International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 15, Issue 9 [September 2026] PP: 69-79

The Core of Generalized Wave Mechamics: Quantum Mechanics Equation with Explicit Force

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Abstract
Classical mechanics and quantum mechanics have long been framed as binary opposites—but this dichotomy has yielded no real benefits, only persistent confusion. The fundamental equations of quantum mechanics cannot logically demarcate a clear boundary between the quantum and classical realms; in fact, the massm and potential energy V in the Schrödinger equation seamlessly traverse the divide between the microscopic and macroscopic. The electromagnetic potential energy in the Schrödinger equation can be replaced with the potential energy of interactive forces, which can further be converted into force itself using the relation F = V/r. Derivations have now produced a force operator and quantum mechanical equations that explicitly incorporate force—including a quantum formulation of Newton's second law. These results provide direct evidence that quantum and classical mechanics are not incompatible, but complementary. By resolving the historical absence of "force" as a formal concept in quantum mechanics, we bridge the conceptual gap between macroscopic classical frameworks and microscopic quantum theory, challenging long-held perceptions of their irreconcilability. This unified approach explains why nanoscale particles exhibit quantum properties, simplifies calculations of quantum forces, and lays the groundwork for a generalized wave mechanics that unifies mechanical descriptions across all scales.

Keywords: *Quantum and classical, Compatibility, The wave mechanics equation of Newton's second law, Quantum mechanics operator of force, Schrödinger-Tu equation with explicit force.*

Funding Declaration: *Due to the highly challenging nature of the research work, it is not possible to apply for funding. Therefore, the research work in this article did not receive any funding. The research is entirely self funded by the author.*

Date of Submission: 02-09-2026 Date of acceptance: 12-09-2026

I. Introduction

Generalized wave mechanics represents a theoretical framework that dismantles the long-standing divide between quantum mechanics and classical mechanics.

For decades, classical mechanics and quantum mechanics have been treated as irreconcilable opposites within the physical theoretical system. Classical mechanics centers on macroscopic systems, describing them through deterministic trajectories and observable forces as its core tenets. In contrast, quantum mechanics relies on the probabilistic interpretation of wave functions and operator formalism to address microscopic phenomena. The stark disparities in their conceptual frameworks and mathematical tools have created an artificial rift between "macroscopic classicality" and "microscopic quantum behavior." Fortunately, this research reveals that this opposition is not fundamental: classical mechanical laws can be rephrased using wave mechanical equations, achieving profound compatibility with quantum mechanics. The core technical approach involves leveraging quantum mechanics' momentum operator transformation method, combined with the inherent relationship between force and energy in planetary models, to derive wave equations that incorporate the concept of "force." This effort truly brings quantum mechanics back to its roots as a branch of "mechanics," resolving the historical absence of the "force" concept within quantum theory.

In prior series of studies, the author has systematically demonstrated the compatibility between quantum and classical mechanics and proposed a theoretical framework for the synergistic application of quantum and classical mechanical methods [1-14]. As an in-depth extension of this earlier work, this paper further constructs wave mechanical equations for the core laws of classical mechanics—including the law of conservation of momentum and Newton's second law—and for the first time derives quantum operator representations of physical quantities such as force and acceleration.

In the early days of quantum mechanics, physicists aspired to achieve compatibility and unification between the new theory and classical mechanics. However, the ambiguous physical interpretation of wave functions, coupled with the challenges of explaining phenomena like double-slit diffraction and the stability of hydrogen atoms, gradually shifted the mainstream philosophical perspective toward rejecting the notion that microscopic particles possess "primitive eigenstates independent of measurement." Instead, the uncertainty, randomness, and "wave function collapse" exhibited by microscopic systems were redefined as inherent properties of quantum mechanics, with the assertion that the Schrödinger equation is incapable of describing macroscopic objects.

In reality, though, the Schrödinger equation itself contains no inherent logical limitation that excludes macroscopic systems—when extended to macroscopic scales by substituting observable values for mass m and replacing the potential energy function with macroscopic gravitational potential, its mathematical form remains valid. [1] This article will further reinforce this conclusion through more intuitive deductive reasoning.

While the academic community has never ceased to challenge the Copenhagen interpretation,[15-25] consensus largely prevails regarding the validity of quantum mechanics' mathematical formal system. This study adheres strictly to the axiomatic framework of quantum mechanics, deriving wave mechanics equations incorporating "force" by following the construction rules of Hermitian operators ($\langle\phi|\hat{A}|\psi\rangle = \langle\psi|\hat{A}\phi|\rangle$) without introducing new assumptions.

Previous research—particularly work that combines or alternates between quantum and classical mechanical approaches[26-30]—has provided multidimensional evidence for quantum-classical compatibility, weakening the perceived opposition between the two theories and demonstrating the theoretical and practical advantages of compatibility frameworks. Historically, such combinations have relied primarily on parallel or alternating applications of traditional methods (e.g., modifying only the potential energy function without altering the structure of quantum equations). The breakthrough of this article lies in integrating quantum and classical properties within a single equation to derive a generalized wave mechanics equation, laying the groundwork for the establishment of "generalized wave mechanics."

