International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 15, Issue 6 [June 2026] PP: 33-37

Preview-Kinematic Backstepping Lyapunov Control for Robust Trajectory Tracking of Differential-Drive Mobile Robots

A multi-trajectory Monte Carlo simulation study under measurement noise and wheel slip

Ha Nguyen Ngoc*, Hoan Ngo Trong
Faculty of Mechanical Engineering, Thai Nguyen University of Technology, Vietnam
**Corresponding author*

Abstract
This study presents a preview-kinematic backstepping Lyapunov controller (P-KBLC) for trajectory tracking of differential-drive mobile robots. The controller is derived from the nonlinear kinematic error model expressed in the robot body frame and combines reference feedforward with feedback of longitudinal, lateral, and heading errors. A Lyapunov stability argument is provided for the nominal controller, and a gain-scheduled implementation is formulated so that the cancellation structure of the proof is preserved. The proposed controller is evaluated against Pure Pursuit, Adaptive Pure Pursuit, Stanley, kinematic PID, LQR, sliding mode control, and a simplified grid-search MPC. The numerical evaluation covers five reference trajectories, three disturbance scenarios, and Monte Carlo trials with measurement noise and wheel-slip perturbations. The results indicate that P-KBLC is not uniformly superior to all alternatives. Instead, it provides a practical compromise between tracking accuracy, nonlinear stability support, smooth control action, and low computational cost. In the considered benchmark, P-KBLC achieves an RMSE close to LQR while maintaining a higher tolerance-satisfaction rate than several geometric and predictive baselines.

Keywords: *Differential-drive mobile robot; trajectory tracking; Lyapunov control; backstepping; gain scheduling; Monte Carlo simulation; wheel slip; LQR; sliding mode control; MPC.*

Date of Submission: 06-06-2026 Date of acceptance: 16-06-2026

I. Introduction

Trajectory tracking is a fundamental problem in autonomous mobile robotics. Differential-drive mobile robots are widely used in indoor transportation, service robotics, warehouse automation, and educational platforms because of their simple mechanical structure and high maneuverability. In a tracking task, the controller must generate feasible linear and angular velocity commands such that the robot follows a time-parameterized reference trajectory with small position and heading errors.

Geometric tracking methods, including Pure Pursuit and Stanley control, remain attractive because of their low computational cost and straightforward implementation [1], [2]. However, their performance can be sensitive to the selected lookahead distance, cross-track gain, and curvature variation of the path. PID, LQR, sliding mode control, and model predictive control provide alternative design philosophies. PID is easy to implement but does not provide a direct nonlinear stability guarantee. LQR yields smooth local tracking when the linearized model remains accurate. Sliding mode control improves robustness but may introduce chattering. MPC can handle constraints and prediction, but it generally requires a larger computational budget [5]-[8], [12]-[14].

This paper investigates P-KBLC as a lightweight nonlinear tracking controller. The purpose is not to claim that the proposed controller dominates all existing approaches under all operating conditions. Rather, the controller is positioned as a practical compromise between tracking accuracy, Lyapunov-based stability support, computational simplicity, and real-time applicability.

II. Related work

Pure Pursuit drives the robot toward a lookahead point on the desired path. Its tracking quality depends strongly on the lookahead distance, which must balance responsiveness and smoothness [1]. Adaptive Pure Pursuit modifies this distance according to speed or tracking error, but the method remains primarily geometric

[4]. Stanley control uses cross-track and heading errors and has been widely discussed in autonomous vehicle tracking [2].

Lyapunov-based tracking laws and backstepping methods provide a more model-based foundation for nonholonomic mobile robots [3], [9], [19], [20]. These methods are particularly relevant when the controller is derived directly from the nonlinear kinematic or dynamic model. Extensions include adaptive dynamic tracking, fuzzy sliding-mode backstepping, and learning-assisted backstepping [10], [11], [16].

Optimization and intelligent-control approaches have also been investigated. LQR and iterative LQR optimize quadratic performance indices around local models [12]. MPC and NMPC account for future states and constraints at the cost of additional computation [5], [6], [13], [14]. Neural-network and NN-PID controllers can learn control actions from data or sample controllers, but their performance depends on training quality and may require additional analysis to guarantee closed-loop stability [15], [21].

