{\partial Y_{lm}(\theta,\phi)}{\partial\phi}-\sum_{l,m,i,j}l_{lm}^{t,ij}\frac{\partial Y_{lm}(\theta,\phi)}{\partial\theta}
\end{aligned}\right\} \tag{6}$$

The coseismic gravitational potential change can be written as:

$$\psi(a,\theta,\phi)=\sum_{l,m,i,j}k_{lm}^{ij}Y_l^m(\theta,\phi)\cdot v_i n_j\cdot\frac{g_0 UdS}{a^2} \tag{7}$$

3.2 Seismic Moment Tensor Representation

The key to calculating displacement and gravitational potential changes for different dislocation types using the normal mode method is to compute the normal mode combination coefficient $C_f^n=M:E_k^*(r_s)$. The seismic moment tensor M_{ij} can be expressed as:

$$M_{ij}=\lambda DS\delta_{ij}(\mathbf{n}\cdot\mathbf{v})+\mu DS(n_i v_j+n_j v_i) \tag{8}$$

where λ and μ are the Lamé parameters, D is the dislocation amount, S is the fault area, $\mathbf{n}$ is the unit normal vector to the fault, and $\mathbf{v}$ is the unit slip direction vector.

To simplify calculations, we place the source at the North Pole. The strain tensor expressed in terms of normal modes can be calculated from the eigenfunctions. The conjugate strain components for spheroidal and toroidal fields are shown in Tables 1 and 2, respectively.

$$M_{ij}=\delta_{ij}v_k n_k\lambda DS+\left(v_j n_i+v_i n_j\right)\mu DS \tag{9}$$

The seismic moment tensor is transformed from the Cartesian coordinate system to the spherical coordinate system with the following correspondence：

$$\left(M_{zz},M_{xx},M_{yy},M_{zx},M_{zy},M_{xy}\right)=\left(M_{rr},M_{\theta\theta},M_{\varphi\varphi},M_{r\theta},-M_{r\phi},-M_{\theta\phi}\right) \tag{10}$$

To simplify calculations, the source is placed at the North Pole, and the strain tensor expressed via normal modes is defined as $\lim_{\theta\to 0}E(\theta,\varphi)$. Strain components for $|m|>2$ are zero, leading to the coseismic strain components E_{ij}^{*} for spheroidal modes at $\theta\to 0$.

Table 1 Conjugate strain components for spheroidal modes

S	$m=0$	$m=\pm 1$	$m=\pm 2$
E_{rr}^{*}	$b_0 U'$	0	0
$E_{\theta\theta}^{*}$	$b_0\left[U'+\frac{(l-1)(l+2)}{2r}V\right]$	$b_0\left[\frac{1}{2}l(l+1)\left(\frac{U}{r}-V'\right)\right]$	$e_{\theta\theta}$
$E_{\phi\phi}^{*}$	$b_0\left[U'+\frac{(l-1)(l+2)}{2r}V\right]$	$b_0\left[\frac{1}{2}l(l+1)\left(\frac{U}{r}-V'\right)\right]$	$-e_{\theta\theta}$
$2rE_{r\theta}^{*}$	0	0	$2re_{\theta\theta}$
$2rE_{r\phi}^{*}$	0	$-2rie_{\theta\theta}$	0
$2E_{\theta\phi}^{*}$	0	0	$ime_{\theta\theta}$

Note. $U(r)$ and $V(r)$ are the radial and horizontal displacement eigenfunctions of spheroidal modes, respectively; $W(r)$ is the horizontal displacement eigenfunction of toroidal modes.