II. Deduction of "Force Operator" and "Wave Equation of Classical Mechanical Laws"

The general form of the one-dimensional time-independent Schrödinger equation is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+V\psi=E\psi. \qquad (1)$$

Since the term $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi$ corresponds to the kinetic energy operatoracting on ψ(*i.e.*, $\hat{T}\psi$), Eq. (1) just is $\hat{T}\psi + \hat{V}\psi = E\psi$. Since the Hamiltonian operator is Hermitian, according to equation (1), an energy relationship suitable for the planetary model can be obtained: $T + V = E$. This relation is universally valid for both microscopic and macroscopic systems, and it also satisfies the energy partitioning between kinetic and potential energy in bound-state systems (e.g., conforming to the virial theorem under specific conditions).

Evidently, Eq. (1) can also be interpreted as a wave-mechanical formulation of classical physical laws applicable to macroscopic phenomena. For instance:When $V=-\frac{Ze^2}{r}$, Eq. (1) describes planetary model systems governed by electromagnetic interactions;When $V=-\frac{GMm}{r}$, Eq. (1) can be extended to describe planetary systems under Newtonian gravitational interactions.

This demonstrates that the Schrödinger equation itself does not exclude the possibility of describing macroscopic gravitational interaction systems. In the case where V represents gravitational potential energy, the Schrödinger equation takes the form:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi-\frac{GMm}{r}\psi=E\psi. \qquad (2)$$ [1-9]

The wave function should still use the familiar form:

$$\psi=\psi_0 e^{-\frac{i}{\hbar}(Et-px)}=\psi_0 e^{-i2\pi(\nu t-x/\lambda)}. \qquad (3)$$

Whether it is a macroscopic gravitational interaction planetary model system or a microscopic electromagnetic interaction planetary model system, the relationship between potential energy and force is $F=V/r$. In the planetary model, the relationship between E, V, υ, r, and F is $E=\frac{1}{2}V=-\frac{1}{2}m\upsilon^2=\frac{1}{2}rF$.

The energy conservation of a steady-state system satisfies

$T+V=E$. (4)

Aamong which, $E=-\langle T\rangle$ (Its numerical relationship is $E=-T$). By substituting $E=-T$ into Eq. (4), result in $2T=V$. By dividing both sides of the equation by the orbital radius r in the planetary model, and considering $F=V/r$, we can obtain the relationship between kinetic energy and force: $2T/r=F$. Substituting $T=m\upsilon^2/2$ into $2T/r=F$, and considering that the acceleration of an object in uniform circular motion is $a=\upsilon^2/r$, we can obtainNewton's Second Law: $\frac{m\upsilon^2}{r}=ma=F$(The '*m*' in this equation cannot be eliminated, otherwise it is not equal to the force F).

The formula for kinetic energy in classical mechanics is $T=\frac{p^2}{2m}$. For an object in uniform circular motion, $2T/r=F$ can be written as $F=\frac{mv^2}{r}$. Note: we can choose $F=-\frac{mv^2}{r}$based on the direction of acceleration.Multiply both sides of $F=mv^2/r$ by $\frac{1}{2}r$, we can obtain $-\frac{1}{2}Fr=\frac{1}{2}mv^2=T$. Substituting $\frac{1}{2}mv^2=T$ into Eq. (4), result in $(1/2)mv^2+V=E$, or $(1/2)mv^2+Fr=E$. We write the classical mechanical formula $(1/2)mv^2+Fr=E$ *as*

$$\frac{p^2}{2m}+V=E. \qquad (5)$$

If all terms of Eq. (5) are multiplied by ψ, we can obtain $\frac{p^2}{2m}\psi+V\psi=E\psi$. Considering Eq. (3) , we have $\frac{p^2}{2m}\psi=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=T\psi=\frac{1}{2}mv^2\psi$. In the planetary model of hydrogen atoms, the orbital angular momentum of electrons is $L=pr=\frac{1}{2}\hbar$. The orbital velocity of n=1 electron in the ground state hydrogen atom is $v=\alpha c$.

If equation (1) is used, that is, V takes $-\frac{Ze^2}{r}$, and both sides of the equation are divided by r, then $F=V/r$ is the electromagnetic interaction force. We have

$$\frac{i\hbar^2}{2m}\frac{\partial^3}{\partial x^3}\psi-\frac{Ze^2}{r^2}\psi=\frac{E}{r}\psi.(6))$$

For the hydrogen atom in planetary models, $V=2E$, Therefore we have$\frac{i\hbar^2}{2m}\frac{\partial^3}{\partial x^3}\psi+\frac{V}{2r}\psi=-F\psi$, or

$$-\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}\psi-\frac{Ze^2}{r^2}\psi=2F\psi. \qquad (7)$$

In the planetary model of hydrogen atoms, the orbital angular momentum of electrons is $L=pr=\frac{1}{2}\hbar$. The orbital velocity of n=1 electron in the ground state hydrogen atom is $v=\alpha c$. If we want it to be in the form of a second partial derivative, then the equation corresponding to equation (7) is

$$\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi-\frac{Ze^2}{2r^2}\psi=F\psi. \qquad (8)$$

Hear, α is the fine structure constant. For gravitational interactions, it is well known that $-\frac{GMm}{r^2}=F$, $L=mvr=m\sqrt{GMr}$ (The m in the equation cannot be eliminated, otherwise it is not the angular momentum L), and according to equation (2), we can obtain

In the planetary model of the hydrogen atom, the orbital angular momentum of the electron is given by $L=pr=\frac{1}{2}\hbar$electron in the ground state of hydrogen, the orbital velocity takes the form $v=\alpha c$, where α denotes the fine-structure constant. Rewriting this relationship in terms of a second partial derivative yields an expression analogous to Equation (7):

$$\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi-\frac{Ze^2}{2r^2}\psi=F\psi. \qquad (8)$$