III. Kinematic model and tracking-error formulation

Consider a differential-drive robot with pose $q = [x, y, \theta]^T$, where x and y are the coordinates of the robot center and \theta is the heading angle. Under the standard no-lateral-slip assumption, the unicycle kinematic model is

$$\dot{x} = v\,cos(\theta), \quad \dot{y} = v\,sin(\theta), \quad \dot{\theta} = \omega$$

$$v = \frac{r}{2}\,(\omega_R + \omega_L), \quad \omega = \frac{r}{L}\,(\omega_R - \omega_L)$$

$$\omega_R = \frac{2v + L\,\omega}{2r}, \quad \omega_L = \frac{2v - L\,\omega}{2r}$$

Let the desired trajectory be $p_{d(t)} = \left[x_{d(t)}, y_{d(t)}\right]^T$. The desired heading, linear speed, and angular speed are computed from the first and second derivatives of the reference trajectory:

$$\theta_d = atan2(\dot{y}_d, \dot{x}_d), \quad v_d = \sqrt{\dot{x}_d^2 + \dot{y}_d^2}$$

$$\omega_d = (\dot{x}_d\,\ddot{y}_d - \dot{y}_d\,\ddot{x}_d)/(\dot{x}_d^2 + \dot{y}_d^2 + \epsilon), \quad \epsilon > 0$$

The body-frame tracking errors are defined as

$$e_x = cos(\theta)(x_d - x) + sin(\theta)(y_d - y)$$

$$e_y = -sin(\theta)(x_d - x) + cos(\theta)(y_d - y), \quad e_\theta = wrap(\theta_d - \theta)$$

The associated error dynamics are

$$\dot{e_x} = v_d\,cos(e_\theta) - v + \omega\,e_y$$

$$\dot{e_y} = v_d\,sin(e_\theta) - \omega\,e_x, \quad \dot{e_\theta} = \omega_d - \omega$$

IV. Proposed control method

The nominal P-KBLC law combines feedforward terms from the reference motion and nonlinear error feedback:

$$v = v_d\,cos(e_\theta) + k_x\,e_x$$

$$\omega = \omega_d + v_d\,(k_y\,e_y + k_\theta\,sin(e\_\theta))$$

where k_x, k_y, and k_θ are positive gains. The term $v_d\,cos(e_\theta)$ follows the desired linear speed while reducing forward motion when the heading error is large. The ω_d term compensates for the desired curvature, whereas $k_y e_y$ and $k_\theta\,sin(e_\theta)$ correct lateral and heading errors.

4.1 Stability analysis

$$V = \frac{1}{2}\,e_x^2 + \frac{1}{2}\,e_y^2 + \frac{1}{k_y}\,(1 - cos(e_\theta))$$

$$\dot{V} = e_x\,\dot{e_x} + e_y\,\dot{e_y} + \frac{1}{k_y}\,sin(e_\theta)\,\dot{e_\theta}$$

$$\dot{V} = -k_x\,e_x^2 - \left(v_d\,\frac{k_\theta}{k_y}\right)sin^2(e_\theta) \leq 0$$

Thus, for positive gains and nonnegative reference speed, the closed-loop system is stable in the Lyapunov sense under the nominal kinematic assumptions. Convergence follows under standard smoothness and persistence conditions, provided that severe saturation and long zero-speed intervals are absent.

4.2 Gain-scheduled implementation

If k_y is scheduled as a state-dependent variable, additional derivative terms may appear in the Lyapunov function. To avoid this difficulty, the implementation schedules only k_x and k_θ while keeping k_y constant:

$$k_x(e_x) = k_{x0} + k_{x1}\,|e_x|, \quad k_y = \gamma_y > 0$$

$$k_\theta(e_\theta) = k_{\theta 0} + k_{\theta 1}\,|e_\theta|$$

$$\dot{V} = -k_x(e_x)e_x^2 - v_d \frac{k_\theta(e_\theta)}{\gamma_y} sin^2(e_\theta) \leq 0$$

V. Simulation methodology

The evaluation considers five reference trajectories: figure-eight, circle, ellipse, sine, and straight line. The use of multiple paths reduces the risk of drawing conclusions from a single favorable geometry.

Measurement noise is modeled by adding zero-mean Gaussian perturbations to the measured position and heading. Wheel slip is represented by multiplicative losses in the executed linear and angular velocities. Three scenarios are considered: ideal motion, measurement noise only, and measurement noise combined with wheel slip.

Each controller-trajectory-scenario combination is evaluated in repeated trials with different random samples. The reported metrics include RMSE, MAE, maximum error, integral absolute error, a control-energy index, the percentage of samples within the 0.05 m tolerance band, and the acceptance rate.

$$e_i = \sqrt{(x_d - x)^2 + (y_d - y)^2}$$

$$RMSE = \sqrt{\frac{1}{N}\sum e_i^2}, \quad MAE = \frac{1}{N}\sum |e_i|$$

$$IAE = \sum(|e_i|)\,\Delta t, \quad E_u = \sum(v_i^2 + \omega_i^2)\,\Delta t$$

VI. Results and discussion

Table 1 summarizes the Monte Carlo results. LQR obtains the lowest mean RMSE in this benchmark, while P-KBLC achieves a comparable RMSE and the highest tolerance-satisfaction and acceptance rates. The geometric controllers show larger average errors, mainly on trajectories with rapidly changing curvature. The simplified MPC baseline is intentionally identified as a grid-search baseline and should not be interpreted as a fully optimized NMPC implementation.