Table 2 Conjugate strain components for toroidal modes

T	$m=0$	$m=\pm 1$	$m=\pm 2$
E_{rr}^{*}	0	0	0
$E_{\theta\theta}^{*}$	0	0	$imb_0\frac{(l-1)(l+2)}{8r}W$
$E_{\phi\phi}^{*}$	0	0	$-e_{\theta\theta}$
$2rE_{r\theta}^{*}$	0	0	$b_0\left[\frac{1}{2}l(l+1)\left(W'-\frac{W}{r}\right)\right]$
$2rE_{r\phi}^{*}$	0	$b_0\left[\frac{1}{2}l(l+1)\left(W'-\frac{W}{r}\right)\right]$	0
$2E_{\theta\phi}^{*}$	0	0	$b_0\frac{(l-1)(l+2)}{2r}W$

3.3 Dislocation Love Numbers for Four Basic Fault Types

(1) Vertical Strike-Slip Fault Rupture (12)

For a vertical strike-slip fault rupture, we have **n**=(0,1,0) and **v**=(1,0,0), meaning all moment tensor components are zero except $M_{12}=M_{21}=\mu DS$. The expression of the seismic moment tensor in spherical coordinates is:

$$M^{12}=\begin{bmatrix}0 & -\mu DS & 0\\ -\mu DS & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\xrightarrow{(r,\theta,\varphi)}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & \mu DS\\ 0 & \mu DS & 0\end{bmatrix} \tag{11}$$

$$\left.\begin{aligned}C_f^{12}&=\frac{imb_0\sqrt{l(l+2)(l^2-1)}}{4}\frac{V}{r}\mu DS\\ C_f^{t,12}&=\frac{-b_0\sqrt{l(l+2)(l^2-1)}}{2}\frac{W}{r}\mu DS\end{aligned}\right\} \tag{12}$$

Substituting C_f^{12} into the displacement formula(1)and gravitational potential variation formula (2), and $C_f^{t,12}$ into the displacement formula, the Love numbers are obtained as:

$$\begin{aligned} h_{l2}^{12} &= \sum_{n=1}^{\infty} \frac{-c_{l2} b_0 \mu u_{nl} a^2 \omega^{-2} \sqrt{l(l+2)\left(l^2-1\right)}}{2} \frac{V_{nl}}{r} \\ l_{l2}^{12} &= \sum_{n=1}^{\infty} \frac{-c_{l2} b_0 \mu v_{nl} a^2 \omega^{-2} \sqrt{l(l+2)\left(l^2-1\right)}}{2} \frac{V_{nl}}{r} \\ l_{l2}^{t,12} &= \sum_{n=1}^{\infty} \frac{c_{l2} b_0 \mu w_{nl} a^2 \omega^{-2} \sqrt{l(l+2)\left(l^2-1\right)}}{2} \frac{W_{nl}}{r} \\ k_{l2}^{12} &= \sum_{n=1}^{\infty} \frac{-2\pi G c_{l2} b_0 \mu \sqrt{l(l+2)\left(l^2-1\right)}}{(2l+1) g_0 a^{l-1}} \frac{V_{nl}}{r} F_{nl} \end{aligned} \tag{13}$$

(2) Vertical Dip-Slip Fault Rupture (32)

For a vertical dip-slip fault rupture, we have **n**=(0,1,0) and **v**=(0,0,1), meaning all components are zero except M_{23}=M_{32}=μDS. The moment tensor components in spherical coordinates are:

$$M^{32} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mu DS \\ 0 & -\mu DS & 0 \end{bmatrix} \xrightarrow{(r,\theta,\varphi)} \begin{bmatrix} 0 & 0 & \mu DS \\ 0 & 0 & 0 \\ \mu DS & 0 & 0 \end{bmatrix} \tag{14}$$

$$\left. C_f^{32} = \frac{-b_0\sqrt{l(l+1)}}{2}\left[V' + r^{-1}(U-V)\right]\mu DS \right\} \tag{15}$$

Substituting $C_f^{22,0}$ into the displacement formula (1) and gravitational potential variation formula (2), the Love numbers are obtained as:

$$\left. \begin{aligned} h_{l1}^{32} &= \sum_{n=1}^{\infty} \frac{c_{l1} b_0 \mu u_{nl} a^2 \omega^{-2} \sqrt{l(l+1)}}{2} \left[V_{nl'} + r^{-1}\left(U_{nl} - V_{nl}\right)\right] \\ l_{l1}^{32} &= \sum_{n=1}^{\infty} \frac{c_{l1} b_0 \mu v_{nl} a^2 \omega^{-2} \sqrt{l(l+1)}}{2} \left[V_{nl'} + r^{-1}\left(U_{nl} - V_{nl}\right)\right] \\ l_{l1}^{t,32} &= \sum_{n=1}^{\infty} \frac{c_{l1} b_0 \mu w_{nl} a^2 \omega^{-2} \sqrt{l(l+1)}}{2} \left[W' - r^{-1} W\right] \\ k_{l1}^{32} &= \sum_{n=1}^{\infty} \frac{2\pi G c_{l1} b_0 \mu \sqrt{l(l+1)}}{(2l+1) g_0 a^{l-1}} \left[V_{nl'} + r^{-1}\left(U_{nl} - V_{nl}\right)\right] F_{nl} \end{aligned} \right\} \tag{16}$$

(3) Vertical Tensile Fault Rupture (22)

For a vertical tensile fault rupture, we have **n**=(0,1,0) and **v**=(0,1,0), with the moment tensor component M_{22}=(λ+2μ)DS. The expression in spherical coordinates is:

$$M^{22}=\begin{bmatrix}\lambda DS & 0 & 0\\ 0 & (\lambda+2\mu)DS & 0\\ 0 & 0 & \lambda DS\end{bmatrix}\overset{(r,\theta,\varphi)}{\rightarrow}\begin{bmatrix}\lambda DS & 0 & 0\\ 0 & \lambda DS & 0\\ 0 & 0 & (\lambda+2\mu)DS\end{bmatrix} \tag{17}$$

$$C_f^{22,0}=b_0\left[\lambda U_{nl'}+r^{-1}(\lambda+\mu)\left(2U_{nl}-l(l+1)V_{nl}\right)\right]DS \tag{18}$$

Substituting $C_f^{22,0}$ into the displacement formula (1) and gravitational potential variation formula (2), the Love numbers are obtained as:

$$\left.\begin{aligned}
h_{l0}^{22,0}&=\sum_{n=1}^{\infty}c_{l0}b_0u_{nl}a^2\omega^{-2}\left[\lambda U_{nl'}+r^{-1}(\lambda+\mu)\left(2U_{nl}-l(l+1)V_{nl}\right)\right]\\
l_{l0}^{22,0}&=\sum_{n=1}^{\infty}c_{l0}b_0v_{nl}a^2\omega^{-2}\left[\lambda U_{nl'}+r^{-1}(\lambda+\mu)\left(2U_{nl}-l(l+1)V_{nl}\right)\right]\\
k_{l0}^{22,0}&=\sum_{n=1}^{\infty}\frac{4\pi Gc_{l0}b_0}{(2l+1)g_0a^{l-1}}\left[\lambda U_{nl'}+r^{-1}(\lambda+\mu)\left(2U_{nl}-l(l+1)V_{nl}\right)\right]F_{nl}
\end{aligned}\right\} \tag{19}$$

(4) Horizontal Tensile Fault Rupture (33)

For a horizontal tensile fault rupture, we have **n**=(0,0,1) and **v**=(0,0,1), with the moment tensor component $M_{33}=(\lambda+2\mu)DS$. The expression in spherical coordinates is:

$$M^{33}=\begin{bmatrix}\lambda DS & 0 & 0\\ 0 & \lambda DS & 0\\ 0 & 0 & (\lambda+2\mu)DS\end{bmatrix}\overset{(r,\theta,\varphi)}{\rightarrow}\begin{bmatrix}(\lambda+2\mu)DS & 0 & 0\\ 0 & \lambda DS & 0\\ 0 & 0 & \lambda DS\end{bmatrix} \tag{20}$$

$$C_f^{33}=b_0\left[(\lambda+2\mu)U_{nl'}+r^{-1}\lambda\left(2U_{nl}-l(l+1)V_{nl}\right)\right]DS \tag{21}$$