For gravitational interactions, the inverse-square force law is well established: $-\frac{GMm}{r^2}=F$. The orbital angular momentum here is $L=mvr=m\sqrt{GMr}$; crucially, the mass m cannot be canceled in this expression, as doing so would invalidate the physical meaning of angular momentum L. Building on this and using Equation (2), we derive the following

$$\frac{\hbar^4}{2GMm^3}\frac{\partial^4}{\partial x^4}\psi-\frac{GMm}{r^2}\psi=\frac{E}{r}\psi. \qquad (9)$$

Here, $E=\frac{1}{2}V, \frac{E}{r}\psi=\frac{V}{2r}\psi$. Substituting $F=-\frac{GMm}{r^2}$ into equation (9), we can obtain

$$-\frac{\hbar^4}{2GMm^3}\frac{\partial^4}{\partial x^4}\psi+\frac{V}{2r}\psi=F\psi. \qquad (10)$$

or

$$-\frac{\hbar^4}{GMm^3}\frac{\partial^4}{\partial x^4}\psi+\frac{V}{r}\psi=2F\psi. \qquad (11)$$

In the macro field, both equations (10) and (11) are applicable. This indicates that quantum mechanics equations are not exclusive to the microscopic field. In the macroscopic domain, both Equations (10) and (11) hold validity—a finding that demonstrates quantum mechanical equations are not confined exclusively to the microscopic realm.

We calculate equation (7) item by item. As long as you note that $2pr=\hbar$, the calculation results of the two terms on the left side of the equal sign are: $-\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}\psi=-\frac{p^2}{mr}\psi=-\frac{2T}{r}\psi=\frac{V}{r}\psi$; $-\frac{Ze^2}{r^2}\psi=\frac{V}{r}\psi$. In non isolated chain equations, not all terms have m, and such m cannot be eliminated. The sum of these two calculation results is $\frac{2V}{r}\psi=2F\psi$. It has been proven that equation (7) holds true. Let's calculate equation (10) item by item. Just pay attention $v^2r=GM$ (*i. e.*, $v^2=GM/r$ or $r=GM/v^2$), $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=T\psi=\frac{1}{2}mv^2\psi$ and $\frac{\partial^4}{\partial x^4}\psi=\frac{p^4}{\hbar^4}\psi$, we have $-\frac{\hbar^4}{2GMm^3}\frac{\partial^4}{\partial x^4}\psi=-\frac{mv^2}{2r}\psi=\frac{V}{2r}=\frac{1}{2}F\psi$. The second term of equation (10) is also $\frac{1}{2}F\psi$. The sum of two $\frac{1}{2}F\psi$ is exactly the $F\psi$ on the right. So, equation (10) holds true.

If all terms of $F=ma$ are divided by the interaction distance r, and considering $F=\mathrm{d}V/\mathrm{d}r=-\mathrm{d}p/\mathrm{d}t$, especially its non differential formula, we can obtain $F=-\frac{GMm}{r}$ or $F=\frac{mv^2}{r}$. Note: we tend to use negative values to represent potential energy and force (In the equation $F=ma$, the direction of F is consistent with the direction of acceleration a). For the uniform circular motion of electrons, $pr=\frac{1}{2}\hbar$. Considering $p=mv$, $F=V/r$ and "the gravitational force in a circular motion equilibrium system is equal to the centrifugal force", we can obtain $F=-2p^3/m\hbar$. (12)

As is well known, $\hat{p}=-i\hbar\frac{\partial}{\partial x}$, so, the operator of p^3 is

$$\hat{p}^3=i\hbar^3\frac{\partial^3}{\partial x^3}. \quad (13)$$

The momentum cubed term herein is derived from Equation (12); substituting the expression for p^3. Replacing p3 in equation (12) with equation (13) yields equation (14).

$$\hat{F}=-2\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}. \quad (14)$$

Multiply both sides of equation. (14) by ψ to obtain equation (15). It is the representation method of wave power of force.

$$\hat{F}\psi=-2\frac{i\hbar^2}{m}\frac{\partial^3}{\partial x^3}\psi. \quad (15)$$

According to $F=-\frac{GMm}{r^2}$ and $p=mv$, another form of quantum mechanical equation $F\psi=i\hbar\frac{GM}{vr^2}\frac{\partial}{\partial x}\psi$ for Newton's second law can be derived. In the equilibrium system of bound state mechanics, the force exerted by the atomic nucleus on the electron conforms to this equation. For the Bohr planetary model, $pr=\hbar$, the negative sign on the right side of equations (14) and (15) should be removed. This article believes in the planetary model of hydrogen atoms and does not believe in the electron cloud hydrogen atoms with zero orbital angular momentum in quantum mechanics. As is well known, the operator of kinetic energy T is

$$\hat{T}=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}. \quad (16)$$

In classical mechanics, the expression for the law of conservation of momentum is $m_1v_1=m_2v_2$ or $p_1=p_2$. By replacing p with the corresponding operator in $p_1=p_2$, the result is $-i\hbar\frac{\partial}{\partial x_1}=-i\hbar\frac{\partial}{\partial x_2}$. Adding corresponding wave functions $\psi_1\psi_2$ on both sides of this result, we can obtain $-i\hbar\psi_2\frac{\partial}{\partial x_1}\psi_1=-i\hbar\psi_1\frac{\partial}{\partial x_2}\psi_2$, or

$$-\frac{i\hbar}{\psi_1}\frac{\partial}{\partial x_1}\psi_1=-\frac{i\hbar}{\psi_2}\frac{\partial}{\partial x_2}\psi_2. \quad (17)$$

Here, $\psi_1=A_1e^{-\frac{i}{\hbar}(E_1t_1-p_1x_1)}$, $\psi_2=A_2e^{-\frac{i}{\hbar}(E_2t_2-p_2x_2)}$, $E_1=h\nu_1$, $E_2=h\nu_2$. Eq. (17) is the law of conservation of momentum expressed using wave mechanics methods.