Controller	RMSE mean	RMSE std	MAE mean	In tol. mean (%)	Acceptanc e rate (%)
P-KBLC	0.0517	0.0240	0.0466	81.97	50.00
LQR	0.0503	0.0087	0.0475	70.99	10.00
Sliding Mode Control	0.0580	0.0112	0.0518	63.31	0.83
PID Kinematic	0.1245	0.0377	0.1057	21.46	0.00
MPC Simplified	0.3122	0.1289	0.2889	6.74	0.00
Stanley	0.3195	0.1328	0.2926	8.21	0.00
Adaptive Pure Pursuit	0.3250	0.1391	0.3010	0.34	0.00
Pure Pursuit	0.3255	0.1397	0.3025	1.19	0.00

Table 1. Monte Carlo summary over five trajectories and three disturbance scenarios.

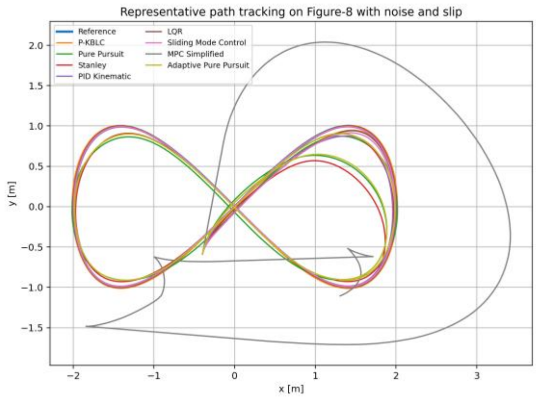

Figure 1. Representative trajectory-tracking result under measurement noise and wheel slip.

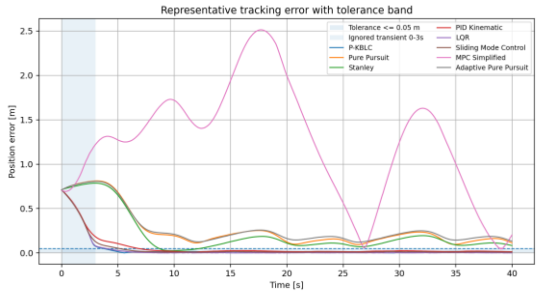

Figure 2. Representative position-error response with a 0.05 m tolerance band.

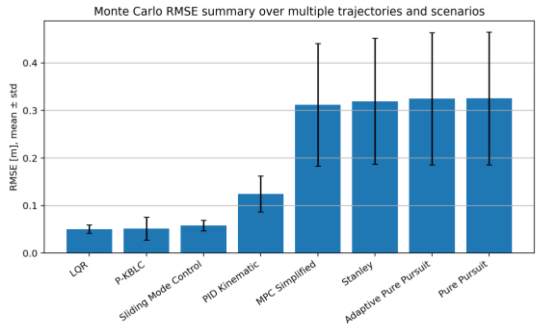

Figure 3. Mean RMSE and standard deviation obtained from Monte Carlo trials.

These results support a cautious interpretation. P-KBLC should not be described as universally superior. Its main value is the combination of nonlinear model-based design, a transparent Lyapunov argument, low computational complexity, and competitive tracking performance. Compared with SMC, the absence of a strong switching term can reduce the likelihood of chattering. Compared with simplified MPC, the controller requires substantially less online search.

VII. Limitations and future work

The present work remains a simulation-based study. Although measurement noise, wheel slip, and Monte Carlo trials are included, actuator dynamics, communication delay, acceleration saturation, odometry drift, and obstacle interactions are not fully considered. Future work will include state estimation using odometry, IMU, and filtering methods, replacement of grid-search MPC with a solver-based NMPC baseline, and validation in ROS/Gazebo or on a physical differential-drive robot.

VIII. Conclusion

This paper presented P-KBLC for trajectory tracking of differential-drive mobile robots. The controller combines reference feedforward with nonlinear feedback in the robot body frame. A Lyapunov analysis was provided for the nominal law, and a gain-scheduled implementation was formulated to preserve the proof structure. Multi-trajectory Monte Carlo simulations under measurement noise and wheel slip showed that P-KBLC offers competitive accuracy and a high tolerance-satisfaction rate while retaining low computational complexity. The method is therefore best interpreted as a balanced real-time tracking option rather than a universally optimal controller.

Declarations

Conflict of interest: The author declares no conflict of interest.

Data availability: The numerical data and computational files used to produce the figures and tables are available from the corresponding author upon reasonable request or may be provided as supplementary material according to the journal policy.

