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Starlike Spanning Trees of Ladders

M.Lewinter and L.Varrelmann

Abstract
It is shown that 3-branched starlike trees on 2n vertices span the ladder, M(n, 2), if and only if the the 3-branched starlike tree is equitable, that is, if the two color sets have the same cardinality.

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I. Introduction

Given positive integers $a \geq 2$ and $b \geq 2$, the *two-dimensional mesh* $M(a, b)$ is defined as $P_a \times P_b$. The two-dimensional mesh $M(4, 3)$ is shown in Figure 1. In particular, $M(n, 2)$, where $n \geq 3$, is called a *ladder*. [1]

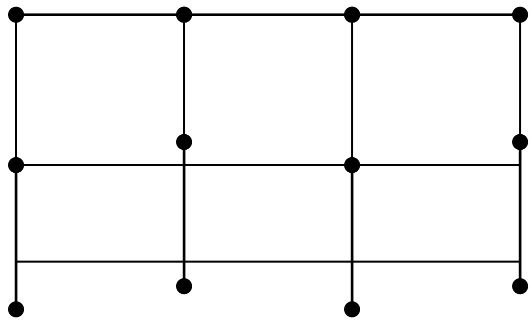

Figure 1: The two-dimensional mesh $M(4, 3)$

A *tree* is a connected, acyclic graph. A tree T is called *starlike*, if it contains a vertex v for which $deg(v) \geq 3$, while all other vertices in T have degree 1 or 2. The vertex v is called the *junction*. If $deg(v) = 3$, then T is 3-*branched*. $T - v$ has 3 components, each of which is either K_1, K_2, or a path.

Let $S(a_1, a_2, a_3)$ denote a 3-branched starlike tree T with junction v such that the components of $T - v$ are paths of orders a_1, a_2, and a_3. The order of S is $1 + \sum_{i=1}^{3} a_i$. See Figure 2. The junction is labeled v.

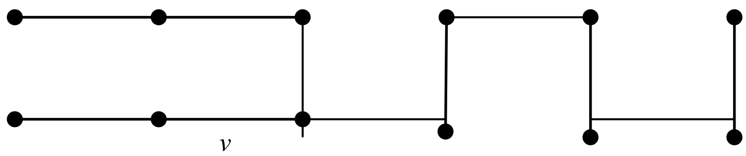

Figure 2: The 3-branched starlike tree $S(2, 3, 6)$ spans $M(6, 2)$.

Does the ladder $M(6, 2)$ have a spanning tree isomorphic to $S(2, 3, 6)$? A glance at Figure 2 answers this question in the affirmative. In fact, the layout of the figure indicates that $S(2, 3, 6)$ is indeed a spanning tree of $M(6, 2)$.

A *bipartite* graph has the property that each of its vertices is colored black or white, and adjacent vertices are oppositely colored. A bipartite graph is *equitable* if the two color sets have the same cardinality. It is *nearly equitable* if the cardinalities of the two color sets differ by one. Otherwise it is *skewed*. [2]

All trees are bipartite but they can be equitable, nearly equitable, or skewed. All meshes are bipartite. *Even* meshes are equitable while *odd* ones are nearly equitable.

Remark: A 3-branched starlike tree T is equitable if and only if exactly two of its endvertices have the same color as the junction. Equivalently, a 3-branched starlike tree T with junction v, is equitable if and only if exactly two of the paths of $T - v$ are even. Then exactly two of the branches are of even length.

II. Main Result

We have the following theorem.

Theorem 1: An equitable, 3-branched, starlike tree on $2n$ vertices, where $n \geq 3$, spans the ladder $M(n, 2)$.

Proof: Let T be an equitable-branched starlike tree on $2n$ vertices. Let the junction be v and let the three endvertices be x, y, and z, respectively. Assume, WLOG, that the junction v and the endvertex z are black. By the remark in Section I, the remaining vertices x and y are oppositely colored. Let R be the *unicyclic* graph $T \cup xy$. Embed the cycle of R as shown in Figure 3, and embed the path with endvertex z in the winding manner shown there. This last embedding which is needed to span the ladder is achievable since v and z have the same color. Upon deleting edge xy, we find that T spans $M(n, 2)$. The same holds if edge xy is horizontal. ■

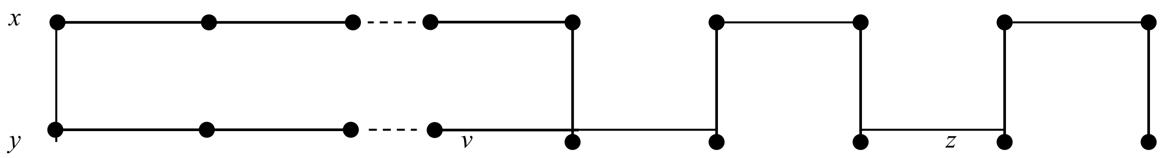

Figure 3: The unicyclic graph $T \cup xy$ spans a ladder.

References

[1] F.Buckley and M.Lewinter, *A Friendly Introduction to Graph Theory*. Prentice-Hall, 2003.
[2] F.Buckley, M.Lewinter, *Introductory Graph Theory with Applications*. Waveland Press. 2014.