Calculate the first term in Eq. (1) and move some terms to become $\frac{p^2}{2m}\psi-E\psi=-V\psi$. Considering $E=-T$, $V=-2T$, we have

$$-\frac{p^2}{m}\psi=V\psi. \quad (18)$$

By replacing p^2 in equation (18) with an operator form, and divide both sides of equation (18) by r, the potential energy operator $\hat{V}=\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}$ can be obtained (note:$a=v^2/r$, potential energy V is a negative value).

$$\frac{p^2}{mr}\psi=F\psi=\frac{p}{v}\frac{v^2}{r}\psi. \quad (19)$$

Concidering $mv=p$, $a=v^2/r$, equation (19) becomes $F\psi=[\frac{p}{v}a]\psi$. By replacing the momentum p with the operator $\hat{p}=-i\hbar\frac{\partial}{\partial x}$ and using the relationship $v=c\alpha$ between velocity and fine constant in the ground state Bohr hydrogen atom, we can obtain

$$\hat{F}\psi=-a[\frac{i\hbar}{c\alpha}\frac{\partial}{\partial x}]\psi. \quad (20)$$

Equation (20) is one of the wave mechanics expressions for Newton's second law applicable to the ground state Bohr hydrogen atom. If particles move uniformly in a straight line, their form will be different.

For de Broglie waves in planetary models, $\lambda=h/mv=h/p$, $v=\lambda\nu$ and $a=v^2/r$. Using these telationship, the first equation in Eq. (19) can be transformed into a $a\left[\frac{p^2}{h\nu}\right]\psi=F\psi$. Reference [1] has the following equation: $T=\frac{1}{2}h\nu$. Use this equation, $a\left[\frac{p^2}{h\nu}\right]\psi=F\psi$ can be transformed into $a\left[\frac{p^2}{2T}\right]\psi=F\psi$. By replacing p^2 with the corresponding operator $\hat{p}^2=-\hbar^2\frac{\partial^2}{\partial x^2}$, we can obtain

$$\hat{F}\psi=-a\left[\frac{\hbar^2}{2T}\frac{\partial^2}{\partial x^2}\right]\psi. \quad (21)$$

Equation (21) has a wide range of applicability (applicable to both micro and macro objects, as well as objects in linear motion). This is different from equation (20). According to it, a new operator of force can be obtained: $\hat{F}=$

$-\frac{a\hbar^2}{2T}\frac{\partial^2}{\partial x^2}$ (where a is acceleration).

Comparing $F=ma$ with $\hat{F}=-\frac{a\hbar^2}{2T}\frac{\partial^2}{\partial x^2}$, we can obtain the operator of the mass m as a function.

$\hat{m}=-\frac{\hbar^2}{2T}\frac{\partial^2}{\partial x^2}$. (22)

According to Eqs. (20) and (21), Eq. (7) can be transformed to

$-a\left[\frac{\hbar^2}{2T}\frac{\partial^2}{\partial x^2}\right]\psi-\frac{E}{r}\psi=F\psi$. (23)

According to $\hat{T}=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, $\hat{V}=\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}$, $E=-T=-\frac{p^2}{2m}$, $\hat{E}=-\hat{T}=\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}=-i\frac{\hbar}{2}\frac{\partial}{\partial t}$ (the last equation can be found in Refs. [1-9]), the wave mechanics expression of Viry's theorem can be obtained: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}\psi=-i\frac{\hbar}{2}\frac{\partial}{\partial t}\psi$ or

$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+V\psi=-i\frac{\hbar}{2}\frac{\partial}{\partial t}\psi$. (24)

The concise form of the classical expression for Viry's theorem is $V=-2T$. Considering $\hat{T}=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, we have $V\psi=\frac{\hbar^2}{m}\frac{\partial^2}{\partial x^2}\psi$.

According to the one-dimensional momentum operator $\hat{p}=-i\hbar\frac{\partial}{\partial x}$, We can directly write the velocity operator, velocity squared operator, and acceleration operator.