Substituting C_f^{33} into the displacement formula (1) and gravitational potential variation formula (2), the Love numbers are obtained as:

$$\left.\begin{aligned}
h_{l2}^{33}&=\sum_{n=1}^{\infty}c_{l0}b_0u_{nl}a^2\omega^{-2}\left[(\lambda+2\mu)U_{nl}^{'}+r^{-1}\lambda(2U_{nl}-l(l+1)V_{nl})\right]\\
l_{l2}^{33}&=\sum_{n=1}^{\infty}c_{l0}b_0v_{nl}a^2\omega^{-2}\left[(\lambda+2\mu)U_{nl}^{'}+r^{-1}\lambda(2U_{nl}-l(l+1)V_{nl})\right]\\
k_{l2}^{33}&=\sum_{n=1}^{\infty}\frac{4\pi Gc_{l0}b_0}{(2l+1)g_0a^{l-1}}\left[(\lambda+2\mu)U_{nl}^{'}+r^{-1}\lambda(2U_{nl}-l(l+1)V_{nl})\right]F_{nl}
\end{aligned}\right\} \tag{22}$$

IV. NUMERICAL RESULTS AND VALIDATION

4.1 Validation Using Synthetic Seismograms

To verify the correctness of normal mode calculation and superposition, we first synthesized fundamental-mode synthetic seismograms and compared them with Mineos results. We used the PREM Earth model (with the ocean layer removed), and the station and source information is shown in Tables 3 and 4. We selected normal modes with periods ranging from 5 to 200 s and superposed all n=0 spheroidal and toroidal modes in this interval. The seismic moment tensor components were obtained from the Global Centroid Moment Tensor (GlobalCMT) catalog.

Table 3 Station information

Code	Station Name	Latitude	Longitude	Epicentral Distance (°)	Azimuth (°)
BJT	Beijing	40.0183°N	116.1679°E	19.123	-135.267

Table 4 Source information of earthquake

Earthquake	Origin Time (UTC)	Latitude	Longitude	Depth (km)
Yunnan	2000-01-14 23:37:07	25.607°N	101.063°E	33