Funding: This research is supported by the Thai Nguyen University of Technology.

Acknowledgments

This research is supported by the Thai Nguyen University of Technology

References

[1] R. C. Coulter, "Implementation of the Pure Pursuit Path Tracking Algorithm," Carnegie Mellon University, Technical Report CMU-RI-TR-92-01, 1992.

[2] G. M. Hoffmann, C. J. Tomlin, M. Montemerlo, and S. Thrun, "Autonomous Automobile Trajectory Tracking for Off-Road Driving," in Proc. American Control Conference, 2007.

[3] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for an autonomous mobile robot," in Proc. IEEE ICRA, 1990.

[4] S. Macenski, S. Singh, F. Martin, and J. Gines, "Regulated pure pursuit for robot path tracking," Autonomous Robots, vol. 47, pp. 685-694, 2023.

[5] F. Kuhne, W. F. Lages, and J. M. G. da Silva Jr., "Mobile robot trajectory tracking using model predictive control," 2005.

[6] T. P. Nascimento, C. E. T. Dorea, and L. M. G. Goncalves, "Nonholonomic mobile robots' trajectory tracking model predictive control: A survey," Robotica, vol. 36, no. 5, pp. 676-696, 2018.

[7] B. Jiang, J. Li, and S. Yang, "An improved sliding mode approach for trajectory following control of nonholonomic mobile AGV," Scientific Reports, vol. 12, article 17763, 2022.

[8] J. Riera, S. Cachiguango, M. Pedraza, G. M. Andaluz, and P. Leica, "Sliding mode control for trajectory tracking of a TurtleBot3 mobile robot in obstacle environments," Engineering Proceedings, vol. 77, article 7, 2024.

[9] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot: Backstepping kinematics into dynamics," Journal of Robotic Systems, vol. 14, no. 3, pp. 149-163, 1997.

[10] F. N. Martins, W. C. Celeste, R. Carelli, M. Sarcinelli-Filho, and T. F. Bastos-Filho, "An adaptive dynamic controller for autonomous mobile robot trajectory tracking," Control Engineering Practice, vol. 16, no. 11, pp. 1354-1363, 2008.

[11] X. Wu, P. Jin, T. Zou, Z. Qi, H. Xiao, and P. Lou, "Backstepping trajectory tracking based on fuzzy sliding mode control for differential mobile robots," Journal of Intelligent & Robotic Systems, vol. 96, pp. 109-121, 2019.

[12] H. Zhang, J. Gong, Y. Jiang, G. Xiong, and H. Chen, "An iterative linear quadratic regulator based trajectory tracking controller for wheeled mobile robot," Journal of Zhejiang University SCIENCE C, vol. 13, pp. 593-600, 2012.

[13] T. P. Nascimento, C. E. T. Dorea, and L. M. G. Goncalves, "Nonlinear model predictive control for trajectory tracking of nonholonomic mobile robots: A modified approach," International Journal of Advanced Robotic Systems, 2018.

[14] J. Zhang, J. Li, and L. Fan, "Trajectory tracking control of nonholonomic wheeled mobile robots using model predictive control subjected to Lyapunov-based input constraints," International Journal of Control, Automation and Systems, 2022.

[15] T. T. K. Ly, N. H. Thai, and L. T. Phong, "Design of neural network-PID controller for trajectory tracking of differential drive mobile robot," Vietnam Journal of Science and Technology, vol. 62, no. 2, pp. 374-386, 2024.

[16] N. He, Z. Yang, X. Fan, J. Wu, Y. Sui, and Q. Zhang, "A self-adaptive double Q-backstepping trajectory tracking control approach based on reinforcement learning for mobile robots," Actuators, vol. 12, no. 8, article 326, 2023.

[17] H. Xie, J. Zheng, Z. Sun, H. Wang, and R. Chai, "Finite-time tracking control for nonholonomic wheeled mobile robot using adaptive fast nonsingular terminal sliding mode," Nonlinear Dynamics, vol. 110, pp. 1437-1453, 2022.

[18] L. Medina, G. Guerra, M. Herrera, L. Guevara, and O. Camacho, "Trajectory tracking for non-holonomic mobile robots: A comparison of sliding mode control approaches," Results in Engineering, article 102105, 2024.

[19] C. Samson, "Time-varying feedback stabilization of car-like wheeled mobile robots," The International Journal of Robotics Research, vol. 12, no. 1, pp. 55-64, 1993.

[20] A. De Luca, G. Oriolo, and C. Samson, "Feedback control of a nonholonomic car-like robot," in Robot Motion Planning and Control, Springer, pp. 171-253, 1998.

[21] H. Najva and A. Saleem, "Neural network-based adaptive controller for trajectory tracking of wheeled mobile robots," IEEE Access, vol. 10, pp. 13582-13597, 2022.