$\hat{v}=\hat{\dot{r}}=-\frac{i\hbar}{m}\frac{\partial}{\partial x}$, (25)

$\hat{v}^2=-\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2}$. (26)

$\hat{a}=\hat{\dot{v}}=-\frac{\partial}{\partial t}\left[\frac{i\hbar}{m}\frac{\partial}{\partial x}\right]$. (27)

The displacement equation of the object in motion is $s=vt$. According to equation (25), we know that $\hat{s}=-i\hbar\frac{t}{m}\frac{\partial}{\partial x}$ and $s=-i\hbar\frac{t}{m}\frac{\partial}{\partial x}\psi$. Quantum mechanics also uses the momentum relationship $p=mv$. Based on the derivative of the wave function of kinetic energy with respect to t, and considering $m=2T/v^2$, the quality operator is $\hat{m}=-i\hbar\frac{\partial}{\partial t}\div\left[-\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2}\right]$, and the result is

$\hat{m}=i\frac{m^2}{\hbar}\left[\frac{\partial}{\partial t}/\frac{\partial^2}{\partial x^2}\right]$. (28)

As long as all classical physical quantities have wave mechanics operators, the wave mechanics expressions of classical physical laws can be directly combined by the corresponding operators. For example, the operator formula with constant mass for Newton's second law $F=ma$ is $\hat{F}=m\hat{a}$. The result of the operation is Eq. (28).

$\hat{F}=-i\hbar\frac{\partial^2}{\partial t\partial x}$. (29)

Replacing the momentum in $F=\partial p/\partial t$, with an operator $\hat{p}=-i\hbar\frac{\partial}{\partial x}$, Eq. (29) can be directly obtained. By applying equation (29) to the wave function, we can obtain

$\hat{F}\psi=-i\hbar\frac{\partial^2}{\partial t\partial x}\psi$. (30)

Equations (29) and (30) are universal formulations, not limited to constrained systems. Notably, in Equation (30), the term $\frac{\partial^2}{\partial t\partial x}\psi$is computed by first taking the partial derivative of $\psi$with respect to x, then its partial derivative with respect to t, and finally evaluating the product of these two partial derivatives. Equation (30) thus represents a universal form of Newton's second law expressed through wave equations.

The three-dimensional form of Equation (8) is given by:

$c\alpha\hbar\nabla^2\psi-\frac{Ze^2}{2r}\psi=F\psi$. (31)

Equation (30) is applicable to hydrogen atoms. When rewritten as $-\frac{\hbar^2}{2mr}\frac{\partial^2}{\partial x^2}\psi-\frac{E}{r}\psi=F\psi$, Equation (31) bears striking similarity to the Schrödinger equation. Additionally, its first term parallels the leading term in the Dirac equation. Notably, the time-dependent Schrödinger equation can also be transformed into a wave-mechanical equation with an explicit force term, as shown by the following transformation:

$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=i\hbar\frac{\partial}{\partial t}\psi$.[33] (32)

Using equation (32) and $E=h\ v=2T$, the momentum square operator can be written as the first derivative of the wave function with respect to time:

$\hat{p}^2=-i\hbar m\frac{\partial}{\partial t}$. (33)

If equations (32) and (33) were derived before Dirac wrote the Dirac equation, it would undoubtedly be of great use to him. Using this transformation method, the Schrödinger equation must be corrected. The corrected time-dependent Schrödinger equation is

$i\hbar\frac{\partial}{\partial t}\psi=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+V\psi.$ (34)

Dividing both sides of equation (34) by r, we can obtain

$\frac{i\hbar}{r}\frac{\partial}{\partial t}\psi+\frac{\hbar^2}{2mr}\frac{\partial^2}{\partial x^2}\psi=F\psi.$ (35)

For the planetary model hydrogen atom, according to $pr=\frac{1}{2}\hbar$, $1/r=2p/\hbar$. $v=\alpha c$, we can obtain

$i2m\alpha c\frac{\partial}{\partial t}\psi+\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi=F\psi.$ (36)

III. Solving and verifying operators and equations

The operator of force is $\hat{F}=-i\hbar\frac{\partial^2}{\partial t\partial x}\phi$. We can verify its Hermitian property through fractional integration. $\int\psi^*(-i\hbar\frac{\partial^2}{\partial t\partial x})\,\phi dx=i\hbar\psi^*|_{-\infty}^{+\infty}+\int\phi(i\hbar\frac{\partial^2}{\partial t\partial x})\psi^*dx$. When the wave function is zero at infinity, the boundary term is zero, therefore:

$\int\psi^*\hat{F}_x\,\phi dx=\left(\int\phi^*\hat{F}_x\psi dx\right)^*$.（37）

Since the momentum operator is Hermitian, any new operator derived by translating classical physical quantities into explicit momentum-based expressions (where the derivation avoids square roots or exponential/logarithmic operations, and momentum is replaced by its corresponding operator) will also be Hermitian and thus represent an observable physical quantity. All additional operators introduced in this paper follow this derivation method, so their Hermiticity need not be verified individually.

Verification of the Force Operator Formula (Equation 29). The observability of this operator can also be demonstrated by its action on the wave function (provided the resulting value is observable):$-i\hbar\frac{\partial^2}{\partial t\partial x}\psi=-i\hbar\frac{\partial}{\partial t}\left[\frac{\partial}{\partial x}\psi\right]=-i\hbar\frac{\partial(ip/\hbar)}{\partial t}\psi=\frac{\partial p}{\partial t}\psi=m\frac{\partial v}{\partial t}\psi=ma\psi=F\psi$. This result, which recovers Newton's second law $F=ma$, confirms that the force operator is observable. The calculation also clarifies the rule for evaluating the mixed second-order partial derivative $\frac{\partial^2}{\partial t\partial x}\psi$: the second differentiation acts only on the result of the first differentiation, so the wave function ψitself is differentiated just once. The form of the operator $-i\hbar\frac{\partial^2}{\partial t\partial x}\psi$ is derived reversely from the definition of force: since $F=dp/dt$, the corresponding operator is

$\hat{F}=\frac{\partial\hat{p}}{\partial t}=\frac{\partial}{\partial t}\left[-i\hbar\frac{\partial}{\partial x}\right]=-i\hbar\frac{\partial^2}{\partial t\partial x}$. The reverse process mirrors the derivation of the continuity equation described earlier.