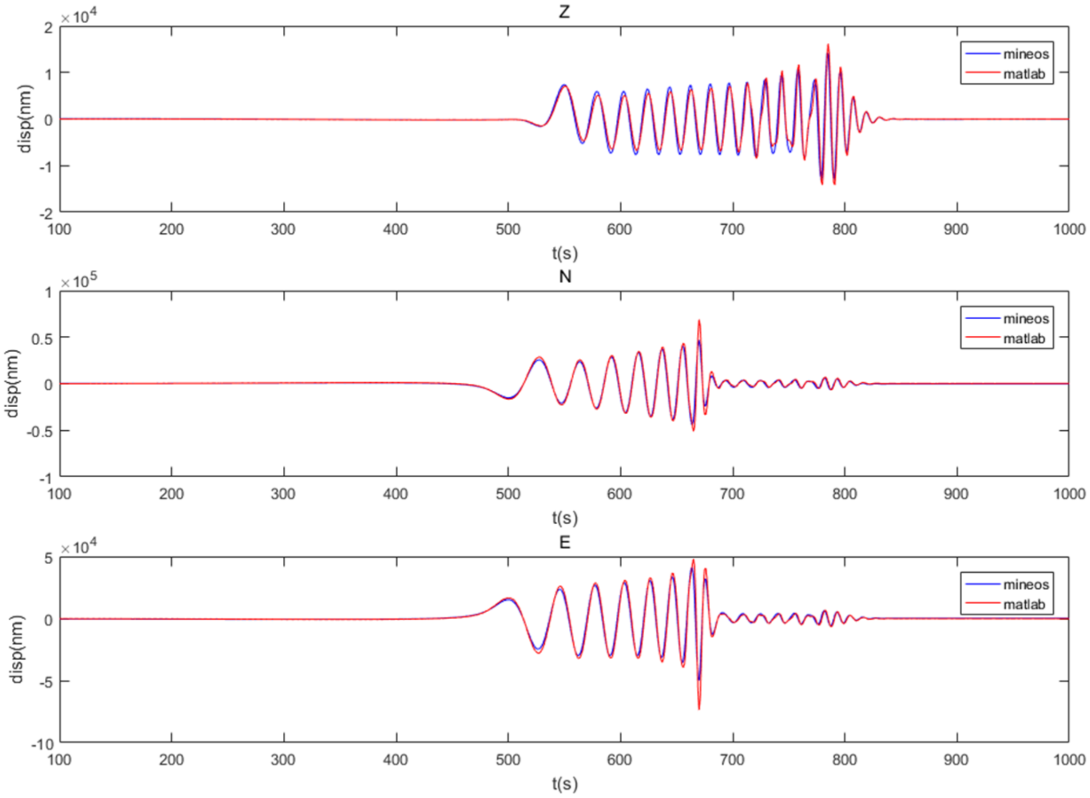

Fig. 3 The Seismograms summed by fundamental modes

Figure 3 shows the comparison between the fundamental-mode synthetic seismograms generated by our MATLAB code and the Mineos results. Z, N, and E represent the vertical, north, and east components, respectively, with red curves for our results and blue curves for Mineos. The three components are generally consistent, verifying the correctness of our normal mode calculation and superposition. Minor local differences arise from numerical calculation details (e.g., numerical precision in eigenfunction interpolation) but have negligible impact on subsequent studies.

4.2 Dislocation Love Number Results

Using the modified PREM Earth model (lithosphere and ocean layers averaged to remove the liquid ocean layer), we computed normal modes with the Mineos software. Spheroidal and toroidal modes of degrees l=2 to 5 with periods longer than 5 s were selected. The radial node number n ranged from 0 to 500 (501 modes per degree) to ensure convergence of the normal-mode summation.

To validate the normal-mode (NM) results, we also computed dislocation Love numbers using the numerical integration (NI) method and obtained reference values (DC) via the reciprocal theorem (Sun et al., 1996). Table 5 lists the results for degree l=2 for the four fault types, while Figure 4 shows the values and absolute differences between NM and DC for degrees l=2 to 5.

As shown in Table 5, for degree l=2, the NM values of h^{ij}, l^{ij}, $l^{t,ij}$, and k^{ij} are in good agreement with the DC references for all four fault types. The largest relative differences occur for the vertical dip-slip case in h^{ij} (about 25%) and for the horizontal tensile case in k^{ij} (about 0.8% in absolute terms), but the absolute discrepancies remain small (~10^{-3}–10^{-4}).

Table 5 Dislocation Love numbers for four fault types at degree l = 2

Fault Type	h^{ij}		l^{ij}		$l^{t,ij}$		k^{ij}	
	NM	DC	NM	DC	NM	DC	NM	DC
Vertical Strike-Slip (12)	-2.018E-03	-1.991E-03	-6.866E-03	-6.728E-03	-1.593E-02	-1.558E-02	-2.196E-03	-2.180E-03
Vertical Dip-Slip (32)	-1.229E-03	-1.643E-03	-3.780E-02	-3.349E-02	3.684E-02	3.394E-02	-9.524E-04	-1.034E-03
Vertical Tensile (22)	4.215E-01	4.088E-01	-6.435E-03	-6.550E-03	—	—	7.517E-04	4.808E-03
Horizontal Tensile (33)	3.712E-01	3.643E-01	1.337E-01	1.347E-01	—	—	5.514E-02	-5.522E-02

Note. NM: normal mode method; DC: reference values derived from the reciprocal theorem.