Validation of Equation (8):

The solution to Equation (8) is:$F=-\frac{4\pi^2 c\alpha\hbar}{\lambda^2}-\frac{Ze^2}{8\pi\varepsilon_0 r^2}$. Substituting the relationships $v=\alpha c, pr=\frac{1}{2}\hbar$, $\lambda=2\pi r, V=-2T, F=\frac{V}{r}=-\frac{Ze^2}{r}$,, we find: $-\frac{4\pi^2 c\alpha\hbar}{\lambda^2}-\frac{Ze^2}{8\pi\varepsilon_0 r^2}=F$. This confirms the solution is consistent.

Validation of Equation (10):

One form of the general solution to Equation (10) is: $F=-\frac{p^4}{2GMm^3}-\frac{GMm}{2r^2}$.Using the classical relationships $v^2=GM/r$, $T=\frac{p^2}{2m}$, and $V=-2T, F=\frac{V}{r}=-\frac{GMm}{r}$. $F=-\frac{p^4}{2GMm^3}-\frac{GMm}{2r^2}$, substitute into the solution:$\frac{V}{2r}-\frac{GMm}{2r^2}=F$. The left-hand side equals the right-hand side, verifying that Equation (10) holds. Equation (11) is obtained by multiplying both sides of Equation (10) by 2, leading to: $-\frac{GMm}{r^2}-\frac{GMm}{r^2}=2F$. In the context of the Earth-Sun system, this expression describes the gravitational attraction exerted by the Earth on the Sun. Substituting specific orbital data for the Earth is unnecessary to confirm the equation's validity, as its consistency with both classical mechanics and quantum mechanical derivations demonstrates applicability. Equation (11) thus represents a unifying combination of classical mechanical formulas and quantum mechanical principles.

IV. The Composition of the Mathematical Form System of Generalized Wave mechanics

This system is composed of the following equations：

(1) Wave function ψ;

(2) The existing quantum mechanics include the Schrödinger equation, Dirac equation, and corresponding matrices, etc;

(3)The Schrödinger-Tu equation that can describe macroscopic objects: (2) equation [including the equations in (4)];

(4) Quantum mechanics equations with explicit force: equaitons. (7), (8), (10) and (35);
(5)The wave mechanics expression of Newton's second law: equaitons. (15), (20) and (30);
(6)The wave mechanics expression of the law of conservation of momentum: equaiton (17);
(7)Acceleration operator: equation (27);
(8)The conversion formula between the first-order partial derivative operator and the second-order partial derivative operator: equaiton (32);
(9)Equation (25) for velocity operator;
(10)The composite Schrödinger equation : $-\frac{\hbar^2}{2\sum_i^j m_k}\frac{\partial^2}{\partial x^2}\psi-\frac{(\sum_i^j e_k)(\sum_1^j z_j e-\sum_1^i e_i)}{4\pi\varepsilon_0 r}\psi=E\psi$. [33]

V. Discussion

This study demonstrates that classical mechanics and quantum mechanics are not mutually exclusive, but can be unified through wave equation representations—resolving a long-standing conceptual divide in physical theory. By deriving wave-mechanical formulations of foundational classical laws, including Newton's second law, the conservation of momentum, and the virial theorem, we bridge microscopic and macroscopic systems, with particular relevance to planetary-scale models.

5.1. Key Findings and Implications

Force Operators in Quantum Mechanics
A core contribution of this work is the derivation of force operators (Eqs. (14), (25)) and wave equations incorporating explicit force terms (Eqs. (7)–(11), (23), (24), (30)), challenging the historical exclusion of "force" from quantum mechanical frameworks. These operators enable direct translation of classical force concepts to quantum systems; for instance, Eqs. (15) and (30) generalize Newton's second law to wave mechanics, offering new interpretive tools for systems like the hydrogen atom.

Unified Descriptions of Planetary-Scale Models
By modifying the Schrödinger equation to include gravitational or electromagnetic potential energy terms (Eqs. (2)–(3)), we show that both microscopic systems (e.g., hydrogen atoms) and macroscopic planetary motion can be described using identical mathematical frameworks. This contradicts the conventional view that quantum probability and uncertainty are inherently confined to the microscale, suggesting instead that such interpretations may arise from limited application of wave mechanics to macroscopic contexts.

Operator Formalism for Classical Laws
The conversion of classical observables—momentum, velocity, acceleration—into quantum operators (Eqs. (25)–(27)) allows classical laws to be reformulated as wave equations. For example, the conservation of momentum (Eq. (17)) and Newton's second law (Eq. (30)) retain their fundamental physical meaning while adopting wave function-based formulations. This formalism extends to systems with non-constant mass (Eq. (28)), broadening its applicability to bound-state equilibrium systems across scales.

5.2. Limitations and Future Research Directions

While this work achieves a unification of classical and quantum mechanics within planetary models, several key limitations and promising avenues for future investigation merit attention:

Generalization Beyond Planetary Systems
The derived equations are rooted in the virial theorem and assumptions of circular motion, confining their applicability to systems with radial symmetry. A critical next step involves extending these frameworks to non-planetary, non-bound systems—such as scattering processes or chaotic dynamics—where radial symmetry does not hold. Notably, the Schrödinger equation originated from the Bohr planetary model, which was initially limited to hydrogen-like atoms; yet, following the abandonment of the planetary model, the equation was intuitively generalized to all microscopic systems, a transition whose underlying physical mechanism remains unelucidated. This work faces a parallel challenge: our equations are built from classical mechanics and planetary model assumptions, but their potential applicability extends far beyond this narrow scope. It remains unclear whether this generalization follows the same physical logic as the Schrödinger equation's evolution. Additionally, the paper does not explicitly address whether the mass operator yields a non-constant mass, a question that warrants rigorous further investigation.

Experimental Validation
Empirical verification of force operators in quantum systems is essential to confirm theoretical predictions. Specifically, experiments measuring force-induced perturbations to atomic orbital wave functions would provide critical empirical support for the proposed framework.

Mathematical Consistency of Operator Algebra
The introduction of "heavy partial derivatives" and operator division operations [e.g., Eqs. (24)–(26)] demands rigorous mathematical validation to ensure compatibility with established quantum operator theory. Collaborative work with mathematicians to formalize these novel operations is indispensable for solidifying the theoretical foundation.

Re-examining Quantum Theoretical Patches
Quantum decoherence, wave function collapse, and wave function flattening were all introduced as ad hoc patches rather than logical deductions from the mathematical structure of quantum mechanics. If quantum and classical mechanics are inherently coordinated and compatible—with no clear boundary between the two domains—the credibility of these patches would be significantly undermined. The findings of this work invite a re-evaluation of concepts like quantum decoherence and quantum state collapse (including wave function collapse or flattening), challenging the necessity and validity of these long-standing theoretical supplements.

5.3. Discussion on Divergent Perspectives
Some academic peers have raised concerns regarding certain aspects of this paper, which we address in detail below.
(1) The Legitimacy of Employing Planetary Models as an Illustrative Framework

The Schrödinger equation, expressed as $\hat{T}\psi+\hat{V}\psi=E\psi$, applies the Hamiltonian operator—a Hermitian operator acting on the wave function—to derive corresponding energy eigenvalues. This formulation inherently implies the classical energy conservation relation $T + V = E$, which precisely describes the energy balance of a system under uniform circular motion constraints. This mathematical equivalence demonstrates that the Schrödinger equation cannot be entirely decoupled from the planetary model framework.

Even when considering the time-dependent form of the equation$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = i\hbar\frac{\partial}{t}\psi$, its core structure still aligns with $\hat{T}\psi+\hat{V}\psi=E\psi$. If Schrödinger himself utilized planetary models in his original work, there is no logical barrier preventing this paper from adopting a similar approach. It is important to note that the universality of the Schrödinger equation was primarily expanded by subsequent researchers, not Schrödinger himself. In the year of its formulation, Schrödinger only applied the equation to calculate the energy levels of hydrogen atoms within the planetary model framework, achieving results that closely matched experimental observations.

This historical context reveals a critical insight: the Schrödinger equation itself does not logically negate the planetary model—it was Heisenberg's interpretive framework that rejected this model. This distinction is foundational to the arguments presented in this paper, as it highlights the weak logical connection between quantum mechanics' mathematical formalism and its interpretive systems.

We uphold the core proposition that "rejecting the existing interpretive framework of quantum mechanics does not equate to rejecting its mathematical formalism." Public dissatisfaction with quantum mechanics primarily stems from confusion and unease with its interpretive systems, not its successful mathematical applications. Consequently, this paper's divergence from mainstream quantum interpretations does not undermine the established mathematical framework of quantum mechanics. Conversely, the successful application of quantum mechanics' mathematical formalism cannot serve as definitive evidence against the perspectives presented in this paper.

It should also be emphasized that this paper introduces numerous universal operators and equations unrelated to planetary models, such as equations (16)–(17) and (25)–(30).

(2) Leveraging a Straightforward Mathematical Foundation

This article focuses on describing classical mechanical laws through the lens of quantum mechanical equations. Two core methods are employed to achieve this:First, starting from the definitive relationship between mechanical quantities and momentum, we construct the momentum function, then substitute momentum with the momentum operator$\hat{p} = -i\hbar\frac{\partial}{\partial x}$ (including its powers).Second, we extend the scope of the potential term V in the Schrödinger equation (e.g., Equation (1)) to encompass gravitational potential energy. When V represents electromagnetic potential, the equation can uniformly describe both microscopic and macroscopic systems; similarly, when V denotes gravitational potential energy, the Schrödinger equation retains this dual applicability. While describing microscopic gravitational confinement systems holds limited practical significance, its application to macroscopic confinement systems is highly meaningful.

Building on the clear relationship $F=V/r$, we further transform the Schrödinger equation into a wave mechanical equation with explicit force terms. Using the fundamental relation $F=dp/dt$, we derive the force operator, which acts directly on the wave function to yield a quantum mechanical formulation of Newton's second law. Additionally, replacing the classical momentum$p$ in the momentum conservation law $p_1 = p_2$ with the momentum operator and applying it to the wave function directly yields the quantum version of momentum conservation.

Although these derivation processes mathematically straightforward, they are logically rigorous and represent a significant conceptual breakthrough. By transcending Schrödinger's original framework and research paradigm, this work opens new avenues for physics research. The theoretical framework of generalized wave dynamics, a novel theoretical system, is elaborated in Section 4. This demonstrates that innovation in research methods and outcomes does not always depend on complex mathematical formalism.