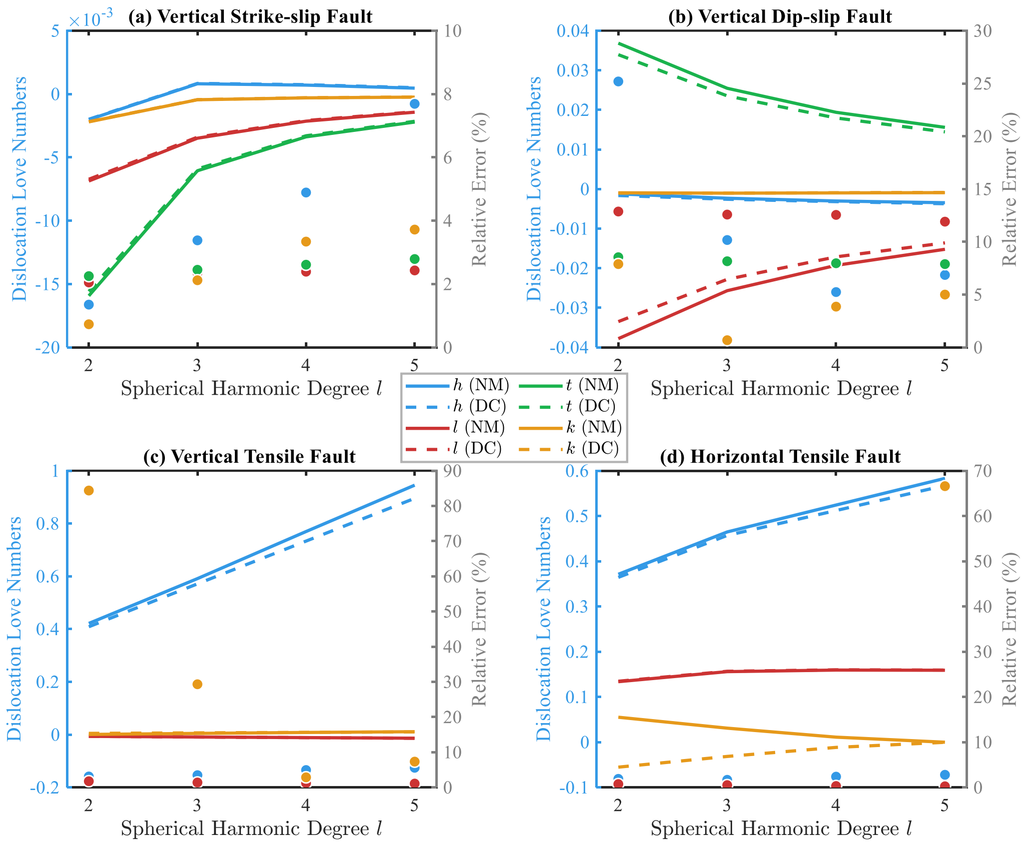

Fig. 4 Dislocation Love numbers for degrees 2 – 5 and absolute differences (NM vs DC).

As illustrated in Figure 4, for all fault types and degrees from 2 to 5, the dislocation Love numbers obtained via the normal-mode method (NM) closely follow the reference DC values, with absolute differences generally decreasing at higher degrees. This trend is readily explained: higher-degree modes are inherently more sensitive to shallow Earth structure and, under a finite number of superposed modes, yield more accurate representations of the deformation field.

Minor discrepancies between NM and DC can be attributed to: (1) truncation of the normal-mode series (theoretical infinite summation vs. finite n up to 500); (2) numerical precision limits in Mineos for highorder eigenfrequencies and eigenfunctions; (3) inherent approximations in the reciprocal theorem reference solutions; and (4) slight differences in the PREM model implementation compared to classic literature. Nevertheless, the overall consistency confirms the validity of the derived normal-mode expressions for dislocation Love numbers.

V. CONCLUSIONS

Based on Earth normal mode theory, this paper systematically investigates the method for calculating dislocation Love numbers and draws the following main conclusions:

(1) We derived practical formulas for calculating dislocation Love numbers for four basic fault types (vertical strike-slip, vertical dip-slip, vertical tensile, and horizontal tensile) using Earth normal modes, and presented detailed expressions.

(2) Using the PREM Earth model and Mineos software, we calculated 2nd to 5th order dislocation Love numbers. Comparison with numerical integration results via the reciprocal theorem shows good agreement, verifying the correctness and validity of the derived formulas.

(3) By synthesizing fundamental-mode synthetic seismograms and comparing them with Mineos results, the three-component seismograms are generally consistent, verifying the correctness of normal mode calculation and superposition.

This study provides an alternative approach to Earth deformation theory independent of traditional numerical integration, helps to further understand the intrinsic relationship between normal modes and the Earth’s global deformation response, and has theoretical significance and application value for studies of coseismic deformations, gravitational field changes, and geoid variations caused by seismic dislocations.

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