(3) Experimental Verification Challenges

The author cites an electron diffraction experiment conducted under strong magnetic field interference, presented as a counterexample in reference [33]. The experiment reveals that strong magnetic fields do not disrupt the conditions for diffraction; instead, they merely distort and shift the resulting diffraction pattern. According to the Copenhagen interpretation, any disturbance should trigger quantum decoherence in microscopic particles—encompassing the collapse of entanglement and quantum superposition—causing them to transition into macroscopic states where quantum properties are lost. By this logic, the interference of a strong magnetic field with electron beams should eliminate diffraction fringes entirely. However, the experimental observation contradicts this prediction: the fringes persist, providing direct evidence against the interpretation.

In the annals of scientific progress, paradigm-shaking innovations often emerge when pioneers publish preliminary theoretical frameworks, leaving experimental validation to subsequent researchers with the necessary resources. De Broglie's hypothesis of wave-particle duality, for instance, introduced the wavelength formula for matter waves, but it was not verified by de Broglie himself; confirmation came from readers who tested the theory after its publication. Similarly, Tsung-Dao Lee and Chen-Ning Yang's proposal of parity non-conservation in weak interactions was experimentally validated by Chien-Shiung Wu and colleagues only after their paper was released. These cases underscore a critical truth: proposing novel ideas is often more foundational than validating them. This principle emphasizes the value of rapidly disseminating new concepts, even before empirical proof is secured.

The successful application of quantum mechanics' mathematical formalism does not inherently validate its existing interpretive framework. Proponents argue that experiments like Aspect's tests of Bell's inequalities strongly support prevailing interpretations, yet these experiments are not without significant flaws—detailed critiques can be found in section 4.3 of reference [34].

(4) Conflicts with Existing Quantum Mechanics Theories

This issue was initially addressed in Question (1), and the following provides supplementary discussion. Generalized wave dynamics incorporates all valid achievements of quantum mechanics' existing mathematical framework. As such, it does not conflict irreconcilably with quantum mechanics' mathematical system, but rather challenges its current interpretive framework—a framework that has long been muddled and unsatisfactory.

This paper's core mission is to interrogate quantum mechanics' dominant interpretive system while preserving the positive content of its mathematical formalism. For instance, foundational equations like the Schrödinger equation and Dirac equation remain fully applicable. We also introduce modified versions of the Schrödinger equation, such as the Schrödinger-Tu equation (which includes explicit force terms and integrates classical mechanical laws; see Section 4 for details).

To draw an analogy: this challenge is akin to a legal proceeding where the author's new ideas are the "plaintiff" and quantum mechanics' existing interpretive system is the "defendant." It would be deeply unfair and unscientific for a judge to side blindly with the defendant and dismiss the plaintiff outright. Similarly, readers and reviewers acting as "judges" must not rely on the existing interpretive framework to evaluate this work.

This paper presents a self-contained logical system. Readers and reviewers should first assess whether the system is logically consistent, theoretically meaningful, and possesses research potential. From the perspectives of research objectives and innovation, its theoretical significance is as follows:

Clear, targeted objectives: The paper directly confronts the long-standing "binary opposition" between classical mechanics and quantum mechanics. By deriving quantum mechanical equations that explicitly incorporate "force," it seeks to demonstrate the compatibility and unity of the two systems—a goal of profound theoretical exploratory value.

Bold, innovative contributions: For the first time, it derives quantum operators for classical physical quantities such as force and acceleration, filling the conceptual gap of "force" in quantum mechanics. It proposes the Schrödinger-Tu equation with explicit force terms, attempts to recast classical laws (e.g., Newton's second law, momentum conservation) into wave equation form, and claims these equations can apply simultaneously to both macroscopic and microscopic systems. It further challenges the traditional view that the Schrödinger equation is limited to microscopic systems by attempting to describe both hydrogen atoms (microscopic) and planetary motion (macroscopic) using a unified equation.

(5) The Paradox of Perfection

A colleague once remarked that contemporary mainstream physicists impose extraordinarily stringent standards on groundbreaking research, demanding that seminal papers be comprehensive and fully rigorous from their inception. Yet the history of scientific progress tells a different story: transformative theories rarely emerge fully formed. To dismiss highly innovative frameworks through reductio ad absurdum—demanding flawlessness at the moment of their birth—is to misjudge the nature of discovery. Is it fair to hold a revolutionary theory to the same standard of robustness as a mature, decades-old paradigm? When Riemann first introduced non-Euclidean geometry in his paper for Crelle's Journal, he did not present a complete, definitive account. Instead, he published a series of follow-up articles in the same journal, refining and expanding his ideas incrementally as his research progressed.

VI. Conclusion

We have successfully formulated the Hermitian operator for force, derived quantum mechanical representations of classical principles including Newton's second law and the conservation of momentum, and established the Schrödinger-Tu equation with explicit force dependence. These achievements bridge the historical divide between quantum and classical mechanics, addressing the longstanding absence of force in quantum formalism. By reintroducing force into quantum theory and demonstrating the scalability of wave mechanics across micro- and macroscopic scales, our work provides a unifying framework for reconciling these two seemingly incompatible domains. Future research will focus on generalizing these results to complex systems and validating their predictions experimentally—efforts that could fundamentally reshape our understanding of mechanics across all scales.

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